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Innovation and Technology for Mathematics and Mathematics Education

Southeast Asian Ministers of Education Organization (SEAMEO)
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in Mathematics

PROCEEDING OF THE 3rd INTERNATIONAL SYMPOSIUM ON MATHEMATICS EDUCATION INNOVATION

Innovation and Technology for Mathematics and Mathematics Education

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Forewords

Mathematics has become more and more international. Inevitably, solidarity across institutions has to be increased at a fast pace. On the other words, mathematics is becoming too large and a diversified subject and it is almost impossible for one person to embrace. Thus, interaction among mathematics students, educators, and experts both at a national and an international level is the clear road for its development. In line with this and the Centre's commitment to enhance the quality of mathematics teachers and education personnel in Southeast Asia, SEAMEO QITEP in Mathematics holds 3rd International Symposium on Innovation of Mathematics Education on 26-30 November 2014.

This symposium is in many ways a very special event. It is the third since the establishment of SEAMEO QITEP in Mathematics, which is particularly a new born Centre. Besides, the 3rd International Symposium on Innovation of Mathematics Education is a collaboration session with the 19thATCM (Asian Technology Conference in Mathematics), 1ST ISIM-MED (International Seminar on Innovation inMathematics and Mathematics Education), and 2nd SENDIMAT (*Seminar PendidikanMatematika*). I believe that this symposium is a great chance to engage diverse group of mathematics students, educators, and experts to review issues, exchange ideas, and share experiences in mathematics education innovation at all levels. For that reason, the Centre is very pleased to address a forum of such importance.

In this opportunity, on behalf of SEAMEO QITEP in Mathematics, I deeply appreciate the participation from all keynote speakers, paper presenters, participants, and other supporting parties. Thank you for making this event possible. Indeed, the four days of Joint the 3rd ISMEI, the 19th ATCM, the 1st ISIM-MED and the 2nd SENDIMAT became a momentous forum for the Centre's journey in disseminating idea and innovation in mathematics education. To enable the papers of the Symposium reach wider community of mathematics teachers and educators, the Centre compiles the papers into this proceeding.

We wish this proceeding may serve as solid references for mathematics teachers and educator to develop and improve their teaching skills, and facilitating their students in such encouraging classroom learning environment in which technology plays vital role for meaningful and joyful learning experiences.

Yogyakarta, November 2014
Prof. Subanar, Ph.D.
Centre Director

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Interactivity and flexibility exemplified with Cabri

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Abstract: *This presentation will support an approach in the development of digital technology, in which student use and teacher use are taken into consideration. This kind of technology enables the development of students centered resources fostering exploration and inquiry by students but also enabling teachers to adapt them in relation with their pedagogical aims and the needs of their students. Such technology scaffolds a progressive process of integration and instrumentation of technology by teachers while teachers may in a first step experiment ready-to-use resources in their classes and then make changes increasing over time as they are more confident and gain experience.*

1. Two kinds of users of technology in the mathematics classrooms

The use of digital technology in mathematics classroom deals with the two main facets of mathematics education: teaching and learning. Studies often have addressed one of these facets: how students use technology or how teachers integrate technology into their practice. This presentation will address both points of views on the way digital technology may favor the use of tasks in the teaching/learning processes. As Thomas and Lin [1] (2013) claim, “the digital technologies may change or challenge traditional student learning trajectories, it can and will also challenge teaching strategies”.

Tasks play indeed an essential role in the teaching/learning processes. “Tasks generate activity which affords opportunity to encounter mathematical concepts, ideas, strategies, and also to use and develop mathematical thinking and modes of enquiry. Teaching includes the selection, modification, design, sequencing, installation, observation and evaluation of tasks.” ([2] Watson et al., p.12, 2013).

This presentation focuses on two critical aspects of technology with regard to tasks: interactivity for students and flexibility for the teachers. Interactivity plays a critical role on the students’ solving processes when they are faced with a task in a digital environment. Information received from the environment may assist students in the use of technology or in the solving process. Flexibility allows the teachers to easily design technology based tasks appropriate for the specific conditions they encounter in their curriculum or in their classrooms.

2. Interactivity for the student

The word interactivity is used to denote interactions between a digital environment and the user. It became very popular and every digital resource, game or environment is told to be highly interactive. Unfortunately the meaning of interactivity became loose and more than often it simply refers to the possibility of pressing keys or buttons or touching objects on the screen.

Here we would like to analyze the notion of interactivity in mathematical and cognitive ways. Interactivity in a mathematical digital environment is a reaction of the environment to the user’s actions related to mathematics. The user without a cognitive additional burden must perceive these reactions and get information on his/her actions. To play a cognitive role, interactivity must take place both for simple actions of the user such as use of a tool or of a feature of the environment and for more complex cases.

2.1 Interactivity at the interface

An example of simple but important interactivity is the readability and mathematical correctness of the tooltips, particularly for learners.

Compare for example the actions and tooltips when creating a simple triangle without names for the vertices in two cases:

In the first case, the user is clicking three times: when clicking for the first point, a message appears "Triangle passing through a new point" for the second point "this new point" and for the third point "and this new point".

In the second case, the user clicks a first time, no message appears but a point is created with name *A* if no other point was created before or with the nth letter in the alphabet if n-1 points are already existing. The user then clicks a second time, no message but a point denoted with the n+1th letter is created, the user clicks a third time, no message but a point with the n+2 the letter is created. Surprisingly the triangle is not yet obtained. The user has to click again on the first point to get the triangle.

In the first case the actions to be performed and the reactions of the environment are consistent with both the intention of the user and the mathematical definition of a triangle. The messages are even doing more. They assist the user by informing him/her about the progress of the creation process. The 'and' in the formulation "and this new point" expresses that it is the last point to create.

In the second case, the reactions are not consistent with the user intention. By choosing the tool "Triangle", the user transmits to the system that he/she wants to create a triangle, i.e. three points connected by segments or three segments, each pair of segments sharing a common endpoint. But in this case the interface expects the user to perform an additional action giving for the second time an information item already given. One vertex of the triangle has to be clicked twice. In this case, the reactions do not assist the user, they are surprising with regard to his/her intentions: four clicks for creating a triangle and letters not intended and not chosen are displayed.

Then when hovering over the triangle or clicking on it, the tooltips are:

- "This triangle" in the first case

- "Segment a: Segment AB of triangle poly 1" in the second case

In the latter case, the tooltips are far too complex for learners. They derive directly from the programming of the software and the underlying logic is related to the internal functioning of the software that users and particularly novice users do not have to know. Why does appear 'segment' twice in the tooltip with two different names? What does mean 'poly'? In the former case, the tooltip is consistent with the intention of the user.

These reactions may seem to be micro reactions and not be important for the user as soon as he/she masters the use of the software. We would like to claim that a learning environment is aimed at helping the students to learn mathematics. Their interface must not puzzle them and not be counterproductive from a cognitive aspect. They must foster the focus of students on mathematics and not be conflicting with a fragile knowledge they are in the process of acquiring. Not only the usability of the software (users' friendliness) is low but also it results in a loss of utility (the extent to which the needs of the user are met; in the case of educational technology, utility is related to the learning potential of the interface).

2.2 Interactivity in the mathematical activity

When students solve problems, the role of feedback to their answers has been stressed by research as a catalyst for changing their solving strategy in case of an incorrect strategy. Students are not only convinced that their answer is wrong but they can extract information from feedback that can be used for searching a more appropriate solving strategy ([3] Brousseau, 1997).

Let us give a simple example in Cabri 3D [4]. The task for the students is to construct a box containing tennis balls (Fig.2.1).

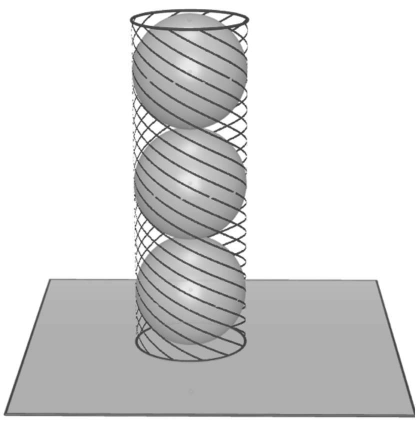


Figure 2.1 The box

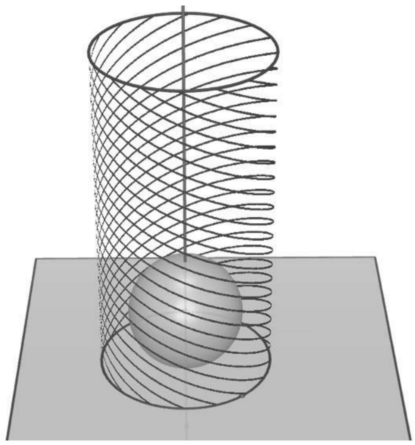


Figure 2.2 The cylinder is enlarged

A first strategy is to construct the cylinder, to put a point on the axis of the cylinder and then to create a sphere with center this latter point and passing through the center of the cylinder base. Then the center of the sphere is dragged until the sphere seems to become tangent to cylinder.

However when changing the radius of the base of the cylinder, the sphere is no longer tangent. The construction is not robust against dragging (Fig.2.2). The drag mode in dynamic geometry environments is a very powerful source of feedback. Information brought in the case of the box is that the center of the ball must be constructed and not simply dragged. It leads to the need of a geometric analysis of the position of the center of the ball.

From a cognitive perspective, feedback gives rise to a conflict between what students expect and how their construction behaves, leading in Piagetian terms to an equilibration process aimed at cancelling the conflict. Overcoming the contradiction is a way of wiping out the conflict.

Let us give another example in 3D geometry in Cabri [5] where the source of feedback is different. Students are faced with the task of constructing the net of a pyramid without folding it. They have permanent access to the pyramid (Fig. 2.3) but must complete the net in a 2D environment (Fig. 2.4). The aim of the task is to require from students an analysis of the nature of the faces of the polyhedron and of the relationships among them.

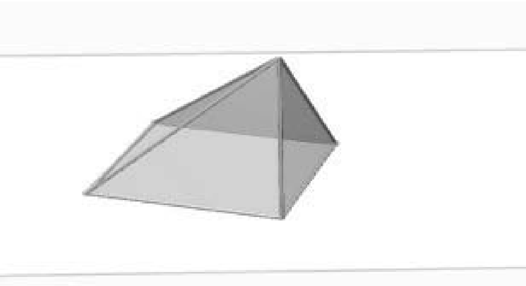


Fig. 2.3 The pyramid

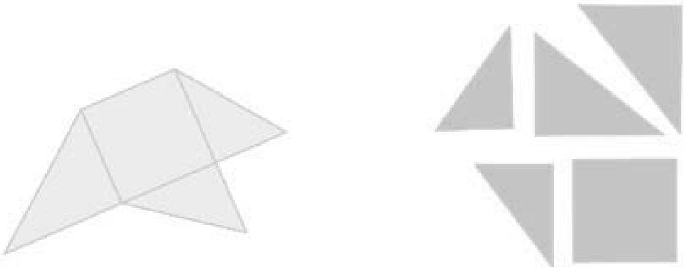


Fig. 2.4 The net to be completed

But after students have completed the construction of the net, they are allowed to check their net by folding it. Folding the constructed net shows whether it is correct but also in case the net is incorrect may reveal some geometric reasons for the inadequacy of the net. An example of a wrong answer is displayed in Fig.2.5. When folding, the student not only perceives immediately that his/her answer is wrong but also may see (Fig. 2.6) that the angles are not appropriate: the acute angle at the bottom is too small. The right angle is too large.

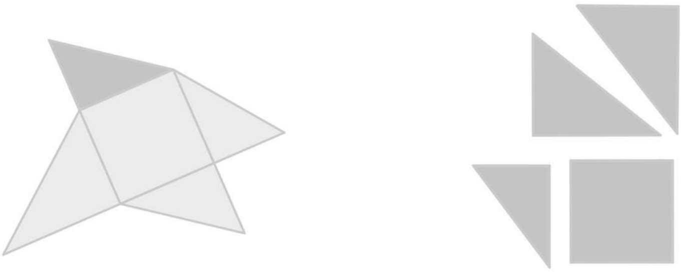


Fig. 2.5 A wrong answer

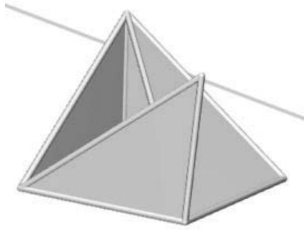


Fig. 2.6 Folding the wrong net

In this perspective, interactivity comes from an interplay between the problem students are faced with and the possibility offered to students to check their answers. A problem is posed to the student with some constraints avoiding an easy solving process resorting for example to pure perception and calling for mathematical solving process. After students have given an answer,

-they are given the possibility of checking their answer through manipulations in the computer environment (like in the example of the net)

-or technology enables them to easily see what results from their answer (like in the case of the drag mode).

Interactivity originates from the coordination of the possibilities offered by the environment and from the choice of the mathematical question or problem to solve. Technology enables

-to design a setting for the mathematical question: simulation of a problem in real world, numerical, graphical or geometrical setting;

-to play on the conditions in which students can solve the task: available tools, possibility or not of dragging;

-to offer means of feedback by enabling students to vary some elements of the question, to transpose it in another setting linked to the initial setting or to free some conditions of the problem setting.

Clearly the design of interactivity is based on didactical choices. But these choices are largely conditioned by the possibilities of the digital environment. Didactical choices and features of the environment are strongly intertwined in the design of interactivity.

3. Flexibility for the teacher

If many innovations taking advantage of technology in mathematics teaching took place during the past thirty years, despite institutional support the use of technology did not widespread, as the enthusiasm of the pioneers would have hoped.

More than ten years ago several studies focused on teacher practice using technology in ordinary classrooms. Observations of teachers showed that integrating technology was not an easy task for teachers. Instead of following textbooks, the teachers must design worksheets [6] (Monaghan, 2004); they must adapt the management of several kinds of time in their classrooms and the relationship between, on the one hand, old and new knowledge and, on the other hand, paper-and-pencil techniques and computer software techniques (Assude, 2005) [7]. Monaghan [6] also showed how technology could affect the emergent goals of the teacher during the lesson. Haspekian's (2005) [8] findings about the management of complexity when a teacher integrates spreadsheet converge supports the research previously quoted: the teacher must simultaneously take into account "the tool features, the instrumented techniques, and the concepts involved." This explains why teachers often preferred to use themselves the power of such technology by demonstrating properties and theorems in front of the students (Ruthven and Hennessy, 2005) [9] and let not students using themselves technology.

Over the ten past years the number of digital resources and applets has grown dramatically freeing the teachers from designing themselves the activities. In the same manner as exercises and problems are available in textbooks for teachers, these latter can now find a huge number of digital resources in repositories. However weaknesses of these repositories have been identified (Trgalova et al. 2011) [10]: difficulty of finding resources fitting to a specific educational context due to the lack of metadata describing accurately the resource content, lack of quality guarantee, difficulty of resource appropriation by teachers. The aim of the European Intergeo project (2007-2010, Kortenkamp et al., 2011) [11] was to set up an international multilingual repository of resources for dynamic geometry environments. The project developed in addition a questionnaire meant for teachers who wanted to assess the quality of repository resources. This questionnaire turned out to be an assessment tool but also a means for teachers to appropriate the resources, to be able to analyzed them and possibly to modify them (Trgalova et al. 2011) [10].

Indeed between on the one hand, open environments giving complete freedom to teachers for designing activities but requiring time and a careful preparation and on the other hand, ready-to-use resources non totally adapted for students, an intermediate approach is possible: providing teachers with flexible resources that they can easily modify and adapt for use in their classrooms.

An activity must meet two needs:

- it must contribute to specific learning;
- it must be adapted to students abilities and competencies, to the actual state of the progression of the mathematics classroom in which it is used.

In attempting to coordinate these two needs, the teacher may need to customize the resource for his/her classroom.

The teacher may want to change

- the formulations to make them consistent with the ones in use in the classroom;
- the context in case the activity is contextualized and students of the teacher are not familiar with; or the teacher may want to motivate students in setting the activity in a context shared by the whole classroom or related to a situation experienced by all students.
- the length of the activity or the number of questions, depending on organization of the time planned by the teacher;
- the learning aim of the activity by changing the value of task variables or didactical variables (Brousseau [3]) affecting the students' solving strategies;
- in relation with the learning aim, the mathematical difficulty of the activity too high or on the other hand too low with regard to students' abilities;
- the aim of the activity in the teaching process: is it meant for assessment, for practicing, for discovering ? Is it meant for home assignment?

The teacher may also want to tune the difficulty of the activities in relation with different groups of students. The teacher may challenge advanced students in increasing the difficulty of the activity. He/she may motivate weakest ones in proposing an easier activity. In addition of tuning the difficulty, the teacher may want to modify the weight of each question in marking the work of the student on the activity.

3.1. Deriving a variety of tasks from one task

Some examples of possible modifications are illustrated below in *Cabri Factory* [12] a collection of mathematics resources for middle school.

In the activity book entitled "In the right place", students must drag a point and place it at the position of the fourth vertex of a particular quadrilateral (parallelogram, rectangle, square, rhombus) in four diagrams. The only available tool is the *Angle* tool that displays the measure in degrees of an angle determined by three points.

In the case of placing the fourth vertex of a parallelogram (Fig. 3.1), the task requires using twice the property of supplementary interior angles to make opposite sides parallel. Students must display the measure of three angles and complete the task in two steps: making a first pair of sides parallel and then the second pair of sides parallel without changing the angle determining the first pair of sides.

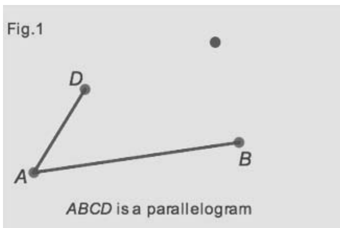


Fig. 3.1 – The blue point must be placed on hidden vertex *C*

This strategy is not spontaneous for students for two reasons:

- they generally do not think to this property as they are attracted by the shape parallelogram in absence of display of lines. They want to achieve the task only by making two opposite angles congruent. The task makes operational the property of supplementary interior angles implying parallelism.

- they try to place the point in only one step by making the opposite angles congruent and the sides visually parallel.

It usually results in a wrong position for the point. In this case, an error message telling the student that the point is not at the right place is displayed as *Cabri Factory* enables the author of the activity to check students' answer whatever the positions of given points *A*, *B* and *C* are. In addition students can access a help page in which they can explore the impact of measures of angles made by two lines and a transversal by dragging one of the two lines. Feedback to students combines information about the wrong position of the point and the possibility of exploring the link between angles and position of lines.

The learning aim may be changed by the teacher. The teacher may want students to practice the congruence of opposite sides for obtaining a parallelogram. In that case, in the teacher mode, the teacher has just to drag the tool *Angle* out of the student page and to drag the tool *Distance* inside the student page. If the teacher prefers to foster the tool *Circle* to reinforce the use of a circle as measurement transferring, he or she makes this latter tool available instead of the tool *Distance*.

If the aim of the teacher is to assess the availability of properties at students after the teaching properties of parallelograms is completed, the two tools *Angle* and *Distance* may both be offered to students. The most popular method will emerge from the students' work. That work is continuously saved and when the work is completed, the teacher may have a look and see which property prevails among students.

The task may be too difficult for some students not having acquired the property of supplementary interior angles for making lines parallel. The teacher may want them first to practice on only one pair of sides. He can draw a parallel line to segment *AB* and passing through *D*, and redefine the blue point as a point belonging to that line.

The teacher may organize a sequence of tasks from this only resource, each activity being derived from the preceding one by changing the value of one or several task variables in order to allow a progress in the appropriation of the relation between parallelism of lines and angles. The two first tasks deal with four points and ask for placing point *D* as the fourth vertex of a parallelogram. In the first task, only one angle must be varied to place the point, in a second task two angles must be varied. The third task deals with a crossed quadrilateral (Fig. 3.2): students must place point *D* so that *ABDC* is a parallelogram. Alternate angles or interior angles can be used to solve the task but alternate angles are more visible since in this case no sides are displayed for one of the interior angles.

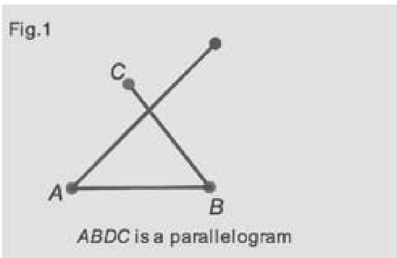


Fig. 3.2 A crossed quadrilateral

The teacher has just to delete segment *AD*, to create segment *BD*, change the label *D* into *C*, and to join point *A* and the blue point with a segment.

Then the number of points can be increased making less visible the sides to make parallel and the related angles. For example, the teacher can ask students to make a hexagon invariant through reflection in a point with the only available tool *Angle* (Fig.3.3).

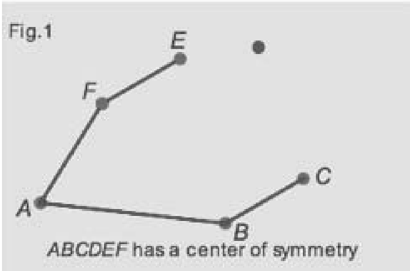


Fig. 3.3 The blue point must be placed on hidden vertex *D*

The teacher has just to introduce a new point *B* and its reflected point *E* with respect to the midpoint of *CF* and change labels of the existing points. The hidden target point is the same as earlier, *ACDF* being a parallelogram.

All these transformations of the activity are possible because of the availability of three modes in *Cabri Factory*:

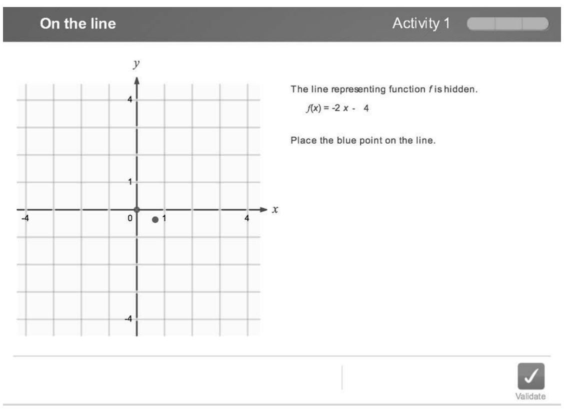
- the author mode
- the teacher mode
- the student mode.

When designing the activity, the author provides the teacher with many geometrical tools (point, segment, line, parallel line, point symmetry, angle, distance…) in order to enable the teacher to modify the activity to a large extent without taking care of feedback evaluating students' answers that is preserved during the transformations done by the teacher. In addition, the teacher mode offers editing facilities, such as editing texts, changing fonts and colors, inserting images, hiding or displaying objects.

All subjects of mathematics give rise to possibilities of change for the teacher. An example about graphing linear functions is presented below.

A coordinate system is displayed as well as a blue point. The expression of a linear function is given. Students must place the blue point on the hidden line representing the function (Fig.3.4). If they succeed, they receive a message congratulating them. If they fail a first time, they have access to a help page (Fig. 3.5) in which the hidden line a point *P* and its coordinates are displayed. Students are asked to calculate $f(x)$ for a given x coordinate of *P* and to look where the y coordinate of *P* is equal to f(x). If they fail a second time, they have access to a solution given for another function. They have a third try for allowing them to reinvest what they understood from the solution.

The expression of the linear function is randomly generated within intervals for the y intercept and the slope that can be changed by the teacher. No tools are available for the student in the activity page.



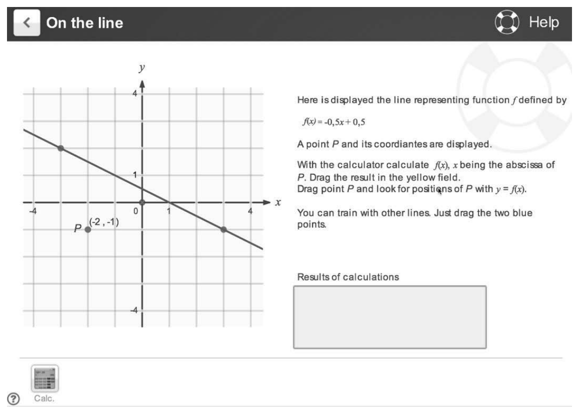


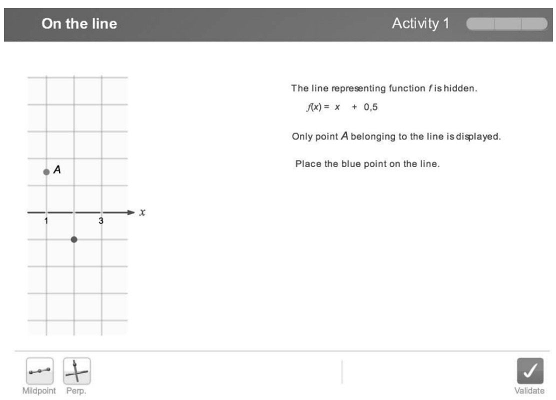
Fig. 3.4 Placing a point on a hidden line **Fig. 3.5** Help page

The learning aim is dual:

- being able to use the expression of the function to place a point on its graph;
- identifying a point with given coordinates in a coordinate system.

After playing several times, students may notice that it is very efficient to use the *y* intercept of the graph.

When students do not have any longer difficulties with the *y* intercept, the teacher may want students no longer use the *y* intercept but rather the slope of the line or the coordinates of a point with *x* coordinate different from 0 and 1. Each axis can be moved even outside the display of the system as in Fig. 3.6. In the following page of the activity book, the point to be placed can only move on a line parallel to the *x* axis (Fig. 3.7).



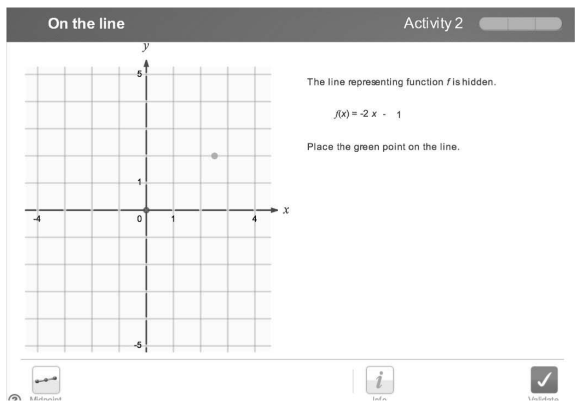


Fig. 3.6 Avoiding the use of the *y* intercept **Fig. 3.7** The green point moves on a hidden line

This new task requires from the student to calculate the *x* coordinate of a point of the graph when the *y* coordinate is known. The teacher can change the parallel line and place it on the *x* axis. In this case students must find the *x* intercept.

The teacher can then challenge students by redefining the green point on line $y = x$. Either students have already been introduced to algebra and they can find out the solution x_0 of the equation $x = \mathrm{f}(x)$ and then estimate the position of x_0.

Or they have not yet been introduced to equations and they can solve the task by varying the green point and for each *x* coordinate of the green point equal to an integer calculate the *y* coordinate of the point of the graph of the function. It is so possible to determine whether the graph is below the green point or above and to identify a small region where the green point should be. Then it is possible to find two close points of the graph of the function in this region and to estimate where a segment joining these two points will cross the line $y = x$.

As seen on the previous examples, flexibility is due to the Cabri technology itself and to the didactical design by the author. Cabri offers the possibility in the teacher mode of easily modifying the choice of the available tools for students, of changing the value of parameters and even of changing questions by preserving the evaluation of students' answers by the environment. In case the question is changed, teachers have only to indicate in a specific field the expected answers. The didactical design is the second factor of flexibility: the author of the prepared activity must carry out a didactical analysis of the task variables and consider the possible variations in relation with the learning outcomes of the activity.

Textbooks offer prepared exercises that teachers extensively use, as they have little time for carefully designing new exercises. The role of the teacher lies in the choice of the adequate exercises and their management in the classroom. Technology may offer the same service but with greater flexibility.

3.2 Living resources extending teachers' experience

Providing teachers with already prepared but flexible resources that can be modified fulfills several functions:

- the teachers may adapt the activities to their pedagogical choices;
- the resources are *living* resources (Geudet et al. 2013) [13]: they are enriched by teachers' experience and extended in various ways according to teachers' choices. The starting activities play the role of a seed.
- The starting activities scaffold in Bruners' [14] sense the experience of teachers novice in the use of technology and contribute to professional development.

Let us discuss the latter point. Several studies show that the ability of designing technology based tasks requires a strong mathematical knowledge for teaching, a good instrumentalisation and instrumentation of the technological tool [1]. Goos (2005) [15] identified a good range of factors influencing teacher technology use that include the availability of appropriate teaching materials and opportunities to learn. Novice teachers find it very difficult to design from scratch activities in the digital environment: they need to resort to existing activities for preparing other ones. Prepared activities and the comments and suggestions for modifications fulfill the function of providing a model of an activity and a didactical analysis of the activities. The tools and parameters made available to teachers fulfill several scaffolding functions [14]:

- enrolment of the teachers: they perceive that they can exert their pedagogical freedom;
- reduction of the steps required to design a new activity: the design of other activities is simplified with regard to a design in an open ended environment;
- maintaining the activity of teachers and controlling their frustration: often novice users have too ambitious goals when starting designing activities and for these reasons may fail to reach their goals. The limitations of the possible changes avoid such a danger.

In this presentation, we wanted to stress how much the development of digital technology for mathematics education must take into account two kinds of users, students and teachers. In particular we have discussed two factors playing a critical role in the usability and utility of a digital environment: interactivity for the students and flexibility for the teachers. Analyzing these two factors we have seen that the development of technology is strongly intertwined with mathematical, cognitive and didactical choices.

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Educating the Educators: Technology-Enhanced Mathematics Teaching and Learning

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***Abstract**: Educational research has shown that teaching quality is one of the most important factors in raising student achievement. There is a compelling need for educators to keep abreast of the important developments that are taking place in educational arena. One of the educational areas that have massive development is on the use of technology to enhance teaching and learning especially in mathematics. Having this development the need for professional development among educators comes in. Being a regional science and mathematics education centre, the Southeast Asian Ministers of Education Organisation Regional Centre for Education in Science and Mathematics (SEAMEO RECSAM) has always been cognizant of the importance of these developments. Its training programmes are planned for in-service teachers, teacher educators and ministry of education mathematics officers are planned to incorporate these developments. As the Centre's mandate it aims to ensure that these participants from Southeast Asians countries as well as those from outside the region are equipped with emerging educational technology tools which can enhance teaching and learning of mathematics. This paper will share the Centre's experiences in continuing professional development among mathematics teacher, teacher educators and officers from the ministry of education on educational innovation and technology.*

1. Introduction

How can those in the field of education best utilise new technologies? We in the education are reminded by phrases such as "emerging technology" and "evolving technologies" that the digital world is continuously changing. There are new devices, new software and altered applications, and shifting practices which keep crossing our horizon as well as quietly appearing in our midst. How do we help educators to keep abreast with these developments to tap it to enhance teaching and learning specifically in mathematics?

SEAMEO RECSAM, being one of the 21 regional centres of the Southeast Asian Ministers of Educational Organisation and since its inception in 1967 was mandated to improve science and mathematics education in Southeast Asia. This paper shares certain selected initiatives of the Centre in recent years on helping educators from this region as well as those from outside the region through its continuing professional development programmes. Mainly, this paper will highlight the measures taken to update and equip these educators with emerging educational technology tools which will enhance the teaching and learning of mathematics in classrooms.

2. The Training Programmes

SEAMEO RECSAM enhances the skills and knowledge of its participants on educational technology tools through its training programmes, namely, Regular Courses, In-country Courses, Customised Courses and Workshops. Regular Courses are designed to address the needs of educators from SEAMEO member countries and its duration is four weeks. In-country Courses are courses conducted in SEAMEO member own county for their teachers and educators for up to five days duration. On the other hand, Customised Courses are tailor-made programmes of varying duration in respond to the special needs of member countries and other agencies which may be held in-country or at the Centre. Workshops are enrichment activities offered by SEAMEO RECSAM to all SEAMEO member countries for their teachers and educators and are conducted at the Centre.

The "emergent educational technology tools" are normally introduced or shared as part of the course content or entirely as a workshop. External experts are invited or the SEAMEO RECSAM specialists will conduct these educational technology tools sessions.

3. The Teaching Principles and Technological Tools

The aim of conducting these emergent educational technology tools training sessions is to ensure that the participants of the courses in the Centre which are normally from Southeast Asian countries are exposed to new technological tools and to equip them with the knowledge to utilise these tools to enhance the teaching and learning of mathematics.

Various teaching principles are also shared in these training sessions. It is basically based on the experts who introduce these technology tools. Among these principles are; Motivation to the Topic, Teaching in Context, Demonstration and Visualisation, Discovery or Exploration, and Intensification of Students' Self-preparation.

The following case studies demonstrate some of these technology tools that have been introduced to our participants.

3.1 Case 1: Geometer's Sketchpad

There is a general consensus that the use of information and communication technology (ICT) in teaching and learning brings about positive benefits in student learning. The findings of the impact survey (as in [5]) provides concrete evidences of ICT having an impact on teaching and learning in the classroom. The Geometer's Sketchpad (GSP) is a proven Dynamic Geometry tool appropriate for elementary school, middle school, and high school, giving teachers and students a visual way to explore the range of mathematical ideas and representations (see [1], [3], [9]).

Among other good features of GSP is that it allows the users to explore simple as well as highly complex theorems and relations in Geometry (see [4]), and has the ability to record students' constructions as scripts. The most useful aspect of scripting ones' constructions is that students can test whether their constructions work in general or whether they have successfully discovered a special case. In addition, the GSP software allows mathematics to be learnt and taught in a more creative way (see [2]).

The Centre managed to convince the Ministry of Education Malaysia to purchase a nationwide license of the Geometer's Sketchpad version 4.0 in 2004. SEAMEO RECSAM has been instrumental in providing support in the form of organising basic to advance training programmes for key personnel on the use of GSP for teaching and learning.

The following Figure 3.1 is one example of a participant's work using GSP for secondary school students to explore the Standard Normal Distribution.

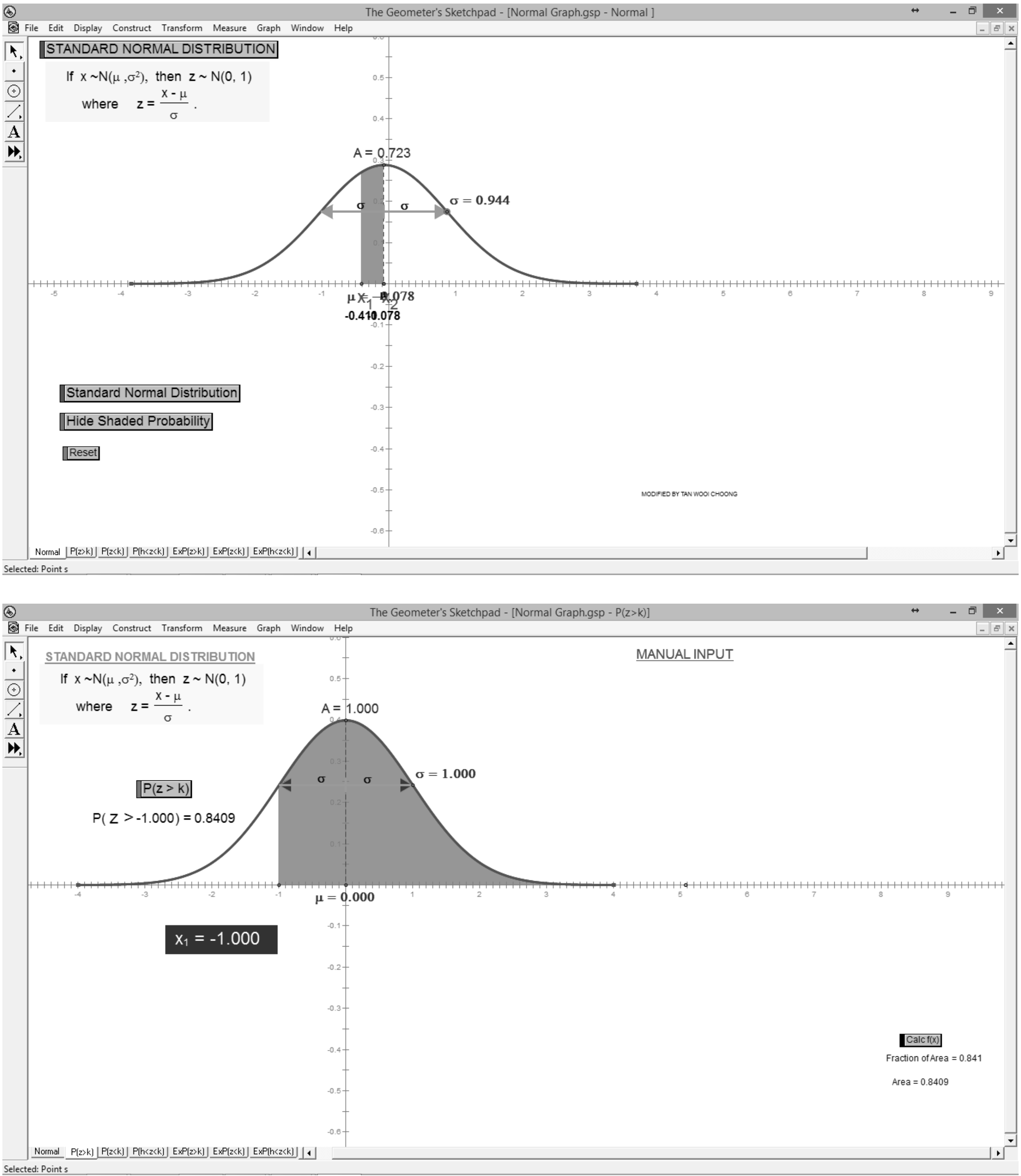

Figure 3.1 Exploring Standard Normal Distribution

In 2011, the Centre carried out a survey on the teachers' use of GSP in secondary schools. A total of 208 selected secondary schools in Malaysia were involved in this study. Although, the majority of the respondents rated GSP as an excellent teaching tool in mathematics classrooms, it also revealed some interesting facts. About half of the respondents used GSP in their classrooms at least once a semester (see [10]). Most of them used it as a demonstrating tool, where whole class instructions was conducted using one computer. A large number of the GSP users (72.2%) also reported facing problem using GSP, which ranged from lack of knowledge, inexperience, time constraint and limitation of ICT facilities available in many of those schools (see [10]).

3.2 Case 2: GeoGebra

In the many of the training programmes that have been conducted in the past, a perennial problem that we have encountered is getting participants to share what they have learned in the course to their colleagues and educators in their country. Another main challenge was getting fund to buy the software involved. This led the Centre to explore other equivalent alternatives. GeoGebra is one alternative source for Dynamic Mathematics Software for learning and teaching of Mathematics at all levels, especially since it is an open-source software. Besides that, there are also easily available ready-made resources which can be downloaded and adopted or adapted for one's own use.

GeoGebra was first introduced in our biannual conference, Conference on Science and Mathematics Education known as CoSMEd and then it was introduced as a regular topic in our courses and workshops. We have not only shared GeoGebra with teachers and educators in the Southeast Asian region but also beyond. Among some of these countries that the Centre has successfully introduced GeoGebra to are Iran, Bangladesh, Afghanistan, Nepal and Bhutan.

Figure 3.2 shows one sample of the participant's work.

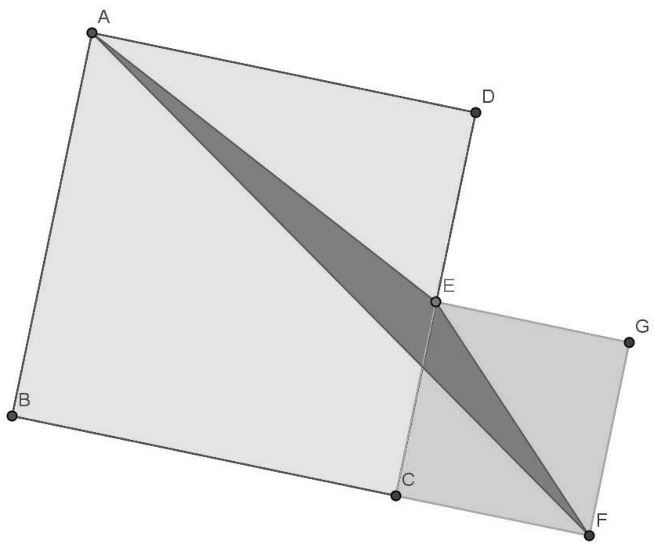

Figure 3.2 The Relationship Between the Area of the Triangle and the Area of the Smaller Square

In this example, students are to determine the relationship between the area of the triangle AEF and that to the smaller square CFGE. The points A, B E are movable and these allowed the students to explore and make conjectures.

Normally, after the end of the workshops or courses, participants are required to provide feedback. For the use of GeoGebra, majority of participants indicated the ease and user-friendliness of this software. Many of these first time users also indicated their request to learn more on using this software in teaching mathematics and this topic recorded the highest increase in the mean value when compared to the other topics in the course. It was also one of the topics that the participants managed to share with their colleagues back home as indicated in their multiplier effect report.

3.3 Case 3: Autograph

Autograph is a dynamic programme that operated in 3 modes, namely; (i) 1D-Statistic & Probability, (ii) 2D-Graphing, coordinates, transformations and bivariate data; and, (iii) 3D-Graphing, coordinates and transformation. Autograph is designed to help teachers and pupils visualise mathematics at secondary to college level, using dynamically linked 'objects'. It is one of the topic shared in a customised course where the participants requested for it. Feedback from our evaluation indicated high mean value for the construct that measured participants' satisfaction while using this technological tool.

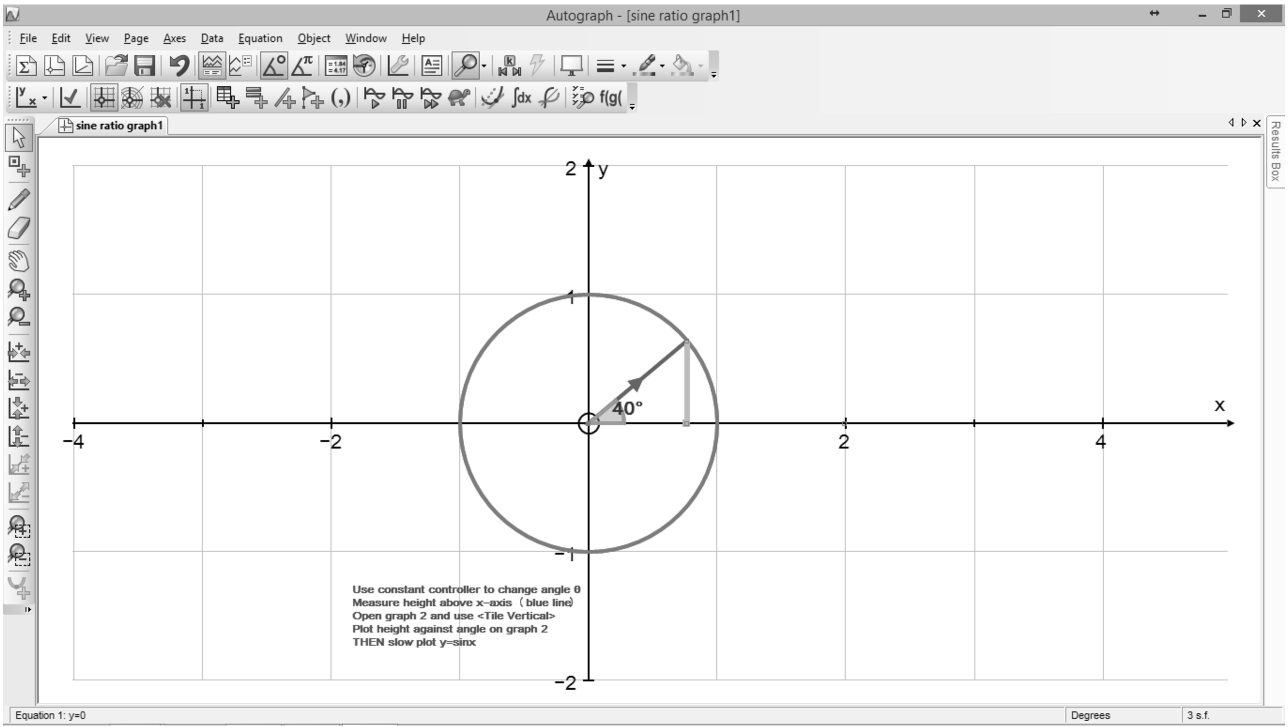

Figure 3.3 Exploring The Sine Ratio Graph

3.4 Case 4: MathDisk

MathDisk is a mathematical tool that can be used to build highly integrative, interactive and dynamic 2D or 3D worksheets. It helps to bring complex concepts to life when students learn through building their own models. Its interface encourages the exploration, construction and evaluation of mathematical models and simulations. The worksheets created with MathDisk can contain multiple graph sheets, equation editors, annotations and a flexible layout which can deliver multimedia-rich interactive version of the textbook which are identical to the text book material. MathDisk also introduces a tool called 'Robo Compass'. This tool provides an opportunity to teach and learn geometry as one would do while using a physical straightedge, compass and protractor. MathDisk is yet to be introduced as a topic and the Centre is considering introducing it in its next fiscal year courses.

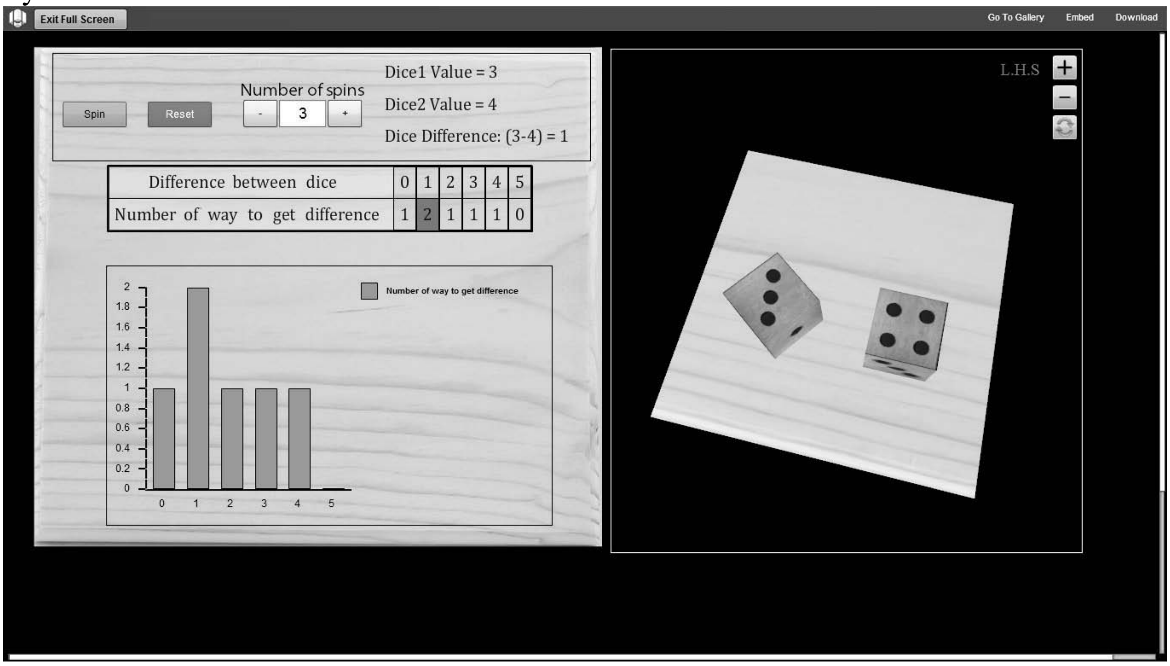

Figure 3.4 Rolling Two Dice

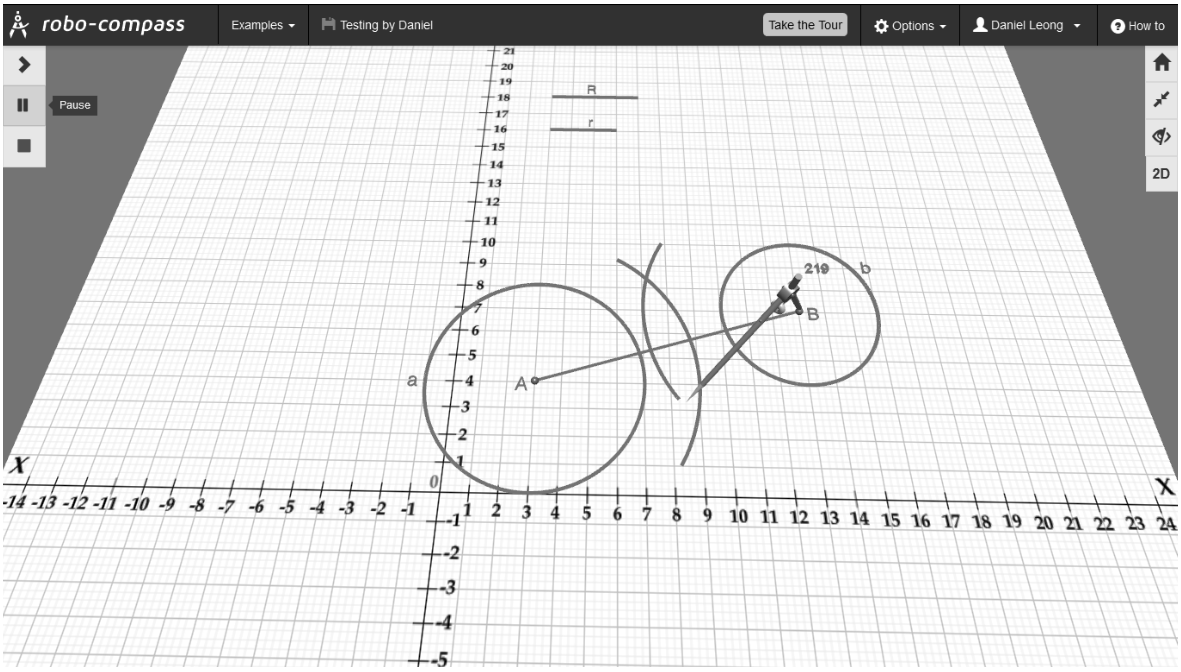

Figure 3.5 Construction with Robo Compass

3.5 Case 5: Camtasia

Camtasia provides teachers the tools to record their computer screen and then turn those recordings into videos. It provides a lot of interesting and attractive features in a fluid interface that makes creating screen capture videos and processing them quickly without much hassle. This technology tool allows users to set the programme to record either the screen or a PowerPoint presentation. Once the user has set the screen to capture a recording in the desired manner, the user can proceed to utilise the various tools included in the programme to create a complete video. These include multiple Transition Options, Voice Narration, a Zoom and Pan Effect, Cursor Effects, Captions, Quizzing, and more. The user can also use built-in tools to enhance and improve the audio quality of their voice recording.

Camtasia was introduced to a school near the Centre as part of their staff professional development programme and since then the school has been utilising it to produce videos of their teachers teaching with PowerPoints and other media. These videos were used during lessons when teachers were not present in the class. In this way, students did not miss any lessons or topics.

Some teachers used part of the videos created for the students to review whenever they were unsure of what they had learned in class. These teachers have successfully taken the first step to flipping their classroom and the Training Programme Division of SEAMEO RECSAM is proud in being a part of this transformation in the Malaysian education system.

3.6 Case 6: Concerto

Concerto is an open source on-line adaptive testing that was developed by the Psychometrics Centre of University of Cambridge and can be accessed from *https://code.google.com/p/concerto-platform/.* It can be used to develop a simple adaptive test. The test is designed by using R script. Table is used to store the data in the form of items, their responses and their difficulty level or any other item parameters depending on the parameter model stipulated in the Item Response Theory. The HTML template is used to create introduction template, item template and feedback template and is used to present as it is to the test takers. The scores are calculated as the test takers advance into the test.

Computer Adaptive Testing (CAT) has gained recognition as a test that is tailored to the test taker's ability. The design of CAT allows pre-calibrated items to be administered from an item bank, hence rendering itself as a customised test that continuously re-estimates the test taker's ability with improved precision of person measure. Since the selection of item difficulty is matched to person ability, the iterative process will converge to provide accurate, reliable and valid measures of person ability (see [6]). As items are individually paced, there is no need for the test takers to be presented with irrelevant items that are easy or difficult, hence reduces and eliminates 'unwanted' behaviour like boredom, fatigue, test anxiety and frustration. CAT provides immediate scoring and feedback to the test takers and cuts down testing time by half (see [8]). In addition, administrating equivalently challenging items reduces over-exposure of items and thus increases test security. Cost of administrating tests is cut down as the need for hiring and training invigilators does not arise, which also reduces measurement error. Besides test takers, CAT also benefits test developers and test publishers as experimental items can be simultaneously validated with testing and, test revision is less strenuous since adding or removing of items does not contribute to test reliability (see [7]).

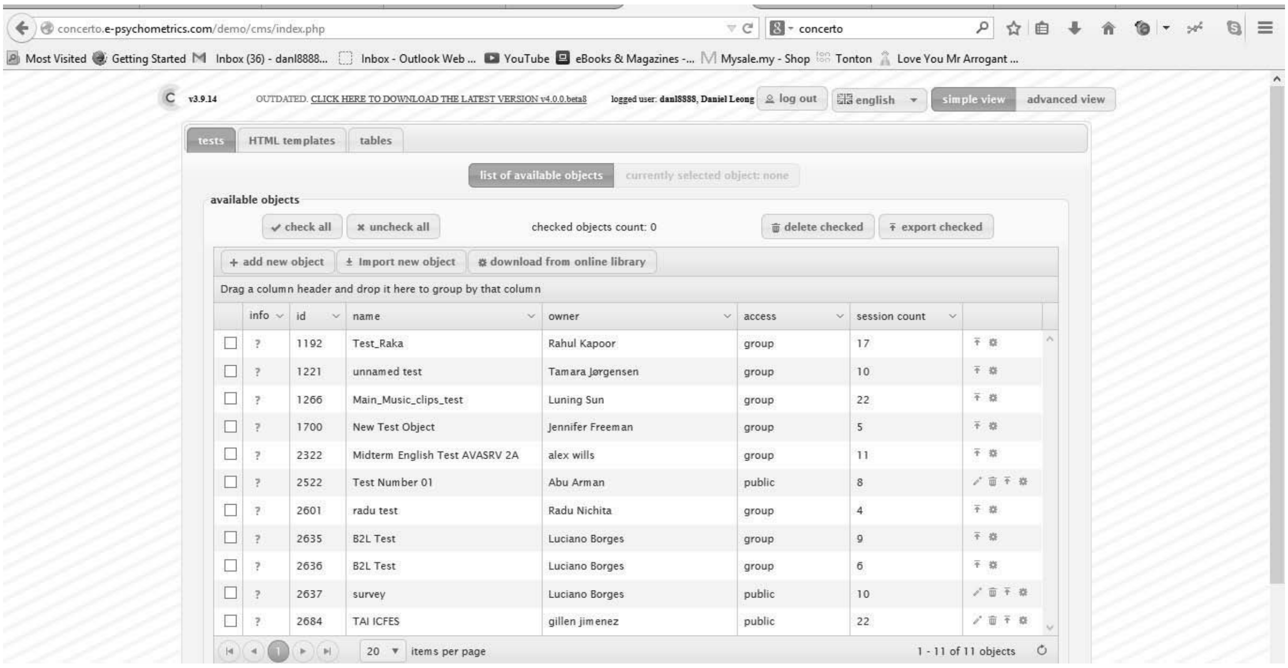

Figure 3.6 Concerto Interface

Concerto was introduced to a cohort of 12 secondary teachers as part of their four-week in-service course. The teachers were guided through hands-on activities to design a simple test that consisted of Mathematics questions. A qualitative study was conducted after the training session. The results indicated the participants strongly agreed that Concerto was an ideal method of assessing students (mean= 4.21 out of 5). The participants felt that on-line adaptive testing was an effective, time-saving and useful mechanism in assessing students as it could accurately estimate students' ability. They expressed their satisfaction in using Concerto as a platform to conduct adaptive testing which they aimed to use in their classes after the course. The participants agreed that Concerto:

- Helped them to be more effective in assessing their students
- Was more productive than paper-pencil test
- Gave them more control in assessing their students
- Saved time when in assessing students
- Fulfilled their expectations about the purpose of an assessment
- Was useful in assessing students
- Gave accurate details about their students' ability
- Was the correct way to assess students as it is based on students' ability

4. Summary and Discussion

This paper has provided selected educational technology tools that were introduced to the participants in our courses and workshops conducted in the SEAMEO RECSAM. Knowledge on incorporating other tools such as the Graphic Calculators, Interactive Whiteboard, tablets and mobile phones are disseminated during the training programmes at the Centre but these gadgets are not discussed here. These technology tools provides affordances to change the way mathematics is taught and learnt; from an extensively teacher directed activity to a more student centred exploratory tool. We also need to be reminded that each and every educational technology has its own strengths and weaknesses. At the end of the day, it is still the teachers that uses these tools are the most essential element in making the difference in the lives of students. The Centre will continue to find means to update the teachers and educators specifically in this region to keep abreast of the important educational technology developments that are taking place in the educational arena.

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Why Technologies Are Necessary for the Current Curriculum Reform? Using the e-textbook produced by dbookPro

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Abstract: *There are two major issues for curriculum reform in these days. Firstly, developing the Skills for 21st Century are one of the main issues for the curriculum reform in the world. Current meaning of ICT might be not the same as future one. On this meaning, it will be a long term issue on the curriculum reform. On this demands, Mathematics are expected subject to develops the competency for producing future scenario thorough searching the unknown-operative variable and for using plausible reasoning. Science, Technology, Engineering and Mathematics (STEM) movement enhance the usage and the way how to use mathematics with technology more. It is a short term issue for the reform. The devices change the meaning of technology in the classroom. This lecture confirm the possibility for using technology in the classroom on these two demands and push the discussion for the future ATCM. dbookPro is the example for e-textbook in current status and a tool for imagine the future. Indonesian edition is distributed by SEAMEO Qitep in Mathematics.*

Integrating Technology in Teaching and Learning Mathematics

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***Abstract**: With the rapid development of ICT and its ramification in our world, especially education, can we envision how will education look like in the future, especially in Indonesia, and in the teaching and learning of mathematics? Employing Zappa's Envisioning The Future of Educational Technology (2013) and NMC Horizon Report 2014 K-12 Edition, this paper will reflect on the effort of ICT integration in teaching and learning, especially in the teaching and learning of mathematics, in Indonesia. Taking stocks of the existing Government's policy on ICT and ICT in Education, also of the facts and figures of Indonesia's ICT profile, this paper discusses initiatives, practices, and studies on ICT in Education, integration of ICT in the teaching and learning of mathematics, what technology and how to integrate in the teaching and learning of mathematics, and some future prediction on the evolution of teaching and learning due to emerging technologies.*

1. Introduction

ICTs have become one of the essential pillars of modern society, therefore, the mastery and understanding ICT basic skills and concepts is imperative. With the rapid development of ICTand its ramification in our world, especially education, can we envision how will education look like in the future, especially in Indonesia, and in the teaching and learning of mathematics? 10 years ago, smartphones were luxury, and tablets did not even exist. This rapid development is also combined with our internal challenges to implement the 2013 Curriculum, which calls for reform in teaching and learning practices. How these challenges impacts mathematics teachers in Indonesia?

Employing Mike Zappa's (2013) Envisioning The Future of Educational Technology (http://envisioning.io/education/) and NMC Horizon Report 2014 K-12 Edition http://cdn.nmc.org/media/2014-nmc-horizon-report-k12-EN.pdf, this paper will reflect on the effort of ICT integration in teaching and learning, especially in the teaching and learning of mathematics, in Indonesia.

2. Policy

The use of ICT in teaching and learning has been encouraged in Indonesia, despite the fact that access to ICT has not been distributed equally to all teachers and students, and schools across Indonesia. The connection (broadband, ISP, etc.), acquisition of hardware (which depends on institutional and financial support), software, and human ware are not yet of equal availabitiy yet. The Government of Indonesia has made serious effort to provide access to ICT – in terms hardware, softwares, and infrastructure to all education institutions for more than two decades. Change is happening incrementally, but these adjustments will need to be intensified and massified at comparable speed with the technology development.

The policy on ICT in Education was part of the the 2001 Presidential Decree on Telematics. Later, issued in 2003, the Law of National Education states ICT as one modality to deliver education both in face-to-face education as well as in a distance education environment. In 2005, the Law on Teachers and Lecturers promotes ICT competency for teachers and lecturers as part of professionalism. In 2006, 441 ICT centers were established in vocational and general high schools. E-dukasi.net is one ICT based program which was then developed by Center for Information and Communication Technology for Education (Pustekkom) to offer multimedia and web-based materials for students of grade 7 to 12, and for vocational schools. Materials for school subjects are completed with exercises and tests.

In year 2006 National ICT Board has prioritize E-education as one of its flagship program (Nandika, 2007). The introduction of ICT to schools has since been carried out by various parties, involving both private and public agents. Further, the national education network (National ICT Backbone) and information system (Jardiknas & Inherent) were also established to serve integrated educational services in provinces, cities, universities, and schools. In parallel to these development, there is also growing interest in open and distance education which was marked by the establishment of Universitas Terbuka in 1984 – mainly serving in-service teachers and teachers' qualification upgrading and establishment of open junior high school and open senior high school; the use of open educational resources, ICT based distance education for teachers (HYLITE), and ICT-based resource sharing and collaboration (e-books, e-journals, e-library).

3. Facts and Figures

As an archipelago Indonesia covers 5,193,252 square kilometers of 17,508 islands with three time zones. It has 80,000 kilometers of shoreline and is the fourth most populous country in the world with an estimated 253,899,536 population. The population represents more than 500 ethnic groups. The country is demographically unique and different, and geographically vast with its people practicing different cultures and traditions.

Picture 1. Map of Indonesia

About 88 percent of Indonesians are Muslims. The rest are Christians (8 pct), Hindus (2 pct) and those of other religions (2 pct). Hence, Indonesia is thus the world's most populous Muslimmajority country. Jakarta, the capital city of Indonesia is also the most populous in Southeast Asia and the 13th most populated city on earth. The official metropolitan area is populated by more than 28 million people.

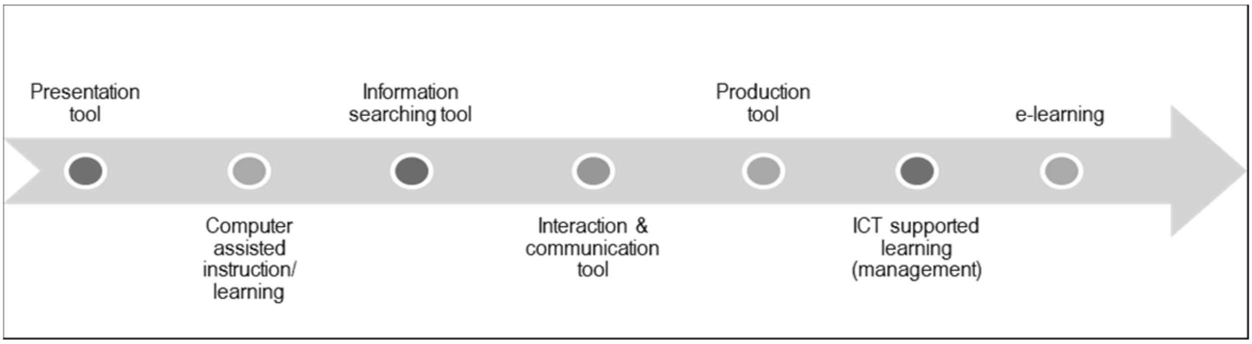
The country is characterized by four core problems: diversity, disparity, scalability, and sustainability (Nizam & Santoso, 2013). Its diversity results from having more than 300 ethnic groups speaking more than 500 languages and dialects. This only means that every district is unique and different from each other. The imbalance in physical development of the country through the course of history contributes to disparity.

Despites the problems, ICT in Indonesia is developing rapidly. Lukman (2013) states that Indonesia sees as many as 41.3 million smartphone and 6 million tablet owners by the end of 2013 and that the number is predicted to increase to 103.7 million smartphone and 16.2 million tablet users by 2017. Meanwhile, there were 74.6 million Internet users, that is, around 25% percent penetration rate. Out of these, 80% are Facebook subscribers. Almost half of Indonesian netizens are below 30 years old, while those above 45 years old make up 16.7 percent of Indonesian netizens. About 86 percent of the netizens access the Internet using smartphones and spend between IDR 50,000 (USD5) and IDR 100,000 (USD 10) monthly for Internet access. They mostly go online to socialize (38 percent), chat (28 percent), listen to music (21 percent) and play games (19 percent). About 17 percent are reading content. The majority of these internet users tend to come from those living in cities and large towns.

4. ICT in Education

Schooling in Indonesia covers around 290.000 schools, 51.3 million students, and 3.7 million teachers, with learner to computer ratio of 136 students for one computer. The use of computers in school started primarily for administrative purposes. Several schools, especially private ones and those in large cities, have developed school websites for promotion and communication between school and its community. However, the application of ICT to teaching-learning activities is prevalent in few schools, usually only international schools or branches of foreign school systems. A survey conducted by the Centre for Information and Communication Technology in Education (PUSTEKKOM) of schools in Jakarta found the use of ICT in Indonesian schools has been part of the curriculum at all school level. Although there is no specific hours dedicated at primary level, there are about 5-10 hours dedicated for ICT learning in junior and senior secondary school (UIS, 2014). In 2013 curriculum, the use of ICT has been integrated across subjects, thus it is inherently integrated instead of a mere standalone course. Regardless the increasing internet penetration rate and use of ICT gadgets and social media, the use of ICT in education, especially in teaching in learning is considerably low, along with low supporting infrastructure (UIS, 2014).

The use of ICT has followed different stages since early years in the early 2000 up to the present years, but mostly indicates that ICT is being used as enhancement to the traditional face-to-face learning as follows:



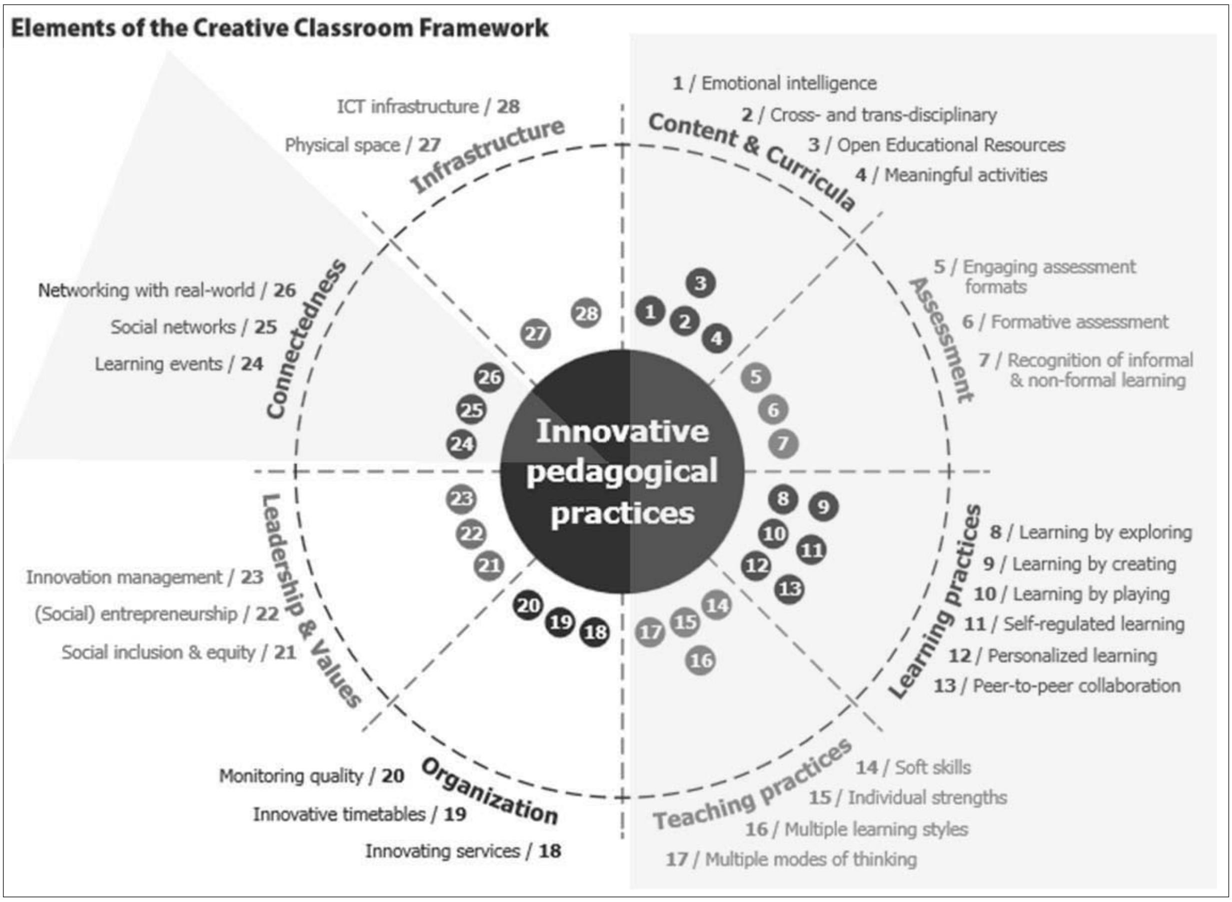
Picture 2: Stages of ICT use (*Adapted from Pannen, 2009*)

The stages depict the incremental progression of the use of ICT as a tool for teaching and learning, from the most simple to the presently most sophisticated use and from single media to the integrated multimedia use to enhance and facilitate what is essentially a face-to-face teaching and learning. From the perspective of teachers, this progression has usually been the case when ICT use is at the (newly) emerging and/or applying stages, instead of infusing or transforming level. The infusing and transforming level usually involve teachers' use of ICT in all aspects of their professional life, and daily life, beyond school. A number of research have been conducted based on this framework and resulted in a general consensus that the use of ICT in teaching and learning brings about positive benefits in student learning (Wahyudi, 2008). A study by Harrison (2002) also provides concrete evidences of ICT having an impact on teaching and learning in the classroom.

The global movement of 21st Century learning as well as the 2013 Curriculum in Indonesia have called for more strategic use of ICT in teaching and learning: an ICT-based teaching and learning, where ICT is no longer a mere facilitator to teaching and learning (ICT-supported teaching and learning or ICT-enhanced teaching and learning), but an integrated component of teaching and learning in various subjects. The questions arise from teachers then: What exactly is the potential of ICT for learning and teaching, how to exploit this potential in education, especially mathematics education, does digital technology really work?

When designed properly, there are numerous potentials uses of ICT in education. As an integrated component of teaching and learning, ICT allows learning experiences which are innovative, accelerated, enriched, and deepened the skills acquisition, motivating and engaging and relating school experience to work practices and authentic context. The integration of ICT can help teachers and students to embark on the student-centered learning instead of content-oriented or teacher-centered learning. Microsoft (2014) elaborates that ICT use in teaching and learning is potential at developing self-regulation, collaboration, knowledge construction, real-world problem-solving and innovation, and skilled communication. These potentials uses of ICT in education indicate that the simple use of ICT as a tool in teaching and learning is no longer adequate, and thus creative and innovative strategy on the use of ICT is called for.

NMC Horizon Report 2014 K-12 Edition by Johnson, et.al. (2014) has summarized the strategy of creating creative teaching and learning through the use of ICT as depicted in the following picture:



Picture 3: Creative Classroom Framework using ICT
(*http://cdn.nmc.org/media/2014-nmc-horizon-report-k12-EN.pdf*)

The use of ICT has enabled teachers to be innovative and creative in developing content and curricula through the use and production of cross and trans-disciplinary open educational resources, to be employed in meaningful activities which challenges students' emotional intelligence. It also enables teachers to innovative and creative evaluator and assessor of learning outcomes, to employ engaging assessment format, various formative assessment, and to recognize prior learning experience. The use of ICT plays key role in innovative and creative learning practices to allow students to learn by exploring, by creating, by playing, by self – regulating, by personalized learning, and by peer collaboration. It also plays important role in innovative and creative teaching practices to include the teaching of soft skills, to cultivate individual strengths, to cater multiple learning styles, and to accommodate multiple modes of thinking. The next important role of use of ICT in teaching and learning is the connectedness, making a classroom has no longer brick walls boundary; it opens the window to the world.

All strategies have been extensively integrating ICT into education, even turning the class into a different form, i.e., the blended learning form, where face-to-face teaching and learning is no longer the most dominated mode of learning. The most recent strategy which integrates ICT into teaching and learning has been the so-called "flip-learning". Examples of this flip-learning have been demonstrated by the Khan Academy (http://www.edutopia.org/khan-academy-discovery-lab-blended-learning-video). The Khan Academy provides numerous learning content – in multiple forms, which students or teachers can download and then learn at their own place, time, and pace, before the face-to-face meeting. Thus, the face-to-face meeting that follows will be used mainly for discussion, simulation, practices, or creation of learning products. As such, the low level learning – memorizing and understanding of learning content can be done at home, prior to the meeting. The meeting at school becomes important session as student will be engaged highly in higher order learning through various meaningful activities.

In Indonesia, a portal similar to Khan Academy is available, i.e., the EdukasiNet and Rumah Belajar. In addition, teachers are welcome to use other sites, as well as Khan Academy itself, to enrich their learning materials, based on the learning outcomes and basic competencies to be achieved. What is needed, then, is the creativity of the teachers to mix and match the rich learning resources available into the teaching and learning process and implement an ICT-based teaching and learning.

In general, the use of ICT in teaching and learning has implications for how teaching is planned and carried through. With the use of ICT, there is need for new approaches to the teaching, new tasks and problems for the students to work on and perhaps new ways of working and learning together. To be able to integrate ICT in teaching and learning, to use ICT properly and creatively, teachers are expected to believe in the effectiveness of ICT as has been proven by numerous studies. Further, teachers should believe that the use of technology will not cause any disturbances in their teaching and learning, on the other hands it will make the teaching and learning more meaningful, fun, and efficient. Finally teachers should believe that they have control over technology, as nothing could replace the role of the human touch of a teacher as a learning designer. Johnson (2014) states "What is the role of a teacher in such a scenario? …we need to put aside the traditional knowledge acquisition model. The main effort of teaching shifts to designing learning environments" that enable the students to realize on their own what they "need to know" in order to to achieve certain competencies.

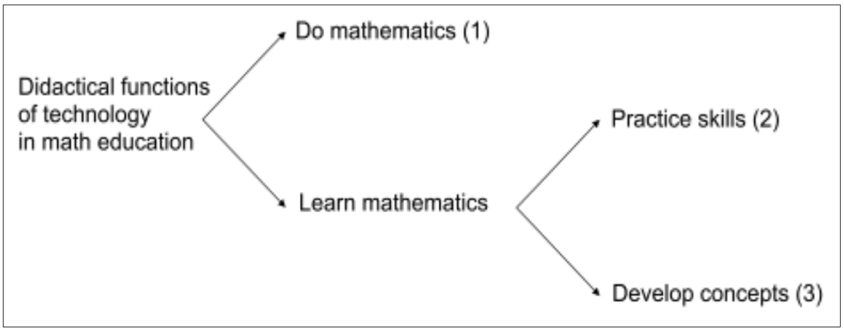
Many research and studies show that most teachers do not make use of the potential of ICT to contribute to the quality of learning environments, although they value this potential quite significantly, because of their beliefs which are contradictory to the expected (Amin, n.d., http://www.nyu.edu/classes/keefer/waoe/amins.pdf). According to MNC Report 2014, key to nurturing the new role of teachers and developing the teachers' belief in the use of ICT is providing them with plentiful opportunities for professional development, to allow upskilling and reskilling.

5. Integration of ICT in the Teaching and Learning of Mathematics

ICT has been considered an essential tool for learning mathematics in the 21st century. The National Council of Teachers of Mathematics (2008) states that schools must ensure that all their students have access to technology. Hoyles & Lagrange (2010) express that "…digital technologies were becoming ever more ubiquitous and their influence touching most, if not all, education systems".

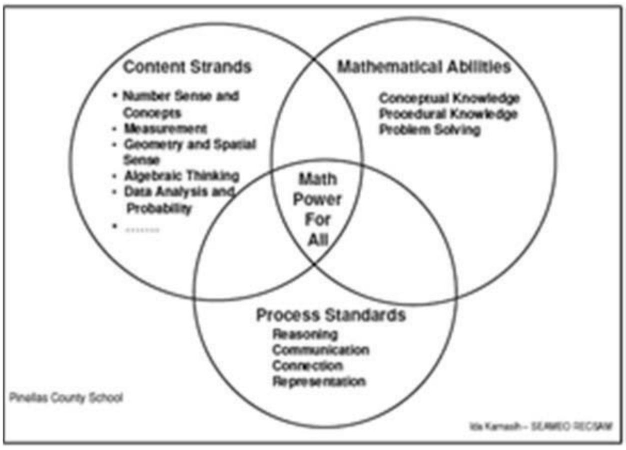
Drijvers, Boon and Van Reeuwijk (2010) distinguishes three main didactical functionalities for ICT in the teaching and learning of mathematics: (1) the tool function for doing mathematics, which refers to outsourcing work that could also be done by hand, (2) the function of learning environment for practicing skills, and (3) the function of learning environment for fostering the development of conceptual understanding, as depicted in Picture 4. Meanwhile, Amarasinghe and

Lambdin (2000) add another aspect of the use of ICT in the teaching and learning of mathematics, i.e., to integrate mathematics with authentic context for a meaningful and contextual learning.



Picture 4: Function of ICT in Mathematics Education

Rahman, Ghazali, and Ismail (2003) describe the main purpose of mathematics teaching and learning as to develop the ability to solve a wide variety of complex mathematics problems. The process of problem solving involves understanding the problem, devising a plan or solution based on deductive or inductive analysis and mathematical model, implementing the plan and looking back (examining the solution). The skills required for these process includes communicating and expressing ideas through symbols, tables, diagrams, and other mathematical illustrations; and mathematical characters – curiosity, motivation, and interest in learning mathematics; resilience and confidence in using mathematics to solve problems (Wardhani & Sutanti, 2008). Karnasih (2008) explains the objectives of the teaching and learning mathematics, i.e., to achieve the mathematical power to include competencies in mathematical communication; mathematical reasoning; mathematical problem solving; mathematical connections; and positive attitudes towards mathematics as depicted in the Picture 5.



Picture 5: Mathematical Power

ICT is able to provide strong support for all the above-mentioned requirements and there are now many outstanding examples of world class settings for competency and performance-based mathematics curricula that make sound use and integration of ICT in the teaching and learning process. The fact is that ICT in various form of its technology has been used for the teaching and learning of mathematics for more than 35 years as quoted by Pustari (2014). Mathematical softwares have been the most ICT used in the teaching and learning of mathematics.

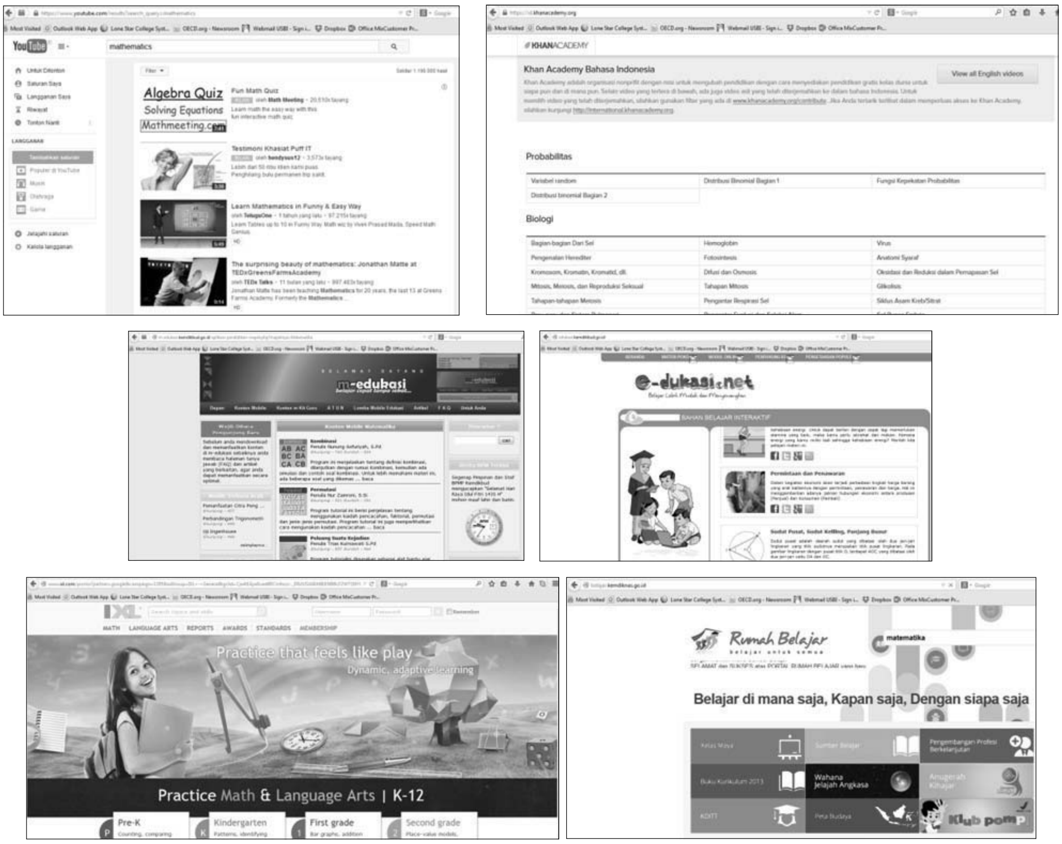
Ekawati (2008) states that integration of ICT in the teaching and learning of mathematics has been able to increase students motivation in learning mathematics, allows individual learning based on individual learning style and pace, and it has been excellent in increasing the cognitive and affective skill as compared to the traditional teaching and learning process. Technology saves time and gives students access to powerful new ways to explore concepts at a depth that has not been possible in the past. Weaker students often are better able to succeed with the help of ICT since they can do more exercises; and drills and practices at their own pace, and based on their individual learning styles.

Wahyudi (2008) states that the challenge for schools and teachers is not whether they use ICT or not but how they use it. ICT must not be considered as a mere technological tool, but it must be linked to educational goals, objectives, and learning outcome to be effective. Furthermore, it has to be well designed into the teaching and learning process – it should be well integrated. Thus, the use of ICT has specific designated function to achieve a certain learning outcome in the learning process.

According to Wahyudi (2008), using ICT in mathematics classroom provide ample learning opportunities for the students at all stages of learning – in the introduction, information presentation and acquisition, as well as assessment and evaluation stage. First, the ICT enable students to learn from feedback. The computer (ICT) often provides fast and reliable feedback which is non-judgmental and impartial. This can encourage students to make their own conjectures and to test out and modify their ideas. Secondly, the ICT (e.g. computers and calculators) enables students to produce many examples when exploring mathematical problems. This supports their observation of patterns and the making and justifying of generalizations. Thirdly, the ICT help students to see pattern and connection. The computer enables formulae, tables of numbers and graphs to be linked readily. Changing one representation and seeing changes in the others helps students to understand connections between them. Fourth, the use of ICT allows students to work with dynamic images that cannot be done within traditional teaching. Students can use computers to manipulate diagrams dynamically. This encourages them to visualize the geometry as they generate their own mental images. Fifth, using ICT (e.g. computers) enables students to work with real data which can be represented in a variety of ways. This supports interpretation and analysis that lead students to higher order thinking skills.

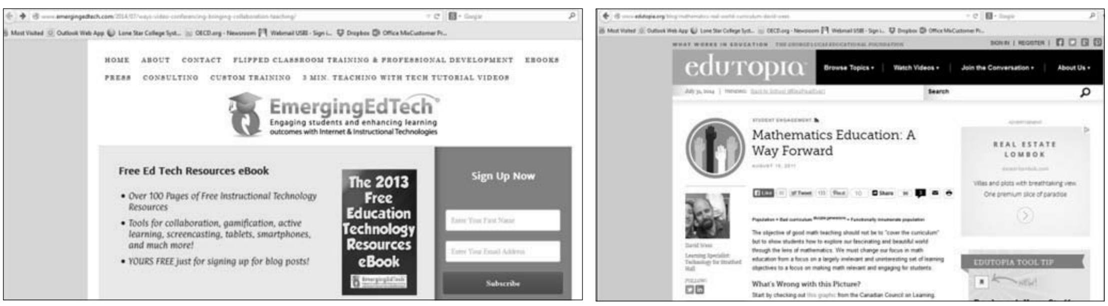
6. What technology and how to integrate in the teaching and learning of mathematics?

Changes brought by the integration of ICT in the teaching and learning of mathematics can be seen at two levels: ICT for learning mathematics, and new strategy for teaching mathematics with ICT. When a teacher uses ICT for learning mathematics, s/he can still design the teaching in an old-fashion way of face-to-face classical teaching, thus ICT is a mere technological tool which can assist students to learn, to do drill and practices, to do exercises on certain concepts in mathematics. In this context, As Pustari (2014) states, mathematical softwares have been the most ICT used in the teaching and learning of mathematics, e.g., Geogebra, Autograph, Maple, Mathematica, MathLab, WolframAlpha, Desmos graphing calculator, Microsoft Mathematics, etc., whether it is a free software or paid ones. In addition, there are also mathematical contents being offered online, and enable teachers to use them in the teaching and learning process, or students to download or directly learn from it at their own time and pace, e.g., Khan Academy, m-Edukasi, Rumah Belajar, Youtube, classroom2.0, IXL, etc.



Picture 6: Mathematics Softwares and Contents from the Internet

When a teacher uses ICT to enable her/him to employ new strategy of teaching mathematics, students will eventually learn mathematics using ICT, as modeled by the flip learning strategy. Thus, with the availability of the mathematical softwares, the teaching and learning of mathematics can be designed differently and creatively, allowing both students and teachers to take advantage of the ICT enabled learning. My favorite websites for teaching and learning strategies have been EmergingEdTech (http://www.emergingedtech.com) and Edutopia (http://www.edutopia.org). Both these sites are sharing ideas on how to teach using various new teachnologies, new emerging technologies, and also new softwares/programs to teach specific content.



Picture 7: EmergingEdTech and Edutopia

There are many other resources available in the internet that mathematics teachers can use. Carefully selected and designed, these resources can transform the teaching and learning mathematics in any classroom or even beyond classroom. Following is an array of softwares that can be used to foster collaboration in any learning process.

Picture 8: Collaboration games, tools, softwares

In addition, there are also examples of the teaching and learning of mathematics using ICT, such as the ones provided by http://www.pil-network.com/Resources/LearningActivities. These examples were demonstrating strategies, tips, and techniques of teaching and learning of mathematics which teachers can replicate or adapt to their own needs in their teaching.

With the abundance of resources available for both teachers and students in teaching and learning mathematics, specific skill is required for teachers and students to be able to select and adapt the resources into their own teaching and learning process, i.e., the so-called "information evaluation skill". This skill will equip teachers and students to select carefully the resources from various perspectives, including validity of the concept or information, the breadth and depth of the information, the language, communication and presentation style, and also the visual presentation (image, animation, navigation, domain composition, etc.). This skill is important to assure the quality of the software/games/presentation (video, etc.) that is being selected for the teaching and learning of mathematics is valid, reliable, and of high quality – it matches the learning outcome, does not contain prohibited materials or images, and suitable for the teaching and learning of mathematics at the intended level for the intended audience.

7. Into the Future

According to Daly (2013), the future of learning with ICT is about access, anywhere learning and collaboration, both locally and globally. Teaching and learning process is going to be social activities. Access will open the door for both teachers and students to the rich and abundance educational resources available in the internet. Learning can take place in the classroom as well as beyond. Face-to-face meeting will be important for assessing and confirming students understanding through discussion with teachers, project and group activities with their peers. However, virtual learning through the internet is also of high importance where students will gain more information from their network – virtual teachers, virtual peers, etc. This arrangement will not be possible one or two decades ago, without the presence of ICT in its various forms.

The NMC Horizon Report 2014 K-12 Edition claims that the development of technology in education will be BYOD, Cloud Computing, Games and Gamification, Learning Analytics, The Internet of Things and Wearable Technology.

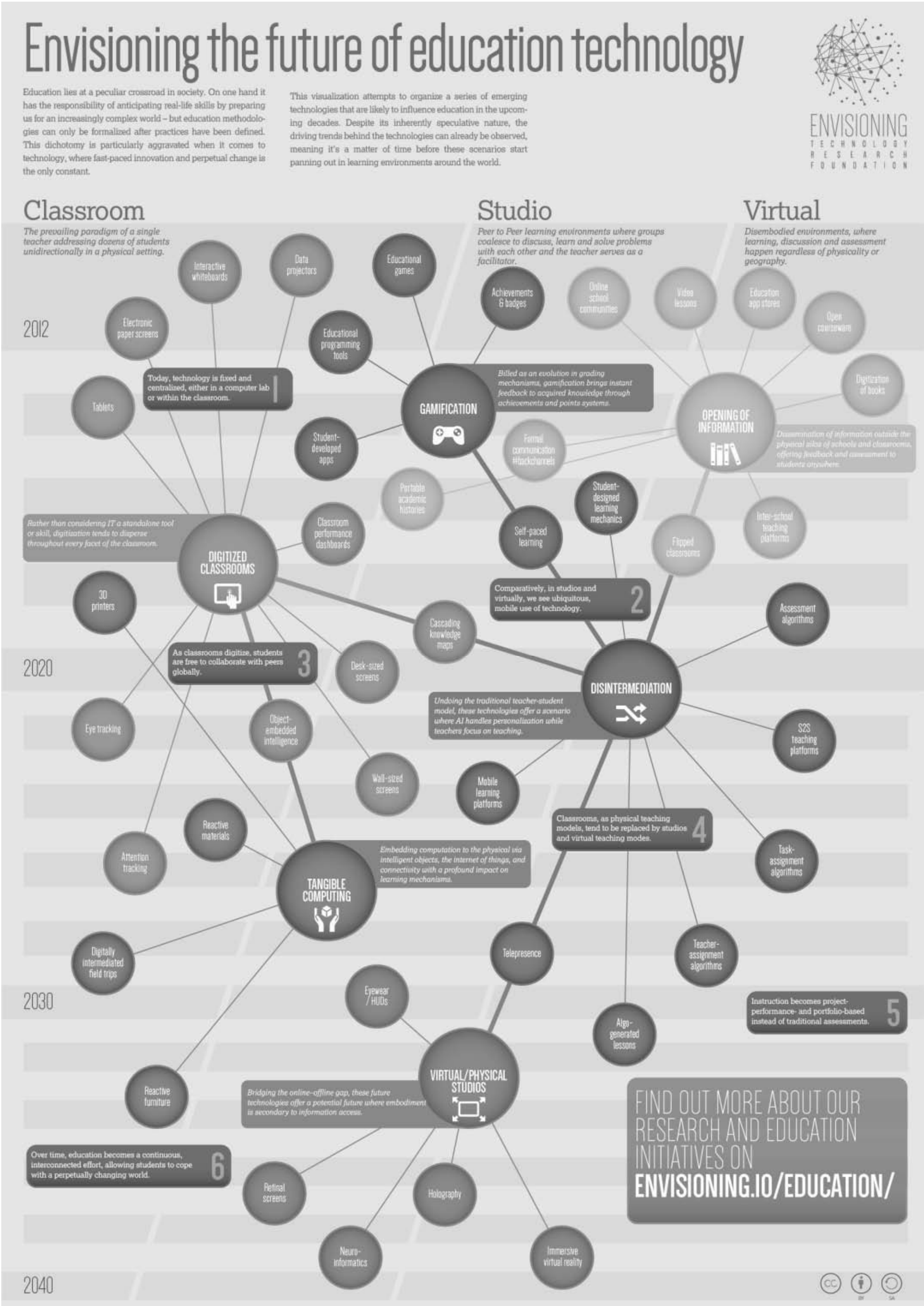
- BYOD refers to Bring Your Own Devices or Bring Your Own Technology (NMC, 2014). As such, the school does not have to provide personal devices anymore. BYOD is still rare in Indonesia, except in big cities where students can afford to have their own devices. Nevertheless, there is also constraining school regulation which does not allow students to bring smartphone or personal mobile phone into the classroom, since some teachers still consider those gadgets as disruptive to the teaching and learning process.
- Cloud Computing refers to expandable, on demand services and tools that are served to the user via the Internet from specialized data centers and consume almost no local processing or storage resources. Cloud computing resources support collaboration, file storage, virtualization, and access to computing cycles (NMC, 2014). Using the cloud, school does not need big server on its own, students may not need to carry flash storage anymore. Everything can be stored in the cloud, to be available anywhere, anytime, and by anybody. At present, cloud computing is gaining its popularity in Indonesia, especially in higher education institution and government and private offices.
- Games and gamification. According to the NMC Report (2014), the gaming industry is producing a steady stream of games that continue to expand in their nature and impact — they can be artistic, social, and collaborative, with many allowing massive numbers of people from all over the world to participate simultaneously. The cognitive, motivational, emotional, and social impact video games on human behavior have also been highlighted. Gamification refers to the integration of gaming elements, mechanics, and frameworks into non-game situations and scenarios for training and motivational purposes, potentially for teaching and learning. In Indonesia, availability of games has also been increasing. The use of games in teaching and learning has been research sporadically, and also studied by SEAMOLEC. Apparently, it still needs some more time to gain support from educators to allow students effectively learned from a well-designed game.
- Learning Analytics is an educational application of web analytics, a science that is commonly used by businesses to analyze commercial activities, identify spending trends, and predict consumer behavior, but currently is being use in education. The aims is for learner profiling, especially about individual student interactions in online learning activities. In Indonesia, much of this has been implemented in higher education institutions offering e-learning courses in a face-to-face setting as well as distance education mode.
- The Internet of Things is a network of connected objects that link the physical world with the world of information through the web. This application is still far from the world of education in Indonesia.
- Wearable Technology refers to devices that can be worn by users, taking the form of an accessory such as jewelry, sunglasses, a backpack, or even actual items of clothing such as shoes or a jacket. A number of devices have been popular in Indonesia, especially the smartphone bangle/watch, and sport devices for bikers and walkers.

From the so many developments predicted by NMC in K-12 teaching and learning, there is only a few that has been used and implemented or integrated into the teaching and learning process in Indonesia. It means that teachers in Indonesia have a lot to anticipate what will be coming soon into their door steps and transforming their teaching and learning practices.

Further to NMC's prediction, Zappa (2013) attempts to organize a series of emerging technologies that are likely to influence education in the upcoming decades. Despite its inherently speculative nature, the driving trends behind the technologies can already be observed, thus it is a matter of time before these scenarios start its way in our education environments. The following infographic, designed by Michell Zappa of Envisioning Tech, examines a few other technologies that could play an important role over the next 30 years (http://www.envisioning.io/education/).

At present stage, according to Zappa, technology is fixed and centralized in a computer lab or classroom. This illustrates precisely the use of ICT in the teaching and learning of mathematics in Indonesia today, ICT is still being used relatively fixed to the teaching and learning in the classroom, instead of taking advantage of its mobility capacity. Gamification and opening of information which indicate the ubiquitous nature and mobile use of technology have been starting along with digitalization of classroom which allows students to do virtual collaboration. Although not as advanced as Singapore or Australia, these trends are already starting in Indonesia, especially in the big cities. Imperative efforts from Government to facilitate this movement to take place more evenly including the remote areas of Indonesia is needed.

With Zappa's prediction, 10-30 years from now, teaching and learning and classroom will be quite different from the way it is today. There will be mass adoption of various forms of technology to leverage expert resources and connectivity. Significant numbers of learning activities are moving to individualized, just-in-time learning, and collaborative approaches. There will be a transition from classroom teacher-centered to hybrid or blended learning that combines e-learning components with less-frequent face-to-face class meetings. Assessment of learning will take into account more individually-oriented learning outcomes and capacities that are relevant to subject mastery and global competency. Teachers will hold a significant role as learning designer – incorporating the available resources into meaningful (virtual) learning experiences for students across the geography. At this point, we would be better readying ourselves as educators to anticipate this transformation, by transforming first ourselves.



Picture 9: Zappa's Envisioning the Future of Education Technology

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Technological tools have enhanced our teaching, learning and doing mathematics, what is next?

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Abstract

In this paper, we give an overview why technological tools have advanced so fast since ATCM 1995, and yet the adoption of exploration activities have not been widely implemented in many mathematics curricula. We also give examples to demonstrate the types of exploration that require the integration of CAS and DGS. We outline the components needed when developing an interactive online system, which is crucial for communications and collaborations on mathematical fields now and in the near future.

1 Introduction

Several speakers at various ATCM occasions have shown that technological tools not only can lead learners to deeper understandings on abstract and complex concepts of mathematics, but also encourage learners to discover more mathematical applications in real life. Indeed, with current technological tools, a mathematical problem or a project can be explored with the help of a computer algebra system (CAS), a dynamic geometry system (DGS) or combinations of both. We have discovered that new concepts or knowledge can be acquired by exploring mathematics with rapidly evolving technological tools. On one hand, many researches have recognized the positive impacts of proper adoption of technological tools in enhancing teaching, learning and doing mathematics and agree that curriculum heavily based on teaching for the test will hinder students' creative thinking skills and opportunities for innovation. However, because mathematics curriculum in many countries still examinations driven, the adoption of technological tools in teaching, learning and research is still far from desired.

In section 2, we describe typical difficulties for teachers when adopting technological tools in a classroom and emphasize the importance of creating video clips. In section 3, we use examples to demonstrate a mathematical problem can be made interesting and yet challenging at the same time. We also give examples to demonstrate why integrating the knowledge of both CAS and DGS is crucial for training students in current environment. Finally in section 4, we outline the needs of an online learning tutorial system which allow students to explore mathematics online, allow students to do online practice problems and receive instant feedbacks through a server. We urge educators, researchers, software and hardware developers to work cooperatively to make doing mathematics online a reality someday. Only when an online interactive mathematics learning environment becomes available, can we deliver dynamic interactive contents to users, and encourage instant discussions on mathematics at anytime and anywhere.

2 Supporting materials for teachers

The benefits of implementing technological tools in classrooms have been long discussed since the first ATCM in 1995. We have seen many governmental initiatives to boost the use of technological tools in math and sciences areas around the world ever since. However, we still see many teachers simply use computers or technological devices for PowerPoint presentations in a regular classroom. The problem exists because many teachers feel uncomfortable using technological tools for discussing or exploring mathematics in a classroom. Simply, it is still a non-trivial task for beginners learning how to use a technological device. So what are the possible reasons?

1. For hand-held devices, there are different brands and different keystrokes, author had always dreamed about all brands of calculators have the same *basic keyboard display*. This way teachers and students can get familiar with a calculator very quickly without worrying about how to manipulate a device. The place where different manufacturing companies may compete is how their computation engines reach an answer or how additional functionality of a calculator are being offered. Just like driving a rental car, consumers are allowed to choose different brands of cars, styles and options but the renter can easily start driving a car without worrying about what the brand of such car is.

2. For computer software packages with different capabilities, we have a lot to choose from, for complete introduction regarding this, we refer readers to *[10]*. In short, we can group them into software packages that can do CAS, DGS, statistics and etc. Any software packages have their own learning curves, it takes time to train teachers to be comfortable using them; not to mention nowadays, we need teachers to be able to integrate different capacities of software together, for example it is important to know how to integrate CAS with DGS. We will see examples in section 3.

The professional trainings for teachers to be competent of utilizing a hand-held device and computer software packages should be an ongoing process but it takes time and resources to attain predetermined objectives. It is known that video clips provide effective way of delivering lectures anywhere anytime such as those from *[7]* Video clips are useful for learners to recap content being covered in a lecture or to review steps when learning how to use a technological tool. It is worth mentioning a project from South Africa directed by Professor Werner Oliver, which contains all learning and teaching contents, to improve the quality of students learning and teachers teaching in school environments where internet bandwidth is not sufficient and also not affordable. His projects are aimed at providing off-line 21st century scaffolding support platforms for learners and teachers that are curriculum aligned and flexibly accessible. One of the aims of the projects is to improve the national pass rate of students who obtain more than of 50% for their final Mathematics school exams as this is normally the minimum requirement for access to science, engineering and technology related study programmes at higher education in this country(currently this rate is only 6%).

The project utilizes syllabus aligned video lessons, student workbooks, cell phone technology for formative assessment and peer tutoring with tablets for also improving teaching and learning mathematics and physical sciences subjects, see Figures 1(a) and 1(b) below:

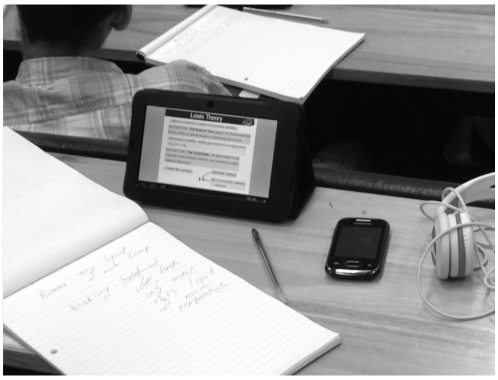

Figure 1(a) Students bring tablets to classrooms

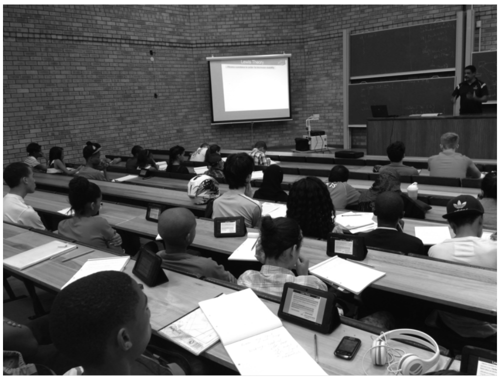

Figure 1(b) Teachers and students have access to contents

Since internet is not widely available currently in South Africa, the project is using an independent offline package which runs in a local Windows or Android browser environment. A local web browser menu system of hyperlinks allow user-friendly and flexible access to (see Figure 2) all math and sciences content components (video lessons, PowerPoint slides, student workbooks and solutions) by merely a click of a mouse or touch of a screen. The content video lessons consist of narrated concept explanations, sets of related examples, tutorial exercise problems and corresponding solutions. It also contains some interactive applets. Although the resource materials of this project is Windows or Android based, we believe this is still very valuable for those countries where internet is not widely available. Moreover, all the content video clips are narrated and animated using dynamic graphical software to enhance visualization of mathematics or science concepts and problems. This makes the model even more attractive as it provides a wonderful teaching and learning tool for teachers and students that is complete and freely accessible on a local device - anywhere, anytime.

Figure 2 Intranet webpages which contains topics in math and sciences

3 Skills needed for students and teachers

Many educators and researchers believe that creativity does not come from rote or repetitive learning but from exploration and play. Those open-ended or out of textbook real-life problems are excellent resources for students to develop their creative thinking skills. The existence of a solution often is not enough. Instead, we may ask how we can approximate a solution. Can we use a technological tool such as DGS to simulate or conjecture what a possible solution may be? Do we have real-life applications for the open-ended project? However, we often see mathematics curriculum (in most part of the world) is geared toward exam-based. Too much emphasis on testing alone when measuring a student's understanding in mathematics will hinder student's interests in mathematics and their abilities for innovation. Here we note that Finnish students do not take a national, standardized high-stakes test until they matriculate secondary school and then only if they intend to enter higher education. Instead, the purpose of assessment in Finland is to improve learning; it is "encouraging and supportive by nature" (Finnish National Board of Education, 2010, "Encouraging Assessment and Evaluation, para. 1). Therefore, we may say that allowing time for exploring is essential for students' learning and success in mathematics for Finnish system. We have seen many presenters showing that evolving technological tools in recent years have allowed learners to increase and enhance ones' mathematics content knowledge. In addition, math can be approached as Fun, Accessible, Challenging and then Theoretical. Learners can expand their knowledge horizons with technology in various stages. Exploration with technology is the key, examination is only one way to measure students' understandings.

We use the following example (see Figure 3) to demonstrate how mathematics can be made interesting and yet challenging at the same time. In the mean time, it also demonstrated some basic skills needed for students and teachers when it comes to solving an open-ended exploratory type of activities in current mathematics current. The following graph is contributed by V.SHELOMOVSKII using *[5]* and its animation can be found at *[6]*.

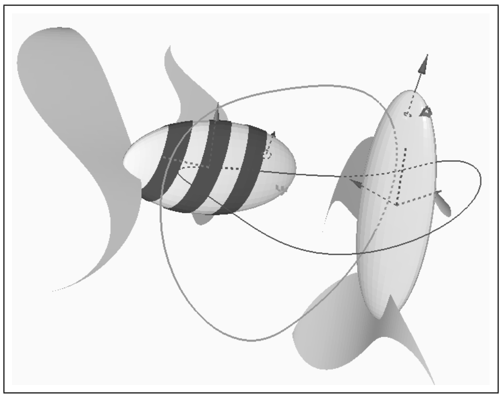

Figure 3 Floating fish with their respective space curves

We can ask the following simple questions to those students who have learned (multivariable) calculus:

1. Please describe how computer program draw a 3D fish in this case. Do we need a two or three variables function or parametric equation?
2. How can we make a 3D fish swim in a space? How many variables do we need in this case?
3. Please describe, in mathematics term, how two fish can swim without running into each other.

The following is a simple example showing basic skills of manipulating a CAS is needed both teachers and students when solving a problem.

Example 1 *Consider* $r = \sin 2\theta$ *versus* $r = \sin 160\theta$, *where* $\theta \in [0, 2\pi]$. *(a) What do you see the differences between these two polar graphs? (b) Would you like to make your conjecture base on what you see? [Do you see the graph of* $r = \sin 160\theta$, *where* $\theta \in [0, 2\pi]$ *almost fill up the whole circle of* $r = 1$*?] (c) Can you prove if your conjecture is true or false? Some may conjecture the area of* $r = \sin n\theta$ *when* $n \to \theta$, *where* $\theta \in [0, 2\pi]$, *is equal to that of* $r = 1$, *can you prove or disapprove this? It is a fun problem for those who just learned the plot of a polar graph, and using a graphics calculator can even trace the direction of the curve such as* $r = \sin 2\theta$. *The Figure 4(a) shows the plot of* $r = \sin 2\theta$ *for* $\theta \in [0, 2\pi]$ *and Figure 4(b) shows the plot of* $r = \sin 160\theta$, *for* $\theta \in [0, 2\pi]$. *Some may ask if we let* $n \to \infty$, *the graph of* $r = \sin n\theta$ *may fill out the whole circle and thus they may conjecture that the area of* $r = \sin n\theta$, *for* $\theta \in [0, 2\pi]$ *and when* $n \to \infty$, *will approach that of the circle of* $r = 1$, *which is* π. *However, we know that the area of* $r = \sin n\theta$, *for* $\theta \in [0, 2\pi]$ *is* $\frac{1}{2}\int_0^{2\pi} (\sin n\theta)^2 \, d\theta$, *and Maple returns the answer of*

$$\frac{1}{2}\frac{\left(-2\left(\cos n\pi\right)^3 \sin(n\pi) + \sin\left(n\pi\right)\cos\left(n\pi\right) + n\pi\right.}{n}. \qquad (1)$$

After taking the limit of equation (1) for $n \to \infty$ using [8] we obtain the answer of $\frac{\pi}{2}$, which means that our earlier conjecture was false.

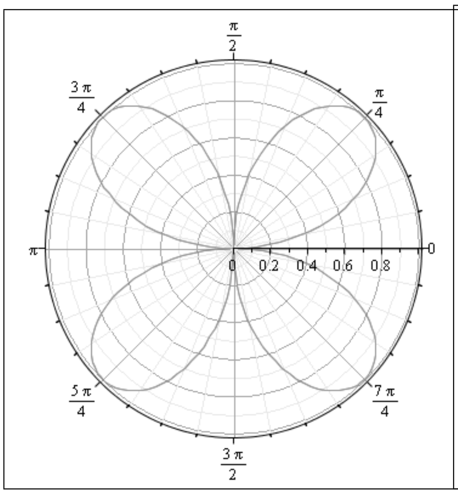

Figure 4(a) Polar graph of $y = \sin 2\theta$

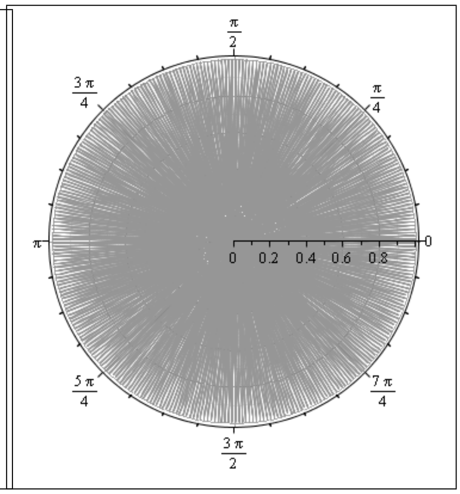

Figure 4(b) Polar graph of $y = \sin 160\theta$

The following problem was posted by Ph.D. students during my research project at Guangzhou University after discussing the problem *[15]* with them.

Example 2 *Consider the following three curves given by $C_1 : y = \sin(x), C_2 : y = x^2 + 2$, and $C_3 : (x-3)^2 + (y-3)^2 - 1 = 0$, see Figure 5(a) below. We need to find approximate points A, B and C on C_1, C_2 and C_3 respectively, so that the total distance of $AB + BC + CA$ achieves its minimum.*

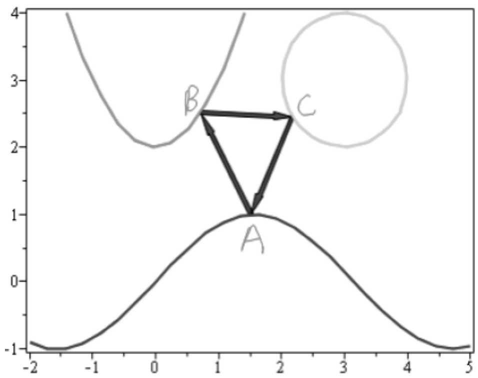

Figure 5(a) Total shortest distance obtained from a CAS

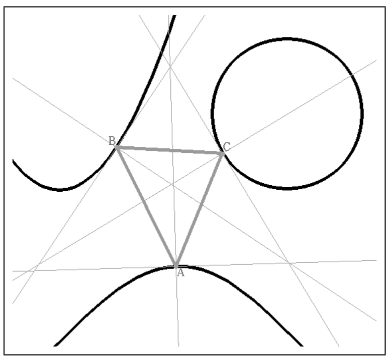

Figure 5(b) Total shortest distance obtained from a DGS

We remark that Ph.D. students have access to Chinese developed DGS so they are excellent in manipulating a DGS; however, some do not have access to a CAS or not comfortable using a CAS, so it is difficult for them to verify if their answers are correct. They first reduced their problem to a simpler case by considering three circles as follows (see Figure 6): $C_4 : (x-2)^2 +$

$(y-2)^2-1=0$, $C_5:(x+3)^2+(y+3)^2-4=0$, and $C_6:(x-3)^2+(y+2)^2-1/4=0$. We need to find appropriate points A, B and C on C_4, C_5 and C_6 respectively, so that the total distance of $AB+BC+CA$ achieves its minimum. Students were able to use geometric constructions (will be published in a separate paper) to show that $AB+BC+CA$ achieves its minimum when we move A to $A' = 1.76761632020814, 1.0273758046571)$, B to $B' = (-1.20628618061282, -2.115358414871)$, and C to $C' = (2.5843451027448, -1.72209532859668)$ accordingly in Figure 6. The normal vectors at A', B' and C' respectively, pass through the ***incenter*** *of triangle $A'B'C'$.*

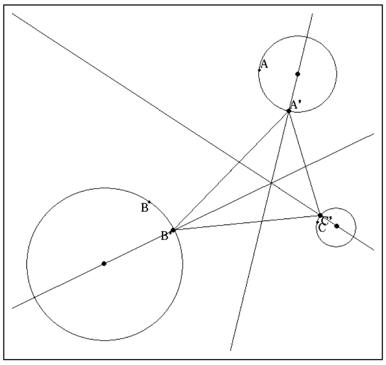

Figure 6 Solution obtained using a DGS

However, because of lacking the access to a CAS, they were not able to extend their result to the original problem (Example 2). In [16], we show that for the general case described in Example 2, the normal vectors at A, B, C respectively should be the angle bisectors of BAC, ABC and BCA respectively and they should pass through the incenter of triangle ABC. We also obtain an analytical solution by using Lagrange multipliers and with the help of Maple (see Figure (a)). We confirm that the analytical solution is consistent with the geometry construction using a DGS [2] (see Figure 5(b)).

It is natural to generalize the $2D$ problem to the following $3D$ case: We are given four surfaces shown in Figure 7, where $S_1: x^2+y^2+z^2-1=0$; $S_2: x^2+(y-3)^2+(z-1)^2-1=0$; $S_3: z-(x^2+y^2)-2=0$ and $S_4: (4(x-3)+(y-3)+(z-1))(x-3)+((x-3)+4(y-3)+(z-1))(y-3)+((x-3)+(y-3)+4(z-1))(z-1)-3=0$. We want to find points A, B, C and D on the surfaces S_1, S_2, S_3 and S_4 respectively, so that the total distance $AB+BC+CD+DA$

achieves its minimum.

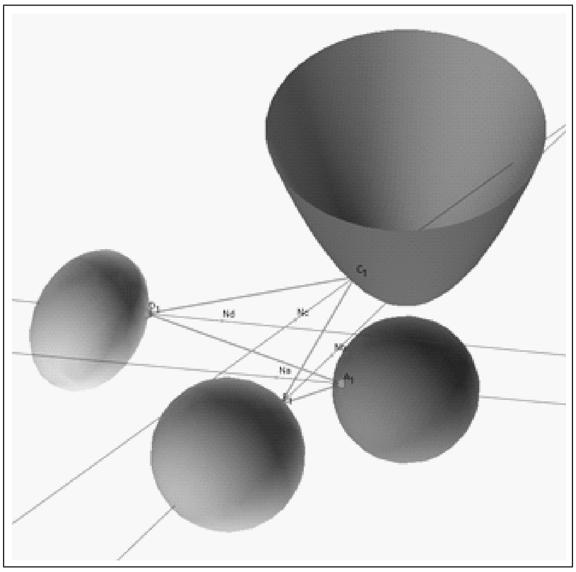

Figure 7 A total shortest distance problem in 3D

Remark: We may think of this problem as a light reflection from one surface to the other. If we think BA as an incoming light toward the point A on surface S_1 and DA as an outgoing light at A, then N_A, the normal vector at A on S_1 should be their angle bisector. Similarly, we should have the followings:

1. The line segments BC, AB and the normal vector at B, denoted by N_B, should lie on the same plane and N_B is the angle bisector for AB and BC.

2. The line segments BC, CD and the normal vector at C, denoted by N_C, should lie on the same plane and N_C is the angle bisector for BC and CD.

3. The line segments CD, DA and the normal vector at D, denoted byN_D, should lie on the same plane and N_D is the angle bisector for CD and DA.

We apply Lagrange Method with the help of Maple to find the shortest total distance $AB + BC + CD + DA$ algebraically. The desired points A, B, C and D on S_1, S_2, S_3, and S_4 respectively obtained from Maple are $A = [s_{11}, s_{12}, s_{13}], B = [s_{21}, s_{22}, s_{23}], C = [s_{31}, s_{32}, s_{33}]$ and $D = [s_{41}, s_{42}, s_{43}]$, where

$$\begin{aligned} s_{11} &= .33389958127263692867, s_{12} = .82675923225542380177, s_{13} = 0.45274743677505224502; \\ s_{21} &= 0.13641928282913798603, s_{22} = 2.0175068585849490438, s_{23} = 1.1268739782018690446; \\ s_{31} &= 0.28488630072773836633, s_{32} = .62315636129160757197, s_{33} = 2.4694840549605319295; \\ s_{41} &= 2.3607938428457871966, s_{42} = 2.4901551912577622423, s_{43} = 1.3195995907854243548. \end{aligned}$$

We show geometrically (see Figure 8) that the blue dotted lines are respective normal vectors at respective points A, B, C and D, and the dark red line segments are respective angle bisectors in Figure 8, they coincide with respective normal vectors when the minimum total distance $AB + BC + CD + DA$ is achieved. In other words, **our geometry constructions coincide with our algebraic analysis using CAS.**

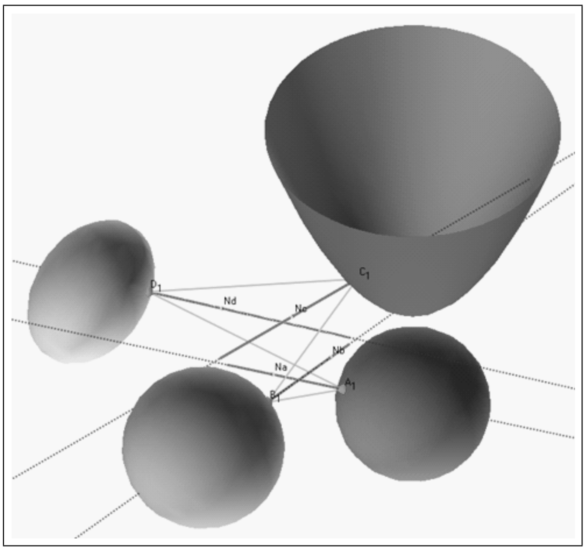

Figure 8 Total shortest distance 3D problem verified using a DGS

The preceding problem demonstrates why integrating the knowledge of both CAS and DGS is crucial for training students in current environment. The following example comes from a university entrance exam practice problem from China. We use the dynamic software *[5]* to demonstrate how the problem can be made accessible to most students and how the problem can be generalized to more challenging ones in 2D and corresponding ones in 3D. We remark that these open ended projects are excellent choices for adopting technological tools for exploring mathematics.

Example 3 *Consider the following Figure 9:*

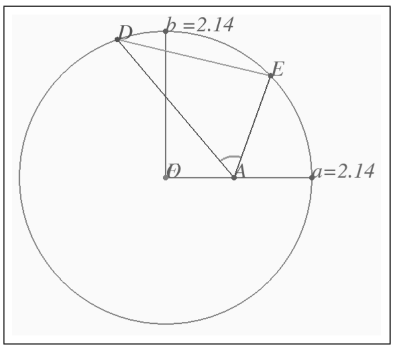

Figure 9 A college entrance practice problem from China

The circle is given with radius r, assuming the center is at $(0,0)$, and $A = (x,0)$ and $x \in [0,r]$. The points D and E are any two points on the circle such that the angle of EAD forms a fixed angle β. For the college entrance exam practice problem from China, students are asked to conjecture the maximum and minimum lengths for DE. The following values were those given at the practice college entrance problem from China: $r = 2, \beta = \frac{\pi}{2}$ and $A = (1,0)$. It is not difficult for one to conjecture after exploring with a DGS that the maximum length of DE occurs when DE is perpendicular to AO and A is on the opposite of DE, see Figure 10(a), and the minimum of DE occurs when DE is perpendicular to AO but A is on the same side of DE, see Figure 10(b).

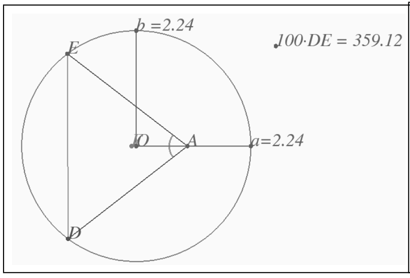

Figure 10(a) Maximum length DE

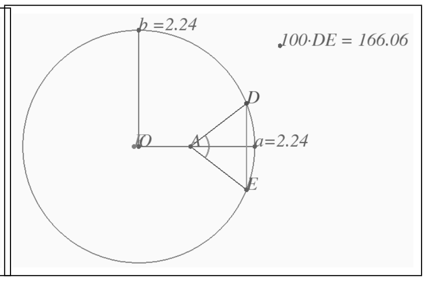

Figure 10(b) Minimum length DE

It is not difficult to generalize this problem from a circle to a sphere in this case. However, the problem should be rephrased as follows: Given a sphere of a fixed radius r of the form $x^2 + y^2 + z^2 = r^2$ and pick the point $A = (d,0,0)$, where $d \in [0,r]$. Let B be a point on the sphere and rotate AB with a fixed angle β to form a cone, see Figure 11. We want to find the maximum and minimum intersecting surface areas between the cone and the sphere.

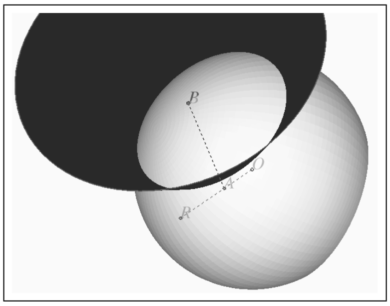

Figure 11 Generalize the circle probem to a sphere

After exploring with [5], it is not difficult to see that the maximum intersecting surface area occurs when the normal vector at B is parallel to the vector OA and A is on the opposite of B (see Figure 12(a)). The minimum intersecting surface area occurs when the normal vector at

B is parallel to the vector OA but A is on the same side of B (see Figure 12(b)).

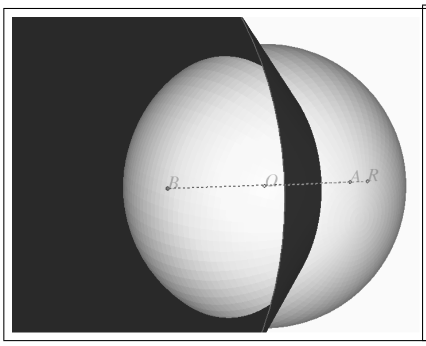

Figure 12(a) Maximum intersecting surface area

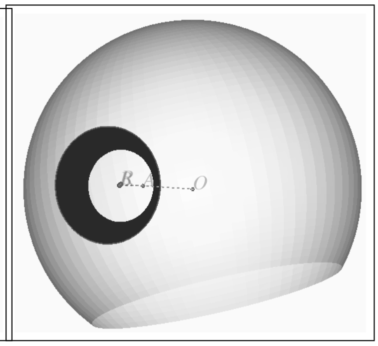

Figure 12(b) Minimum intersecting surface area

We may now ask students to extend Example 3 to an ellipse of this form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and use a DGS to make conjectures where the maximum and minimum lengths of DE could be, see Figure 13. We note that the answers will vary depending on the values of a, b, the fixed angle β and the position of the point A. Complete solutions on the ellipse case have been discovered, see *[17]*

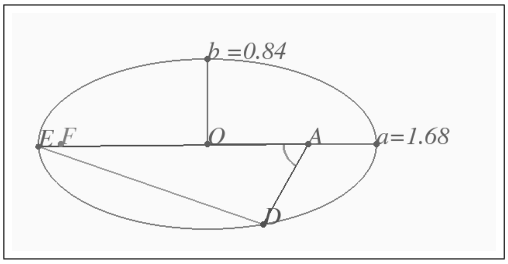

Figure 13 Generalize the 2D problem to an ellipse

The problem becomes even more challenging and is still an *open problem* if we extend the ellipse to an ellipsoid. For example, given an ellipsoid of the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$, let $A = (x, 0, 0)$ with $x \in [0, a]$. Pick B be on the ellipsoid and if the cone determined by rotating a fixed angle β around AB, see Figure 14 below:

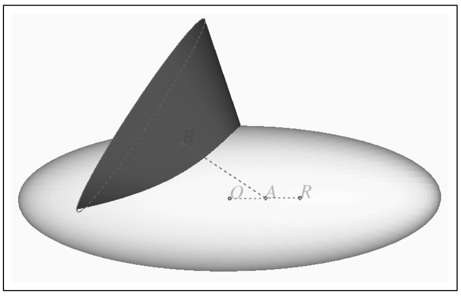

Figure 14 An ellipsoid and a cone

We may ask the question: For a fixed point A and fixed angle for the cone, find the point B that will result in the maximum or minimum intersecting surface area between the ellipsoid and the cone. Just like the case for ellipse, the answers vary depending on the values of a, b, c, the fixed angle β and the position of the point A.

We summarize a general strategy of how we may approach solving a problem and how we can generalize it to higher dimensions with the helpful of a DGS and CAS.

1. We use DGS or similar technological tool to simulate animations in 2D.
2. We make conjectures through your observations from step 1.
3. We verify our results using a DGS or CAS for 2D case.
4. We extend our observations to a 3D scenario with technologies if possible.
5. We prove our results for 3D case using a DGS or CAS.
6. We extend our results to finite dimensions or beyond if possible.

It is interesting to note the following scenarios while author conducted lectures to students of different backgrounds:

1. For students who have access to a CAS but not DGS: It is impossible for them to reproduce different animations with geometric motivations. They can only modify the CAS worksheet author provides them when validating their algebraic answers.
2. For students who have access to a DGS but no CAS: Students will use their favorite DGS to reproduce author's problems and make conjecture about the validity of the solutions; however, their conjectures cannot be verified since they have no CAS to verify solutions analytically.
3. For those students who have no access to either a CAS or DGS, they can only appreciate the graphical representations of authors' lectures, they have no available tools to experiment on their own.

4 Delivering interactive online contents

It is interesting to see that the internet technology has advanced rapidly since the first ATCM in 1995. However, delivering interactive online content still has room to improve. To develop a truly online dynamic learning system for mathematics, we need computation engines that are capable of doing both CAS and DGS. With such a system in place, we will be able to create online interactive textbooks, we can also allow open-ended exploratory type of problems such as those described in Section 3. We describe some of the desired components for online interactive learning in mathematics and related subjects so people can easily discuss mathematics.

1. Allowing instant exploration using CAS such as Wolfram|Alpha *[13]*, the system provides an excellent system where users can explore various areas using Mathematica engine online. For example, we can explore the torus knots (see Figure 15 *[14]*). Author has seen teachers from China developing teaching contents using *[13]*. More interactive approaches using *[13]* can be found in the invited speech by *[1]*. After all, this is a CAS system where users have no way of experimenting problems when a DGS system is desirable.

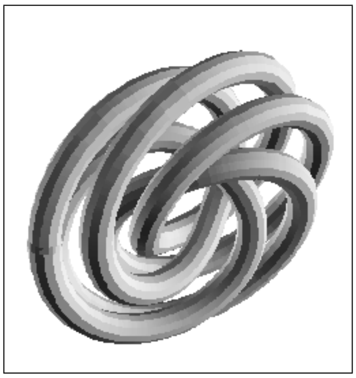

Figure 15 An interactive torus knots using Wolfram Alpha

2. For exploring DGS online, we direct readers to the page of *[12]*, where *[4]* is used as the DGS engine to describe a parabola Caustic Rays emanate from D, and their reflections in the parabola form a caustic (see Figure 16). The family of rays by pressing `Show`, then the caustic curve by pressing `Show` again. You can drag C and D. However, this is a DGS

based not CAS based so we cannot experiment problems when CAS are needed.

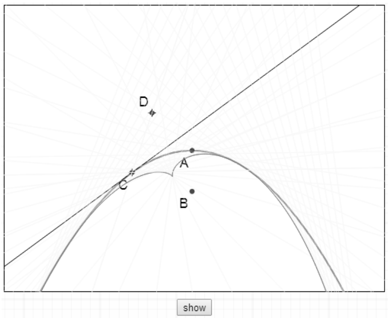

Figure 16 An online interactive caustic using Geometry Expression

3. Using applets: If we look at the webpage *[9]*, it describes an online content in Calculus subject using Java applets. For example the following Figure 17 gives a dynamic way of learning the spherical coordinate system, users can drag ρ, θ and ϕ, and can see the effects of these parameters on the position by dragging the red dot. However, we know that applets using Java script is platform dependent, which also presents a problem for developers when they consider designing interactive online contents.

$$\begin{aligned} x &= \rho \sin\phi \cos\theta, \\ y &= \rho \sin\phi \sin\theta, \\ z &= \rho \cos\phi \end{aligned} \tag{2}$$

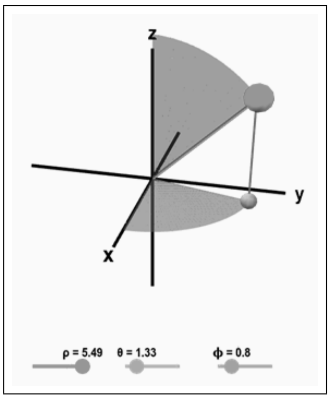

Figure 17 An interactive online content using Java applet

4. Many book publishers are interested in an online assessment system because these are what schools and universities desire for assessment purpose. Ideally a good system should

allow teachers to create randomized questions containing graphs, and students are able to see their results with feedbacks instantaneously. Therefore, a system requires an authoring tool for teachers to design problems and an users' testing system. The following shows a question generated by the Exam Builder of Scientific Notebook (see *[11]*), which is a Window-based and has been available in mid 90s. However, there is no compatible online testing system so far. We observe in Figure 18, the quantity a in $\sin ax$ can be randomized using the authoring system; accordingly the graph of $\sin ax$ will vary accordingly because there is a built-in CAS. After choosing the answers and clicking '`click to grade`', we receive a feedback from the system as seen in Figure 19. This is a Window based program which author adopted during 1997-1998 while visiting Singapore, and yet it is surprising that a comparable 'engine-based' online system is still not available as of now. The invited talk by *[3]* introduces a mathematics interactive learning environment, it remains to be seen if the system can fulfill the possibilities of online discussions on those open ended problems that are discussed in Section 3.

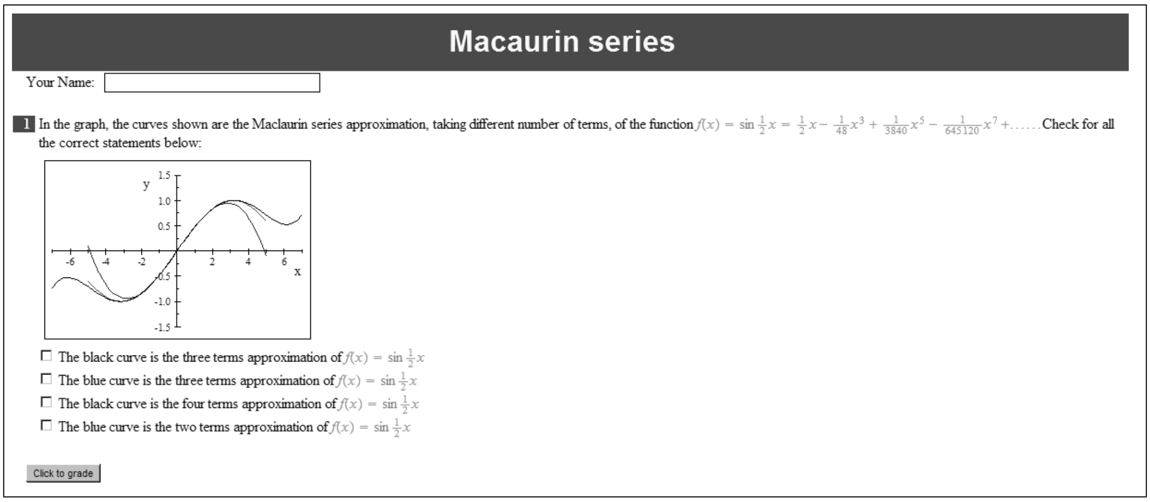

Figure 18 An authoring tool from Exam Builder

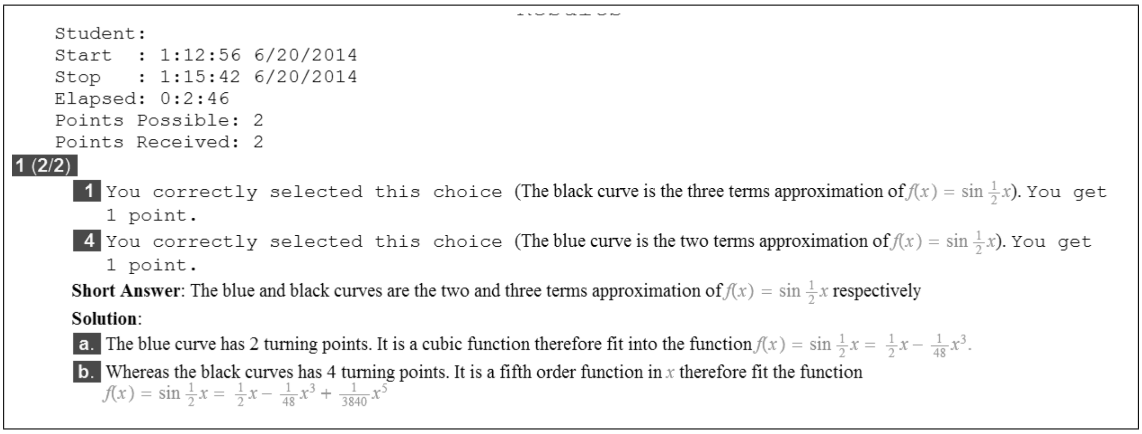

Figure 19 A feedback form using Exam Builder

5 Conclusions

It may be true that many math textbooks (including those from US) are not based on exploration, and schools buy the books that are aligned to the test questions instead of the books that promote deep mathematical understanding and creativity. This is rather a true reality but common sense tells us that teaching to the test can never promote creative thinking skills in a classroom. We know the continual efforts for all ATCM communities to address the importance and timely adoption of technological tools in teaching, learning and research can never be wrong. Therefore, we encourage ATCM communities continue creating innovative examples by adopting technological tools for teaching and research and influence your colleagues, communities and decision makers in your respective countries. We also think by developing technology based textbooks which describe contents in continuos, connected and creative fashion is important. Selecting examples that can be explored from middle to high schools, university levels or beyond, and they should be STEM related by linking mathematics to real-world applications. Allowing users to explore mathematics with technological tools online is crucial step for developing a social media of discussing mathematics through internet. Many agree that examination should not be the only way to measure students' understandings in mathematics. Technology has become a bridge to make us rethink how to make mathematics an interesting and a cross disciplinary subject. Through the advancement of technological tools, learners will be able to discover more mathematics and its applications through an interactive online learning environment.

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PARALEL PAPERS

USING MATHEMATICS SOFTWARE AND INTERNET ACCESS FOR TEACHING IN MATHEMATICS

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ABSTRACT

Curriculum 2013 has put the teacher as a facilitator and would help students to construct their knowledge. As a fasilitator teachers position students at the center of activities. Teachers have to improve the resource of learning suche as using technology. In recent years, the mathematics software and internet has increasingly been used to provide significant toll and resources for student to learn mathematics and learn about mathematics, as well as significant resources for teacher to support these. Limited facilities in school and the limited experience of many mathematics teachers with the software using and internet for mathematics purpose have hampered in practice. This paper offers some mathematics software and internet access which can be used for teaching and as the resource of learning. Limited knowledge about software and lack of skill has made the teachers keep in using traditional lecture as while in this new time, the software of mathematics and access to internet have significantly increased. For this purpose, there are two kinds of the mathematics software, (i) teacher as a developer of lesson media and (ii) teacher as a user of the software. For internet resource it can be indentified into six categories; they are (i) interactive resource, (ii) reading interesting material, (iii) reference information; (iv) Communication; (v) problem solving; (vi) web quest.

KEYWORDS: mathematics software, internet access, mathematics websites

RATIONALE

Curriculum 2013 has put teachers as a facilitator and they would help students to construct their knowledge. As the effect, the students will be the center of the lesson activities. Teacher can not deny to use the technology as the learning resources. Using slide presentations with with attention grabbing graphics are the classroom's technological zenith. Today we live in the age where Radio, Television, Video, Computer, Telephone, include both mobile phone and fixed line, satelite system and the equipment associated with these technologies such as e-mail, video conferences, blog and chat rooms. In this era of globalization and information, the information communication and technology (ICT) plays significant roles in education, particularly in teching and learning mathematics and sciences.

Unfortunately, this developing can not be followed by the developing of teacher's skill in using technology. They often lack the skill and knowledge necessary to use technology and implement it in their classroom. It will effect to the student's skill ini preparing their future, their life after high school gradution. More than others, the limited time in class have students to improve their knowledge at home, and it can be done by access the information at home, it can be used to assist the teacher at their own studying.

WHAT CAN TEACHER USE

In line with its development, the technology has influenced the process of education. At the same time, the educational technologies also have influenced learning theories, the method using for teaching and issues such as media design.

Teacher as director of learning – teaching activities should follow the development of education technologies to improve the quality and quantity learning. It hope it can affect the effectiveness of learning – teaching process which giving positive effect to the result of teaching process.

Particularly in mathematics, the teacher can use two ways to assist in teaching using technology, those are using mathematics software and using internet access.

MATHEMATICS SOFTWARE

Mathematics software is a software used to model, analyze or calculate numeric, symbolic or geometric data. (http:/en.wikipedia.org/wiki/software; 27/10/2014)

We can use mathematics software to build a self studying and drill or practice at home for students, the students can learn it by their self, leading by great tutorial, and it can be used also for preparing and mastering the concept before moving to the next.

There are a lot of software in mathematics, some are very good with high price but we also can find the freeware with good way in use. Because almost all school have Microsoft in their computer, so using Microsoft software is a good idea then the school will not need an extra cost anymore.

Here, the project can be split into two main sections, first is aimed at creating classroom ready resources wich can be used by teacher and students to enhance the learning. Second one is aimed to empowering teacher in creating their own interactive resources.

EXCEL IN CLASSROOM

One of Microsoft software is Microsoft excel. It is ideal for looking at number and data handling work. It is also possible to create workbooks in investigating aspect of algebra and space. Using macro feature without much experience of visual basic.

Microsoft excel is a wonderful application that can be used to achieve the students learning outcomes, so as an educator, a teacher needs to understand the usefulness of this application. The students also can use this in their real life after they finish high school because in most of cooperates they use this application to analyze data and generate and create graphical representations. Unfortunately this good application sometimes cannot be followed by teacher's skill.

MICROSOFT MATHEMATICS

This application was made by Microsoft at 2010 and free download. This application provides a graphing calculator that plots 2D and 3D, step-by-step equation solving, and useful tools to help student with mathematics and science studies. We also can solve the arithmetics, matrix, integral, differentials, trigonometry and some problems in science. (Joyner and Slein. 2007)

MICROSOFT MATHEMATICS ADD-IN FOR WORD AND ONENOTE

*Microsoft mathematics Add-i*n for *Word* and *OneNote* makes it easy to plot graphs in 2D and 3D, solve equations or inequalities, simple algebraics expression in your *Word* documents and *Onenote* notebooks.

With the *Microsoft Mathematics Add in for Word and OneNote* we also can perform mathematical calculation. It is also provide an extensive collection of mathematics symbols and structures to display celarly formatted mathematical expression. We also can insert commonly expression and mathematics structures by using equation gallery.

In ordering we can use it for:

1. Compute standard mathematical functions such as roots and logarithms
2. Compute trogonometric functions such as sine and cosine
3. Find derivative and integrals. limit, and sums and product of series
4. Perform matrix operations such as inverses, addition and multiplications
5. Perform operation in complex number
6. Plot 2D graph in Cartesian and polar coordinates
7. Plot 3D graph in Cartesian, cylindrical, and spherical coordinates
8. Calculate statistical function, such as mode and variances on list of numbers
9. Solve equation and inequalities
10. Factor polynomials or integers
11. Simplify or expand algebraic expressions

As free download, we can download install it into our computer as follow the steps

1. Click the download button next to the *MSSetup exe* file and save the file into hard disc
2. Make sure that all instances of *Word, OneNote* Quick Launcher are closed
3. Double click the *MSSetup.exe* program file on hard disc to start the setup program
4. Follow the instructions on screen to complete the installation
5. After the installation is complete you will be offered to install *DirectX*, it is recommended that you instal It because it will provide better display of graphics

Gettting started with *Add-In*

After you install thie download, start *Word* or *OneNote*. You will see the new equation, computation, and graphing options on the *Mathematics Tab (http;/www.microsoft.com/en-us/download/details)*

MICROSOFT EDUCATION LABS

Microsoft Education Labs can help teacher in creating worksheet. The teacher can generate a worksheet by using this application. This application is also free download.

GRAPH

Graph is open sources and can be downloaded for free. It is a powerful and feature rich application that offers a convenient and effective solution to draw mathematical graphs. In a coordinate system. The *Graph* offers wide variety of customization options and manipulations possibilities for the created graphs. The colors, size, or line styles of the graph can be tweaked to your liking. It can plot standard function, parametics function and polar functions, while the most common function like sin, cos, log, are provided as built-in by default. This powerful tool can also display graphs for equation and inequalities and allows adding shadings, point series and trendlines to graphs according to preferences.

It is also possibleto create our own constants and functions, including add text label, animation to show the exact change that the function encountering tweaked constant.

In the end we can save the graph in various image format such as BMP, JPEG, WMF, EMF, PNG or PDF and SVG. Also can be exported into other program like *MS Word*

AUTOGRAPH

Autograph can be used to create graphics in Geometry. It is different from *Graph*, for full version of *Autograph* we have to pay for cost. However we still can use a free version.

GEOGEBRA

Geogebra is an interactive geometry, algebra, statistics and calculus application, intended to for learning and teaching mathematics and sciences from primary school to university level. *Geogebra* is available on multiple platfroms with its desktop applications for windows and also for android apps, Ipad and the windows phones. It’s a web application based in HTML5 technology. (Http;/www.Geogebra.org)

The teacher can use *Geogebra* from Elementary School until University level for teaching Mathematics and science. Using *Geogebra*, we can construct the point, vectors, segments, line, polygons, conic sections, inequalities, implicit polynomials and also functions. We also can make an animation using slider tool to construct the dynamically graphics and constructions. Teacher and students also can use *Geogebra* to make conjectures and understanding about geometry theorems provement.

SNIPPING TOOL

When we use *Microsoft Office 2007 or newer* version, we can find *Snipping Tool* for capturing the screen. It is different from Print Screen because in *Snipping Tool* we can capture the area in screen only where we need.

D o not worry for user older version, we still can used other tool such as *MWSnap*, but we need to download it before install it into computer. *MWSnap* is a freeware, so we can use for no cost.

There are others important software using for capture such as *CamStudio, Camtasia, ScreencastO-Matics*. We can use these for recording screen activities and save it as video file. Of course, it is editable video.

INTERNET ACCESS RESOURCES IN TEACHING MATHEMATICS

Nowadays the technology is not something special anymore, it commonly knows and almost every live aspect has connection with the technology. In learning, particularly in mathematics and science the technology and internet cannot stand-alone and be separated anymore.

As we know, the strength of net can perform the ability to connect greatly numbers of people. It is also will increase the learning resources for students and teachers as well, collaboration and communication among them.

It is up to the teacher to decide and manage how to use the internet as the resources of learning. The net tools will become a powerful resource when we can use it intelligently

WHAT CAN TEACHER DO ABOUT?

In this paper, the writer divides the aim of using internet into two main sections. The first is aimed to download the classroom ready resources that are flexible and interactive. It is hoped that each material will have a number of uses over the range of ages and abilities and that they can enhance the learning of students in the classroom.

The second section is aimed at following interactive resources by following the activities in a website.

Using internet connection, in other hand, of course will not as smooth as in our mind. We also have to consider how big bandwidth we need. It will be a problem at school when there are many numbers students have to use internet connection at the same time. Can the school bandwidth handle it? Beside the bandwidth, it is also about the manpower to handle it because some teachers are still lack with this technology.

Other big problem is about upgrading equipment of school, it is, of course, it will high cost. Most of schools have at least thirty-six students per class and it will need high cost and high bandwidth. When the teacher gives student a project for being done at home, they will have another problem because when they need help, they parent cannot help because they do not recognize the application used. Also sometimes the lack of teacher in powering application or to create a lesson using the application.

Teacher can use free download media then use it in classroom by projector or interactive board or share with students. If we take this way, the school does not have to prepare high bandwidth and a lot of computer. And also, the student can explore it at home.

YOUTUBE

When we are searching video in internet, the Google or others search engine will direct us to YouTube. As a video hub we can find thousand or perhaps billions of video. Everybody can share his or her video in YouTube, as well as everybody can watch or download any video from this site.

We can find many channels in YouTube about education. Beginning from the basic concept to complex experiments. Youtube enable students and educators to discover what they want to learn, as if the educators bring the professors, scientists or expert educators into classroom. The educators can use it into curricula activities or extracurricular.

As a free channel, youtube also can be used by irresponsible people to share any video or material, which expose something not good for students. In this case, the educators should control or direct the student how to use it wisely.

As the good reaction of youtube about education, they have launched YouTube recently related to this section which is only educational content is permitted to share.

YOUTUBE EDU

We can search many contents about education here. The teacher can access this site freely from the classroom. Many professors, schools, universities and educational institutions upload the educations videos here.

When the school internet bandwidth is not enough to access it for all students, the teacher or education personal can download it first then access together using projector and board.

GEOGEBRA

When the teachers need interactive mathematics software, they can choose *Geogebra* as the resource. Teacher can use *Geogebra* to construct the point, vectors, segments, lines, polygons, conic sections, inequalities, implicit polynomials, and functions. All of them can be designed into dynamical model. We can use slider to construct dynamics model, it is good to proof the theorems and formulas.

Geogebra is a freeware and can be downloaded in *http;/Geogebra.org/download*. When teacher is still lack of knowledge about this software, they can download some materials from the site. There are many materials have been uploaded by users into website. All are free download.

INTERNET ACCESSING

When the school has good internet connection and enough bandwidth to access internet, the teachers can use internet access in classroom.

We can find many free websites, which offer the lesson in internet. There are interactive sites, references material, communication, problem solving, and web quest. Therefore, we can choose what we need to use in classroom or at home for students.

MAKING ONLINE CLASS

Internet makes the distance seems nothing, we can communicate anytime and anywhere. Space is not problem anymore. Limited time at school can be solved by using online class. The teacher can construct and upload the lesson into online class and the students as the learner can access anytime they can.

Teachers can put their student into classroom and control how many times they access the material.

Nation Minister Education of Indonesia has build *Rumah Belajar* as a one of resource site. It is free account and the teacher can use it for learning process, and put the students into. We can create an account in *http;/www.belajar.kemdiknas.go.id*

Different from *Rumah Belajar* we also can use some others free site from international resources such as *Quipper School at http;/www.quipperschool.com*

CONCLUSION

In the age of integration of technology in the classroom, many people may think that the blackboard is an old-fashioned instructional tool and should be combined with the digital media. Using mathematics software's and online lesson can improve the teaching in classroom. The lack of teacher knowledge about software can be covered by using application software or using download media from the credible websites.

For the school with have goo internet facilities, they can bring internet access into classroom. By using online classroom, the teacher can construct their media and upload it into their account, then the student can access it anytime anywhere. Especially, when the school uses curriculum 2013 at their school, where the students become the center of learning.

By putting the materials into website, the student can access it and conduct an active learning at students as the curriculm 2013 has arranged.

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The Inquiry Based Learning Assisted GeoGebra to Enhance Students Learning Out Comes in Geometry Transformation at Mathematics Education University of Bengkulu

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Abstract: *This study aimed to enhance students learning out comes geometry through the implementation of inquiry based learning assisted GeoGebra. This study was classroom action research. The subject was the third semester students at Mathematics Education Study Programme of the University of Bengkulu in the academic year 2013/2014 consisting of fourty students. The data were gathered by using test, observation and interview. The steps in conducting the research were planning, implementation, observation, and reflection. The result showed that through the implementation of inquiry-based learning approach, student learning outcomes in terms of understanding the problem, solving the problem and providing a deep understanding have increased. The use of GeoGebra in the inquiry-based learning has facilitated students exploration and investigation to obtain the solution of the problem. The students were challenged to solve the geometric problems. They were given the opportunity to draw the image digitally and stimulated to perform each step in the discovery of construction.*

Key words: inquiry based learning. GeoGebra, geometry transformation

1. Introduction

Students of Mathematics Education as the mathematics teacher candidates are expected to have content and pedagogical knowledge in mathematics. An understanding of the content of geometry as one of the content in school mathematics should be learned by the students. Prospective teachers need to improve not only mastery of the content of geometry but also pedagogical knowledge, skills, and creativity that related to problem solving. Understanding the concept of properties and the properties of relationships between geometric objects, geometric construction skills, and problem solving skills of geometry should have been developed in this course. From the observations on the geometry class in Mathematics Education Bengkulu University, it was found that students have lack of problem solving competence especially understanding the problem solving, modeling problems, and interpreting the solution. One of the instructional strategies that could be used to solve the problem above is inquiry learning. The stage of inquiry learning generally includes presentation of the problem, hypothesis formulation, data collection, organization and data processing, analysis, inference and interpretation.

This study aimed to enhance students learning out comes using inquiry based learning assissted by Geogebra. GeoGebra is one of the software used in the study of mathematics. The GeoGebra software is dynamic mathematics software that combines geometry, algebra, and calculus. This software is developed for learning school mathematics by Markus Hohenwarterat Florida Atlantic University. This software can be used in the process of solving the problem because it has features as media demonstration and visualization, construction tools of mathematical concepts, and tools for students to find a mathematical concept. This research sought to answer the following questions:

1. How is the implementation of inquiry learning GeoGebra assisted geometry to improve learning out comes for students?
2. How is the description of the learning outcomes of geometry that achieved by students?

2. Learning Geometry

The development of geometry began with human efforts to understand about the size of the angle, length, circumference, and the area of an object that surrounds them. The geometry grew and developed both in terms of geometry and contents of applications in real life. The developments of geometry involved many areas out side geometry. In Walle, (2008) showed the relationship among the measurement of geometry, algebra, proportional reasoning, and numbers. For example, an understanding of fractions can be represented using geometric shapes. In learning geometry, Prince & Felder, (2007) suggested several strategies for geometry instruction: (1) Encouraging students to see the things from the favorable geometry; (2) Using the geometry as a means of problem-solving activities; (3) illustrating the properties of geometry; (4) Using models for illustration and comparison; and (5) Encouraging students to see, touch, and manipulate geometry models in order toget a visual impress. These strategies could be used to introduce new concepts, to expand and to reinforce the concept that has been known by the students in different ways, so the students were motivated to think analytically, critically, and creatively.

Ingeometry instruction at different levels of education, Walle, (2008) classified the content of geometry into four categories: (1) The shape, properties and relationships of shapes and geometric properties of two-dimensional and three-dimensional, (2) transformation or a change of location geometry objects that include translation, reflection, rotation, symmetry, and similarity; (3) the location of an object that refers to the geometry coordinate geometry; (4) Visualization geometry include the introduction of shapes in the environment, the development of relationships between objects of two-dimensional and three-dimensional, and the ability to draw and recognize shapes from a variety of viewpoints.

Learning geometry discussed about objects can be found in Euclidean geometry, analytic geometry and transformation geometry. The Euclidean geomatry discussed about the properties and the relationship between geometric shape such as point, line, plane, and space. Furthermore, Euclidean geometry also cover the shape, properties and relationships of shapes and geometric properties of two-dimensional and three-dimensional and the introduction of the forms in the environment, the development of relationships between objects of two-dimensional and three-dimensional, and the ability to draw and recognize shapes from various viewpoints. The analytic geometry developed by René Descartes was a discussion of algebraic geometry by using coordinates, such as properties and form a straight line graph of the equation in the Cartesian coordinate system. Analytic geometry course in schools aimed at promoting the geometric description of the object and the geometric position to determine how the geometry of objects located in the area or space. Geometry transformation aims to study the changes in the geometry of the location of objects. Changes in these locations included the translation, reflection, rotation, symmetry, and similarity in the two and or three dimension.

3. Inquiry-based learning

One of the best ways to motivate students is inductive teaching, where teachers begin the lesson by presenting a specific challenge to the students, such as the result data of experimental should be interpreted, a case to be analyzed, or even real-world complex problems to be solved (Gary, Yung, & Wai, 2006).

There are various types of inductive teaching such as discovery learning, inquiry-based learning, problem-based learning, project-based learning, case-based teaching, andjust-in-time teaching. The main characteristics of inquiry-based learning (Borich, Hao, & Aw, 2006) areas follows: (1) The lesson begins with an interesting and challenging questions that are designed to improve the outcomes of high order analysis, synthesis, and evaluation, but it does not make students frustrated; (2)The question is open-ended so that students are given the opportunity to provide a wide range of alternative answers, in order to focus the learning process on the way to find something (how to know) not what you know (what to know); (3) Learning is student-centered; and (4); The process of inquiry is a spiral rather than linear; and (5) Students are judged not only on the answer, but also how to obtain the answer by using the experience and knowledge of the students and supported with learning resources to develop a new idea presented in the questions to make learning more meaningful. The inquiry-based learning gives students the opportunity to discover for them selves all the concepts they learned, students understand the concepts being studied better and the concept becomes long lasting to remember. It also involves optimally throughout the students' ability to seek out and investigate something systematically, critically, logically, and analytically, so that they can formulate their own inventions with confidence. Further, in Borich, Hao, & Aw, (2006) describes the process of inquiry-based learning in a spiral as shown below.

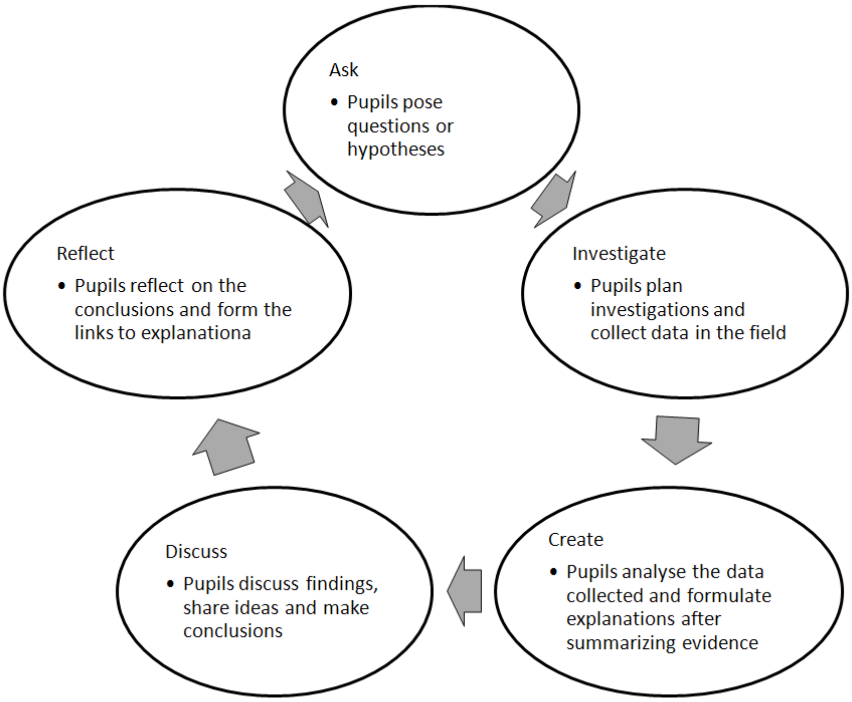


Figure 1. Inquiry-Based Learning by Boriset al. (2006: 32)

The use of computer technology in the learning process should be able to help students explore questions and ideas in a variety of way. In Borich, Hao, & Aw, (2006) stated that the inquiry-based learning, use of technology as cognitive tools to help students to present, share, and communicate ideas. Futhermore, which must be considered in the use of technology in inquiry-based learning was"how to use available technologies effectively?". This means that every available technology can be utilized effectivelyfor aprocessof inquiry.

The use of software Geo Gebrais as a learning tool which gives advantages including: (1) Constructing shape of geometry can be generated quickly and accurately than by using a pencil, a ruler, or the compass; (2) the existence of animation features and manipulation movements dragging GeoGebraprogramcan providea clearervisual experience for the students in understanding the concepts of geometry; (3) Can be used as a feedback/evaluation to ensure that the paintings that have been made correctly; and (4) Let the teacher/student investigate or show properties applied on an object geometry easily.

In Walle, (2008) said how the software was used for the teaching of mathematics would depend on the topic, grade level, and the software it self. As for some of the considerations in the use of software were as follow: (1) the software should support the learning objectives or units. (2) Provide specific instruction on how software was used and planned so that students can freely explore or practice using software (3) Combine activities using software with no computer activity. (4) Create a plan of arrangement for the use of the software. GeoGebra assisted learning is learning that began with preparing the material relevant to the concept to be learned and the learning of the students work in groups with the teacher as facilitator.

4. Learning Outcome

In Sudjana, (2009) said that the learning out comes were the abilities of the students after they received a learning experience. BenjaminS. In Sobel & Maletsky, (1999) argued that the taxonomy of educational objectives that must always refer to the three types of domains that were attached to self-learners, namely: the domain of cognitive thought processes, (2) the domain of values or attitudes (affective), and (3) the domain ofskills (psychomotor). Cognitive domains related to intellectual learning outcomes of six aspects are knowledge or memory, comprehension, application, analysis, synthesis, and evaluation. Affective domains related to the attitude of five aspects are acceptance, answers or reactions, judgement, organization, and internalization. Psychomotor domain of learning outcomes related to the skills and abilities to act. Three of these the domain became the object of assessment of learning out comes. In Dimyati & Mudjiono, (2006) argued that the learning out comes were the result of an interaction act of learning and teaching acts after the learning process. The act of teaching that teachers do included with an evaluation of learning out comes. As for students, learning out comes was the culmination of the learning process.

5. The research method

The research was conducted in Mathematics Education Study Program Faculty of Teacher Training Education University of Bengkulu. The subject of this study is the Mathematics Education Program students in third semester who took the courses Geometry Transformations in the academic year 2013/2014. This research was conducted using classroom action research design through the procedure planning, implementation, observation, and reflection.The instruments in this study are questionnaires, tests, and observation sheets. As for determining indicators of success were improved learning outcomes, the ability to analyze problems, the ability to use the strategy of solution, and the ability to determine the answer.

6. Results and Discussion

The results of the research conducted in two cycles as the following explanation. Learning the inquiry approach implemented either manually or paper and pencil approach to the inquiry withGeoGebraapproach as a tool in the form of a worksheet. It started by giving the following problems.

> A young man was in a tent near the lake. When he was walking around out side the tent carrying an empty bucket, suddenly he saw his tent was fire. To extinguish the fire, the young man first ran to the lake to fill the bucket with water and then he ran to bring a bucket of water and poured water on the tents. How did a trip he do in order to have the shortest distance?

Based on the observation, at the beginning there were 22 students from37 students in the classroom did not understand the problem, the answer was also nodoubt, the students confused. Then the students were guided by guiding questions to understand the problem like: How to position people with tents? What was known? What was asked? Most students know what was known and what was being asked, and then some students described the sketch. By emphasizing that the problem was to determine the shortest path to walk to get water into the lake to get to the tent to put out the fire. Some alternative sketches made in determining the shortest path as follows

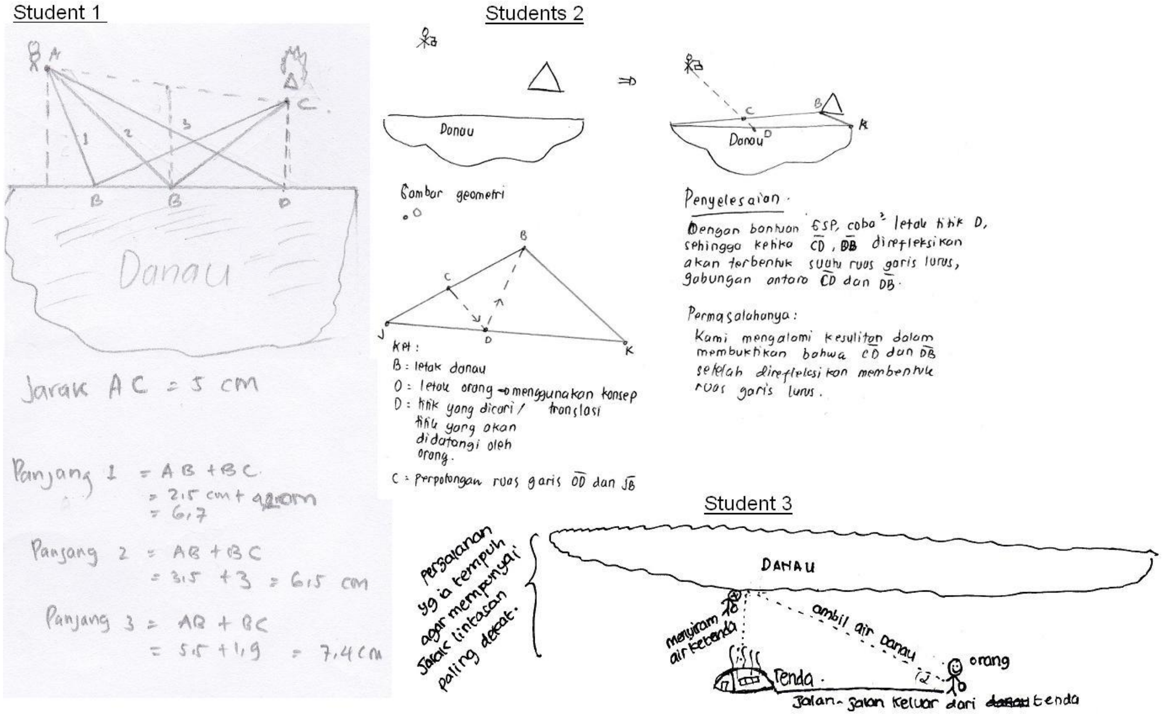
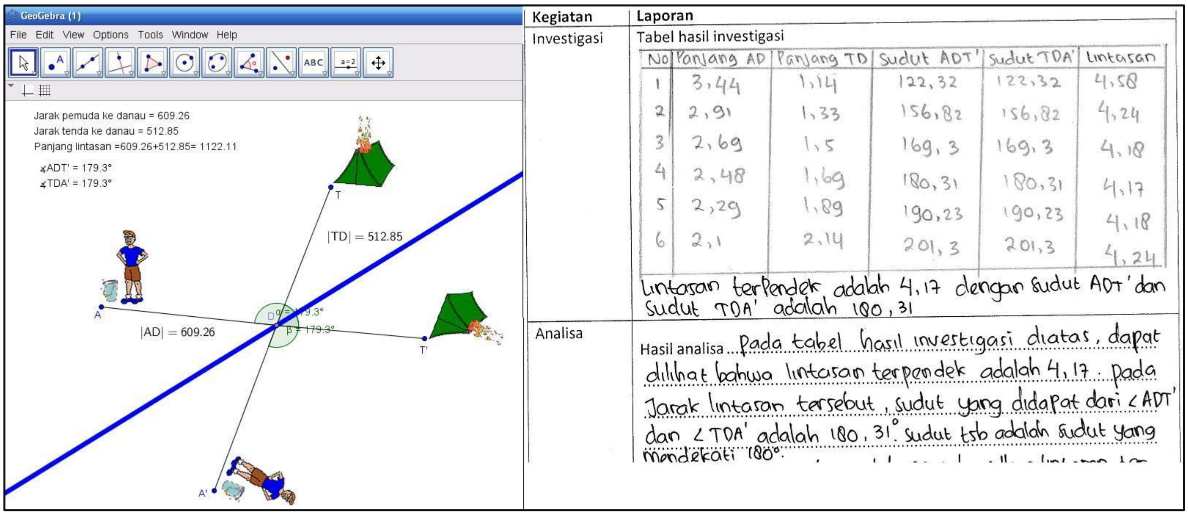


Figure 2. Alternative Sketches of Student to Solve the Fire Tent Problem

Students were actively discussing and drawing various alternative possible answers, but no students found the shortest path even though it was guided by questions. Next the students used the software GeoGebra to draw, every move the position of the direct path students could see the size numerically to obtain the shortest path, but students cannot give any reason why that position was the shortest path. When students were asked to see that the boundary surface of the lake as a reflection line, students easily found the image that occured when the shortest path from the lake to track the tents in line with the position of theshadow path runs into the lake. Next, lecturers and students demonstrated the properties of reflection, and its application to solve the problem. Futhermore, students were given an open problem made more simple, involving the number but still stimulate students to work. Students completed a conventional modeling with paper and pencil and then they used GeoGebrain the topic rotation. The figure below shows how geogebra helped student to solve the problem.



Kegiatan	Laporan
Investigasi	Tabel hasil investigasi

No	Panjang AD	Panjang TD	Sudut ADT'	Sudut TDA'	Lintasan
1	3,44	1,14	122,32	122,32	4,58
2	2,91	1,33	156,82	156,82	4,24
3	2,69	1,5	169,3	169,3	4,18
4	2,48	1,69	180,31	180,31	4,17
5	2,29	1,89	190,23	190,23	4,18
6	2,1	2,14	201,3	201,3	4,24

Lintasan terpendek adalah 4,17 dengan sudut ADT' dan sudut TDA' adalah 180,31

Analisa

Hasil analisa ... Pada tabel hasil investigasi diatas, dapat dilihat bahwa lintasan terpendek adalah 4,17. Pada jarak lintasan tersebut, sudut yang didapat dari ∠ADT' dan ∠TDA' adalah 180,31°. sudut tsb adalah sudut yang mendekati 180°.

Figure 3. Investigation Using Geogebrafor the Fire Tent Problem

The students easily got the data and found the association between the length of path and angles of reflection. After they got the data and collected it in the table, they analyzed it and made a conclusion for solving the problem.

In the second cycle, there was teaching rotation that began with giving the following problems.

> There are two congruences quares which side is 6cm in lengt has shown below.One vertex of one square located at the intercept of diagonal of another square. If OABC square rotated with the center point O which is the center of the square, what is the largest area of the shaded region?

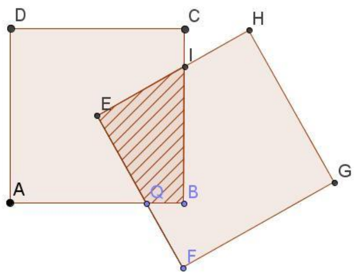


Figure 4. The Area of Intersection of Two Congruent Squares.

In groups of 3-4students, they dicussed tounderstand the problems and find solutions, but each individual student was asked to report process of finding solution.

The observation of the classroom when learning took place showed that students understood the problem easier. Besides, students participated in the discussion and problem solving actively and creatively. Students were given the opportunity to solve problem by their own. The three approaches could be distinguished, who either work using manual drawing, using simulation, and using geometry software. Most of the groups in class used direct drawing on the worksheets while three groups in the class used paper cuts as simulation, and last two groups tried to solve the problem by using dynamic geometry software. Most of students found the same answer that the area of the shaded square was a quarter or directly get the results numerically. Only few of respondents said that the biggest is a quarter square. Students draw using a rotation with the center point of intersection of the diagonal with a special angle was the angle when rotated at an angle of 90 degrees in the position it was clear that the area shaded by a quarter of a square. Then the square shadow played again with the rotation angle of 90°from the original shape rotated 180°also produces a quarter of the area shaded square, then shape with a big square played any rotation angle.By measuring the area at position three different rotation angles, resulting the shaded area was the same. So that the students believed that the area of the shaded was the same. The result of students work could be shown by the following figures.

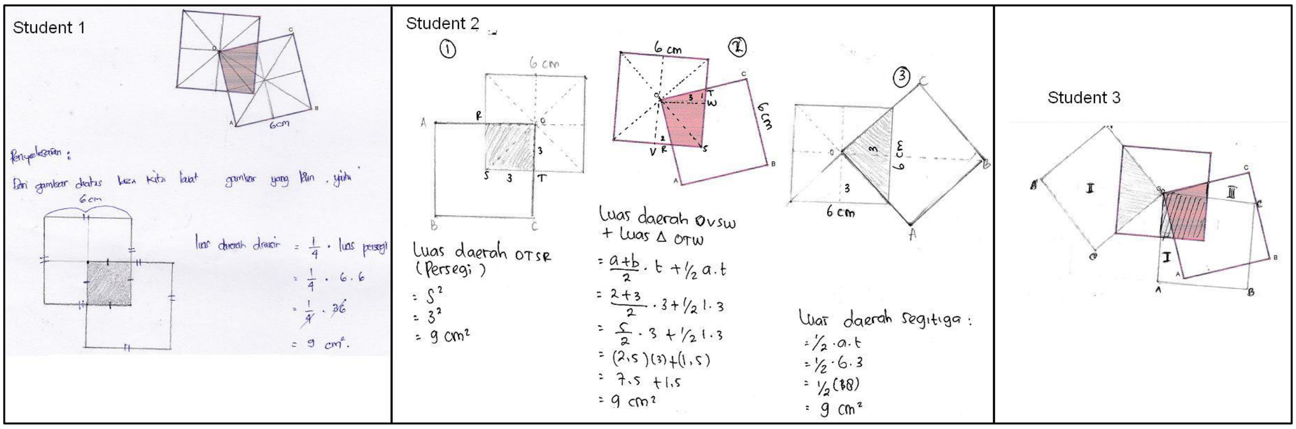


Figure 5. The Results of Students Work to Solve the Second Problem

The figure above shows three ways to solve the second problem which students did. The students made the sketches by imagination the rotation and used some segments to help them made the rotation.

Students investigated further how the relationship between theorig in shape and the shaded areas overlap with areas overlapping rotation results. Students were guided with questions to lead the student found the relation. After going through various trials of each square side eventually students found that, other than through a half turn rotation turns displacement shaded areas could be done in other ways. The way it was to reflect on one side of thesquare through O, then the results reflected shadow again with the other side on the same square and through O so that the shadow was the same, and the image obtained when the shaded area in the center rotate half a turn with O. Students could conclude that the rotation of the half-round relationship equal to the product of two reflection where the angle between the two reflections at 90degrees or intersect at point O could be expressed symbolically as $R(O, 180) = M_{oc}M_{ob}$.

Then they continued to investigate by constructing the shape using GeoGebra software. By rotating the shaded area in the central area of O, it could be read that the area was the same, namely ¼ square areas. Students could easily show the relationship between the angles of rotation by the angle between the two reflections. They could show that a large rotation with rotation angle twice the angle between two large reflection was equal to the product of two reflections with the second point intersection between two line reflection as the center point of rotation.The figure below shows the example investigation that a student done by Geogebra.

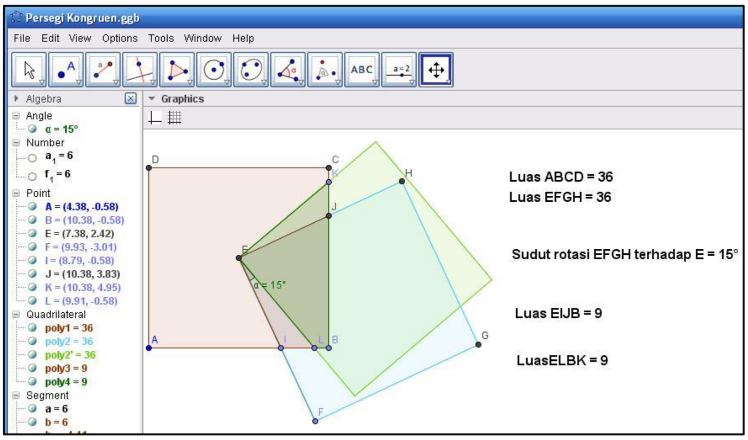


Figure 6. The Example of Investigation Process Using GeoGebra for Solve the Second Problem

After they made the experiment by geogebra, they had to put the data the table and analyze them to find the solution. The next figure is the example of the investigation results in this activity.

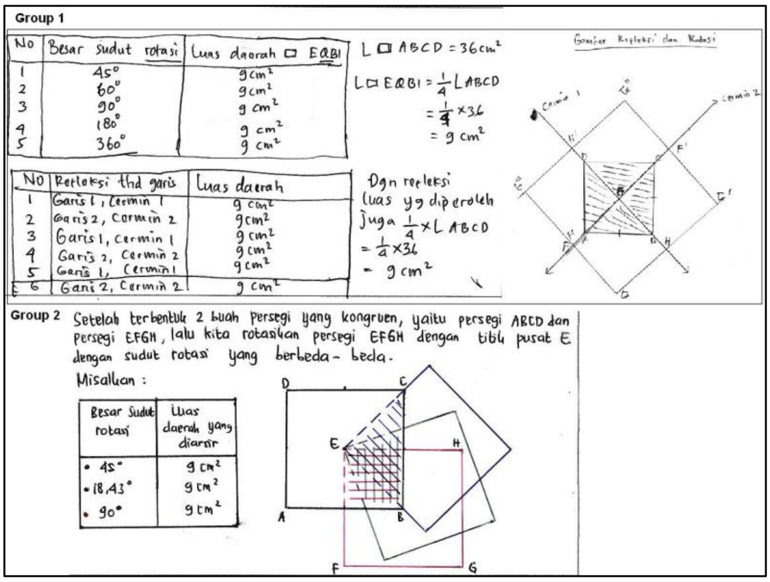


Figure 7. The Result of Investigation Process Using Geogebra to Solve The Second Problem

At the end of the research, students were given a test to determine the abilityto analyze problems, the ability to use the strategy of solution, and the ability to determine the answer. The problem of the test is:

> There are two houses A and B located near two crossed paths. A man wants to create another way to relate house A with a point P on the first path and then relate point P with point Q on the second path and finally relate point Q with house B. How do you relate A – P – Q – B in the shortest way? Explain!)

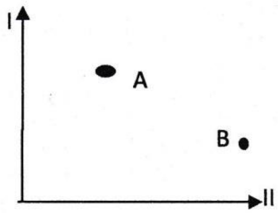


Figure 8. Sketch Test Problem

There are some alternative ways to solve the problem both of the right way or the wrong one. Next figure show two results of the test, at the left is the right answer and the right is exactly true.

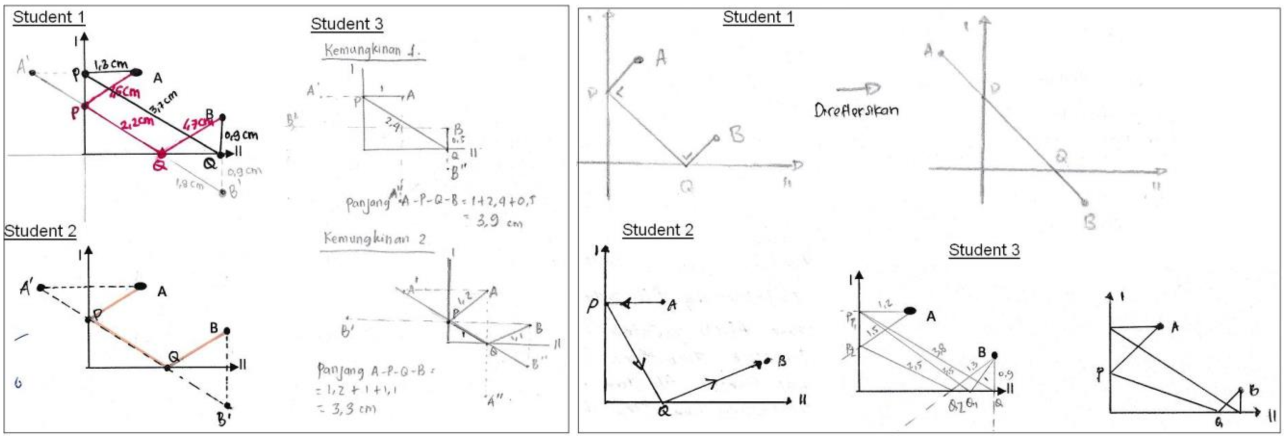


Figure 9. The Sketches of Answers for Test Problem

Through giving the problems in learning the students were challenged in solving the process to get their solution. For example, a student commented, that the given problem was quite challenging, requiring a high logical reason, appealing to be solved by experimenting drawing with the correct scale to help them get the right answer. By some explanation from the lecturer, the students quickly could work adequately and were satisfied with the results.The results of the test after learning showed that the average value was increasing. In general, the students stated that the use of geogebra were very helpful because it simulate moving images accompanied by an accurate measure.

Inquiry approach in the course of geometry transformation required the involvement ofstudents' ability in understanding and solving the problem in order to find the answer. The material was not given directly, but through a process of students' involvement to find the answer. The process of inquiry learning with the GeoGebra assisted saved more time than common learning, providing a direct measure with highly accutared result to help students' work. In this case, the lecturer will give more guidance in small groups or individual students while working in front of a computer to work with, so they could find their own experiences.

The results of observations of two cycles of inquiry approach in the geometry transformations material resulted improving the learning out comes, increasing the understanding of the problem, and increasing the creativity in solving problems to get the right answer,but the process of finding required a longer time thanthe common lesson. This, the thinking process was mostly done by the student while the lecturer gave some questions as a guide to find the answer. In understanding the problem at the beginning of the lessons the students faced the difficultness to understand the question of what an open problem without involving a number of unusual faces. The ability of students to understand the problems varied, most students were confused with open problems. But all students were able to link the problem with describing it geometrically.

Both cycles, the first and second cycle, in solving the problem most students used strategy to simplify the problem, drawing, and discussed in the group. Students tended to work in a way to make the analogy counting numbers or pictures that was easy to do, then simulate to find the answer. There were not many reason mentioned by students during the process of learning, but their work was more concerned with the end of the result. When solving the problems with the GeoGebra assisted was done, there was repetition of steps were taken, because the students needed to equip themselves with the skils first to operate GeoGebra as a learning tool. GeoGebra has encouraged students to have inisiative to discover for them selves how the steps to go through to get the answer of a problem. This was in accordance with the principles of inquiry approach that focused on the process of finding come from the students themselves.

The use of GeoGebra in transformation geometry provided opportunities for students to perform open exploration of a given task, to train skilled students visualize images digitally, and to stimulate students to perform discovery of every step of the progress. Paper and pencil to draw were not limited dynamic form. Advantages that exist in GeoGebra software make students are able to try to draw the appropriate movement for transformation geometry as the geometry moves object as translation, reflection, and rotation.

Finally, students could use GeoGebraas an effective tool to help them to solve the problem of transformation geometry. Similarly, student scould also present the results of their work featured in the computer screen as the accurate size. By using GeoGebra, students are helped to solve the problem and to do some searching exploration of the problem solving approach hat supports the use of it. Students, either individually or in groups, involved actively and seriously in doing their works. In looking for solutions, the students conducted discussions and draw a picture to find the answer. There was a tried out the software, perform simulations, and drawing with paper and pencil, so they learned actively.

7. Conclusions and Recommendations

Based on the result and discussionof learning proses above, it could be concluded that ;(1.) The use ofGeoGebra in the inquiry pproach to the study of geometry transformation could help students to facilitate the exploration, as a survey found out, and was challenged to solve the problems that were assigned in understanding the concepts than with paper and pencil drawings; (2) Through the implementation of inquiry approach to increase student learning out comes in thinking skills to understand the problem, initiatively itself to solve the problem and provide a longer memory because more attention through the settlement process. By providing as much opportunity as possible for the student to carryout a direct investigation of the content being studied so students could find the concept itself was not based on memorization of mathematical formulas. By understanding the concept of the material given, even there were varies of type, students could solve the problems well.

In implementing the inquiry approach, there should be appropriate selected topic because this approach took longer time and more emphasis to get students'inititiative on the process of finding solution rather than the end result. Thus, each subject should not be taught with inquiry approaches. Also the study needed further research to develope model of open questions that stimulate students to find a new concept to be studied. It was not easy to teach the concept or principle indirectly. There must be an ongoing process to find the students' solution.

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Integration of Autograph in Improving Mathematical Problem Solving and Mathematical Connection Ability Using Cooperative Learning Think-Pair-Share

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Abstract. *The aim of this study was to investigate students' mathematical problem solving and mathematical connection ability in cooperative learning setting using Dynamic Software Autograph. This experimental study was conducted at high school in learning statistics. The collection of the data was done using observation sheets, documentation, attitude scale, and performance tests. Repeated measure tests were delivered to students for four times. The result of the analysis showed that: (1) Using Dynamic Software Autograph in teaching learning statistics with cooperative learning Think-Pair-Share improved students' problem solving and mathematical connection ability; (2) Students' activity during teaching learning processes continuously improved; (3) The result of analysis of the questionnaire showed that most students like learning statistics using cooperative learning with dynamic software Autograph; (4) Students were very active and showed positive attitude toward learning using cooperative learning Think-Pair-Share using dynamic software Autograph.*

Keywords : Dynamic Software Autograph, Cooperative Learning, Mathematical Problem Solving, Mathematical connection

1. INTRODUCTION

The rapid advancement of sciences and technology has brought about many changes, challenges, and competitiveness in how people learn and adapt themselves in the process of teaching and learning. Numerous studies have shown the positive impact of integrating technological tool in the teaching and learning process of mathematics in the classroom. But, in reality, there are numerous problems encountered in its implementation in the classrooms. To overcome these problems, mathematics educators should keep up with the teaching challenges by preparing not only themselves but also their students to improve their competitiveness. In this era of information and technology, they need to prepare, understand, and apply their knowledge and skills in the workplace. In fact, there are a number of potential benefits of using the computer as a tool for instruction in an educational setting. Firstly, technological tools help to support cognitive processes by reducing the memory load of a student and by encouraging awareness of the problem-solving process. Secondly, tools can share the cognitive load by reducing the time that students spend on computation. Thirdly, the tools allow students to engage in mathematics that would otherwise be out of reach, thereby stretching students' opportunities. Fourthly, tools support logical reasoning and hypothesis testing by allowing students to test conjectures easily (Kurz, Midleton & Yanik, 2005). Instructionally, computers allow for a record of problem-solving processes to be recorded and replayed as a window into children's thinking, computer also offer interactive learning in promoting students' high order thinking skills and showing them the role of Mathematics in interdisciplinary setting in objective to value the connections between Mathematics and other disciplines. Technology indeed has changed the way classrooms operate, integrating multimedia during learning, online accessibility thus making teaching and learning more interactive and participatory (Martin, 2006).

In educational practices, the use of technology in Indonesia is still in the initial stage. Although data on the actual use of technology in schools have never been comprehensively surveyed, the use of computer is primarily for administrative purposes. It has not yet been widely implemented as an integral part of teaching- learning mathematics at schools. The teaching-learning process is still being dominated by conventional teaching that emphases on practice in manipulating expressions and practicing algorithm as a precursor to solving problem. Due to these problems may, as has been extensively discussed in the Indonesia's newspaper, most teachers focus almost exclusively on teaching how to solve the specific mathematics most likely to be found in the national examination. Therefore, their approach doesn't even attempt to foster true understanding of the Mathematics concepts and developing thinking. Most of teachers used expository in Mathematics instruction, spent time for problem solving only 32% from all time in class, teachers talk more than students, almost of the test use routine-problem and not challenging for student, most of the teachers teach depending with the textbook, and do not have the skills to make a activities, good questions or tests.

Enhancing the ability of the students in mathematical problem solving and mathematical connection can be implemented by using technology such a dynamic software as a tool to help students link the concrete and the symbolic of mathematics. The use of dynamic software can make this easier. One of the dynamic softwares introduced in mathematics classroom is Autograph, that has features to help students solve problems in statistics. The "drag" and various "zoom" options can be used as appropriate to get a clearer picture to make students get more'visible' with the statistical results and the free interaction of students with the computer makes the learning could be more opportunities to do exploration and investigation in developing their thinking.

2. Integrating Technology in Mathematics Learning

Technology becomes a more prevalent part of education culture with each passing year. Schools cannot ignore the impact of technology and the changing face of curriculum. The calculators, computer, and other technologies are as important tools for generating discussions in the mathematics classroom. The technology principle in Principles and Standards for School Mathematics states that *"technology is essential in teaching and learning mathematics, it influences the mathematics that is taught and enhances students learning"* (NCTM, 1989: 24). Wright in Alexiou-Ray, Wilson, Wright & Peirano (2003) reports higher student achievement, self concept, attitude and teacher-student interaction as a result of interactive learning has made possible via technology. Kerrigan in Alexiou-Ray, et.al. (2003) also has found the benefit of using mathematics software and websites to include promoting students' high-order thinking skills, developing and maintaining their computational skills, introducing them to collection and analysis data, facilitating their algebraic and geometric thinking, and showing them the role of mathematics in a interdisciplinary setting.

Resnick (2001) stated that teachers should maintain that new technology changing not only what students should learn, but also what they can learn. This means that concepts and ideas that might have been accessible can now be experienced and understood in the context of learning with information and communication technology. Technology not only influences how mathematics is taught and learned but also affects what is taught where a topic appears in the curriculum.

Introducing Dynamic Software Autograph

Dynamic software is often employed as a fertile learning environment in which students can be actively engaged in constructing and exploring mathematical ideas (Cuoco & Goldenberg, 1996).

Autograph is a Dynamic Software for teaching mathematics such as calculus, algebra and coordinate geometry, Autograph is a dynamic and very versatile software for teaching and learning secondary mathematics developed by Douglas Butler (2005). It is designed to help students and teachers visualize mathematics at secondary/college level, using dynamically linked ‘objects’.

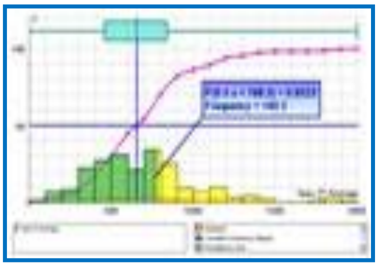


Figure 2. 1D for Statistics

Autograph leads the way in the use of dependent, selectable mathematics objects to help student get to grips with the principles of probability and statistics, coordinate geometry, in both 2D and 3D. Autograph is able to sketch curves (both implicitly and explicitly defined) solve simultaneous equations, plot derivatives, etc. Autograph is a new dynamic PC program operating in 3 modes, those are 1D for Statistics & Probability, 2D for

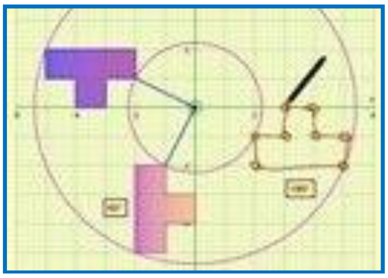


Figure 3. 2D Geometry

Graphing, Coordinates, Transformations and Bivariates Data and 3D for Graphing, Coordinates and Transformation in three dimensions.

Autograph has two levels of operation, '*standard*' and '*advanced*' operation. The 'standard' level has a greatly simplified interface and reduced set options for the user.

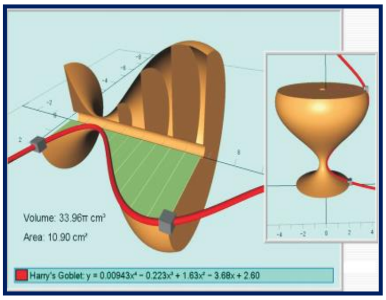


Figure 4. 3D Algebra

At the 'Advanced' level, more challenging problem and investigations can be explored. Autograph is the ideal solution for the instructor looking to bring mathematics to life. Whether through true-to-form animations or through student-driven exploration, its powerful features and point-and-click technology will engage all levels of students. By using technology, users can observe how functions, graphs, equations, and calculations work. It also enables users to change and animate graphs, shapes, and vectors already plotted to encourage understanding of concepts. In mathematics class the use of mathematical software enable students to visualize and further understand mathematical phenomenon in real life.

There is plenty of evidence now that teaching secondary and college level mathematics with dynamic software can be more effective, can be more efficient, and above all can be more enjoyable — for both teacher and student. Autograph leads the way in the use of dependent, selectable mathematical objects to help students come to grips with the basic principles of probability and statistics, and of coordinate geometry in both 2D and 3D.

The interactive features of Autograph allow students to become engaged in the mathematics through teacher-led lessons or in explorations where the answer are found by the students themselves. Teaching by integrating Autograph in schools might increase the effectiveness and the quality of teaching. As mathematics class needs lots of interaction, reasoning, observation the above view clearly indicates that interactive software like Autograph can be useful in teaching and learning mathematics effectively. Autograph "is an extremely useful educational tool for both mathematics teachers and students which help teachers to present the content for the whole class easily and students understand better due to its visual demonstration" (Tarmidzi, Ayub, Abubakar & Yunus, 2008:186).

The introduction of the computer into the school classroom has brought a new technique to teaching, the technique of simulation. In the context of teaching statistics, computer simulation enables students to generate data having the essential characteristic of variability and to follow processes run at a convenient speed which may be faster or slower than in the real instance. The student can now be placed in a decision-making role, a role in which he has to make statistical inferences based on data presented to him and then to take the appropriate actions. It is an essential feature of the interactive simulation that the next set of data generated and presented will depend on the student's previous decisions.

3. Mathematical Problem Solving and Mathematical Connection

Mathematical Problem Solving. Problem solving is an integral part of all mathematics learning. In everyday life and in the workplace, being able to solve problems can lead to great advantages. However, problem solving is not only a goal of learning mathematics, but it is also a major means of doing so. NCTM emphasizes that problem solving has a special importance in the study of mathematics. Furthermore, NCTM states that problem solving should underlie all aspects of mathematics teaching in order to give students experience of the power of mathematics in the world around them. The council sees problem solving as a vehicle for students to construct, evaluate and refine their own theories about mathematics and the theories of others. Unfortunately, from of all of the reform recommendations being made by the NCTM, making mathematical connections is among the more difficult to achieve, yet is so helpful in motivating students in the early grades. Mathematical connections can relate mathematical topics to students' daily lives and to other mathematical topics but are probably most important in relating mathematics to other curriculum areas. The NCTM standard stated two general types of connections, that is modeling connections between problem situations that may arise in the real world or in disciplines other than mathematics and their mathematical representations; and mathematical connections between two equivalent representations and between corresponding processes in each. Many countries emphasized the role of modeling and applications in mathematics education. It became a major objective that students should be able to apply mathematical problem-solving and reasoning skills and attitudes in real-life and scientific situations. It may be concluded that there is a strong relation between mathematical connection and the ability of the student to solving a problem.

Problem solving is one component of higher level thinking. To think effectively, students also need to develop their meta-cognitive skills of planning, monitoring and evaluating so they can plan, monitor and evaluate their academic work. Cooperative learning is an especially effective method of spontaneously activating meta-cognitive aspects of thinking, learning and problem solving.

Studies comparing cooperative learning with competitive and individualistic learning demonstrated that cooperative learning promotes higher achievement than the other two methods. Not only students solve problems more successfully and learn and retain concepts, but cooperative learning also results in more use of higher level thinking, more frequent discovery, generating new ideas and solution strategies, and more transfer of what is learned about problem solving in groups to individual problem solving situations (Johnson, Johnson & Holubec, 1994). These benefits are a result of students internalizing concepts and problem solving through their discussions and explanations of problem solving strategies and approaches with their peers. Giving explanations to other students requires deeper understanding than just putting an answer on a worksheet. Finally, many research found that cooperative group instruction shows significantly better results in mathematics achievement and problem solving skill.

Many studies in the field of educational technology have demonstrated how the personal computer assisted cooperative learning provides better results than traditional learning in a competitive, individualist setting. Personal computers are especially suited, among other roles, to that of initiating and sustaining cooperative and collaborative learning. Recent curriculum reforms suggest that students need to utilize distinct technological tools in their process of learning mathematics.

In this study, mathematical problem solving is defined as the ability of the students to use problem solving approaches to investigate and understand mathematical content, apply integrated mathematical problem-solving strategies to solve problems from within and outside mathematics, recognize and formulate problems from situations within and outside mathematics, and to apply the process of mathematical modeling to real world problem situation. All of this ability carried out with the following steps: (1) understanding the problem; (2) devising a plan for finding a solution; (3) implementing the plan; and (4) looking back at the answer.

Mathematical Connection. Mathematical Connections is the ability of the student to: (1) recognize equivalent representations of the same topics, (2) relate procedures in one representation to procedures in an equivalent representation, and (3) use and appreciate the connections between mathematics and other disciplines. The cooperative learning with the Think-Pair Share method can be describe with the activity of 2 students with one problem share their ideas or questions, each person speaks, listens and gives feedback, and then the students will asses individually.

Cooperative Learning Think-Pair-Share (TPS)

Cooperative learning Think-Pair-Share (TPS) was originally developed by Lyman (1981). Think-Pair-Share allows for students to contemplate a posed question or problem silently. The student may write down thoughts or simply just brainstorm in his or her head. When prompted, the students pair up with a peer and discuss his or her idea (s) and then listens to the ideas of his or her partner. Following pair dialogue, the teacher solicits responses from the whole group (Wikipedia).

The think-pair-share strategy is a cooperative learning technique that encourages individual participation and is applicable across all grade levels and class sizes. Students think through questions using three distinct steps: (1) Students think independently about the question that has been posed, forming ideas of their own and then may write some thoughts in response to the question.; (2) Students are grouped in pairs to discuss their thoughts and changing their ideas with others; (3) Students share their ideas with a larger group, such as the whole class to refine their ideas.

Some studies showed that students need many opportunities to talk in a linguistically rich environment. Researchers have found that students' learning is enhanced when they have many opportunities to elaborate on ideas through talk (Pressley 1992). The think, pair, share strategy increases the kinds of personal communications that are necessary for students to internally process, organize, and retain ideas (Pimm, 1987). In sharing their ideas, students take ownership of their learning and negotiate meanings rather than rely solely on the teacher's authority (Cobb et al. 1991). Additional benefits of using the Think-Pair-Share strategy include: (1) the positive changes in students' self-esteem that occur when they listen to one another and respect others' ideas; (2) Students have the opportunity to learn higher-level thinking skills from their peers, gain the extra time or prompting they may need; and (3) gain confidence when reporting ideas to the whole class. The think, pair, share strategy can be used in a variety of contexts. However, to be effective, students must consider a question or issue and they should derive some benefit from thinking about it further with partners, such as when there are multiple correct answers to a question.

4. METHOD

The purpose of this study was to investigate: (1) whether the implementation of the use of dynamic software Autograph in cooperative learning type think-pair-share can enhance students' mathematical problem solving and mathematical connections; (2) the level of enhancement in students' mathematical problem solving and mathematical connections with the implementation of the use of dynamic software Autograph in cooperative learning type think-pair-share; (3) the student activity during the use of dynamic software Autograph in cooperative learning think-pair-share; and (4) student perception toward the use of dynamic software Autograph in cooperative learning type think-pair-share.

The target population of the study was 188 students of Grade XI in 4 classes in a Senior High School, Medan Indonesia. One class of science was selected randomly from the 4 classes. The sample of this study was determined with random sampling technique, with the name of the class as object of the drawing. One class of 34 students was selected as a sample in this study consisted of 27% high, 50% medium and 23% low. The students in this study were taught and learned statistics and probability by cooperative learning Think-Pair-Share method using Autograph.

The design of this study was one group-within treatment design. A within-subjects design is an experiment in which the same group of subjects serves in more than one treatment. Three treatments were conducted to the students. Students were tested after 2, 4 and 6 treatments.
In this study, each student was provided with one computer installed with Autograph software. In this phase, the students were required to explore and be familiar with the software and its functions in learning statistics. In the second phase, students were introduced to the basic concept of Statistics topic followed by the` teaching and learning using the software. This phase involves instruction using the constructivist approach where students actively explore and discover concept of statistics using modular activity using Think-Pair-Share approach.

The instructional materials for this study consisted of four sets of 2-hour lesson plans for teaching and learning of Statistics. The lesson plan was developed based on the following topics: introduction to statistics, measure of central tendency: mean, median, mode, bar graph, line graph and circular graph. Modules were developed to enable the students to learn statistics using cooperative learning Think-Pair-Share helped with Autograph with variety of contexts in learning statistics. To be effective, the teacher encouraged students to consider a question or issue and they should derive some benefit from thinking about it further with partners, such as when there are multiple correct answers to a question.

Before the study, the instruments were pilot tested to obtain the reliability, validity and the difficult level of the test. These instruments then were administered by giving the pre-test and the treatments for 8 meetings. Prior to the data collection, the students were taught on how to launch and use Autograph, so that the students would be able to use Autograph in learning statistics. At the end of the treatment, pot test was administered and then the data was analyzed.

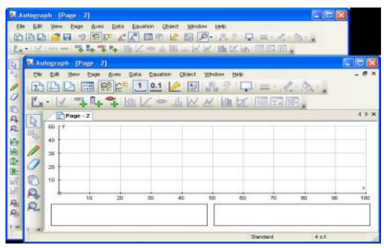
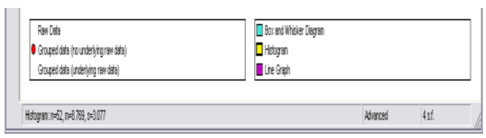
Before testing the hypotheses, the normality of the data of all the tests (mathematical problem solving and mathematical connection) was tested using *chi-square test.* After data analysis, it was found that all of the data were normally distributed. Testing the homogeneity of variance from the score of the pre-test and the post-test by using F-Test, it was found that the data were homogenous. Finally, ANOVA test was conducted to investigate the effect of using Autograph in enhancing the students' ability in mathematical problem solving and mathematical connections.

Two kinds of instruments were applied in this study, test and questionnaire. The test was used to measure the student's ability on the mathematical problem solving and mathematical connections. The questionnaire was used to find out students' perceptions on learning using dynamic software Autograph. The indicators of mathematical problem solving includes: using problem solving approaches to investigate and understand mathematical content; applying integrated mathematical problem solving strategies to solve problem from within and outside mathematics; recognizing and formulating problems from situations within and outside mathematics; and applying the process of mathematical modeling to real world problem situation. The indicators of mathematical connection: recognizing equivalent representations of the same topics; relating procedures in one representation to procedures in an equivalent representation; and using and appreciate the connections between mathematics and other disciplines. In evaluating the ability of mathematical problem solving and mathematical connections from the sample, analytic scoring rubrics was used in which the components of mathematical problem solving and mathematical connections were defined and scored separately based on the scoring purposes of these variables. These rubrics were divided into two folds, the scoring rubric for mathematical problem solving and mathematical connections. The questionnaire with five-point Likert Scale was used to investigate students' perception on the use of dynamic software Autograph. The observation sheets were used to know the student's activity during the learning process in using dynamic software Autograph in cooperative learning. The observation was conducted to observe the teaching learning process in the classroom and math lab. Three mathematics teachers acted as the observers.

Using Autograph 3.2 in Learning Basic Statistics

1. To launch Autograph, double click on the desktop Autograph 3.2 icon or you can launch Autograph from Start => Programs => Autograph 3.2

2. Select ADVANCED LEVEL from the Level Selector.

1	To launch Autograph, double-click on the desktop icon, or you can launch Autograph from Start => Programs => Autograph 3.20	
2	Select STANDARD OR **ADVANCED LEVEL** from the Level Selector	
4	The 1D Page : The Top Level Right Click MENU	
5	The **DATA KEY** (On the left) and the **OBJECT KEY** (on the right)	

6

The Edit Grouped Data Set dialogue box

- *with or without an underlying raw data set*

CLASS INTERVALS - *choose one of the three options:*

♦ **Enter Min, Max and Class Width**

The convention adopted is **a** ≤ **x** < **b**. If the final value of the final class is needed tick

♦ **Integer Data**

Use this if the classes are all integers enter, e.g. Lotto frequencies

(1-49), or dice scores (1-6). Equivalent to a class width = 1.

DATA TYPE

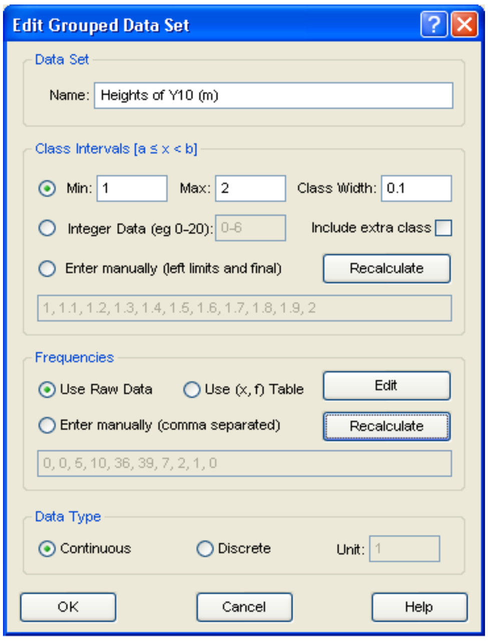
♦ **Continuous:** classes will be plotted at

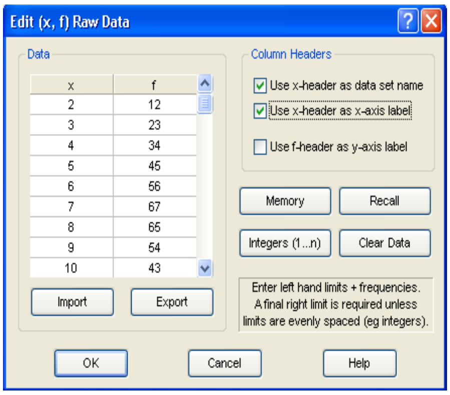
their exact value.

♦ **Discrete, by entered unit (e.g. 1):**

classes will be plotted shifted to the left

by the unit/2. Required for a Line Plot.





The Edit Raw Data Set dialogue box

- can be grouped or remain ungrouped.

Either: Enter the data one at a time, separated by "Enter".

♦ Or: Click on the top cell and paste a single column of data from a spreadsheet. If the top row is text, this will be taken as the "column header".

♦ Or: use Import - Export

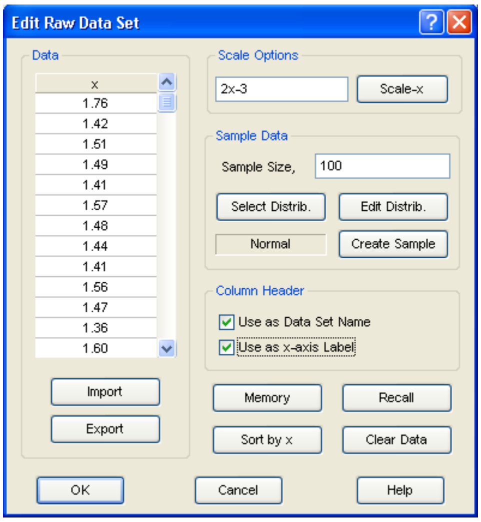
A single column of data can be imported from a saved CSV file (comma-separated value), which is compatible with Excel.

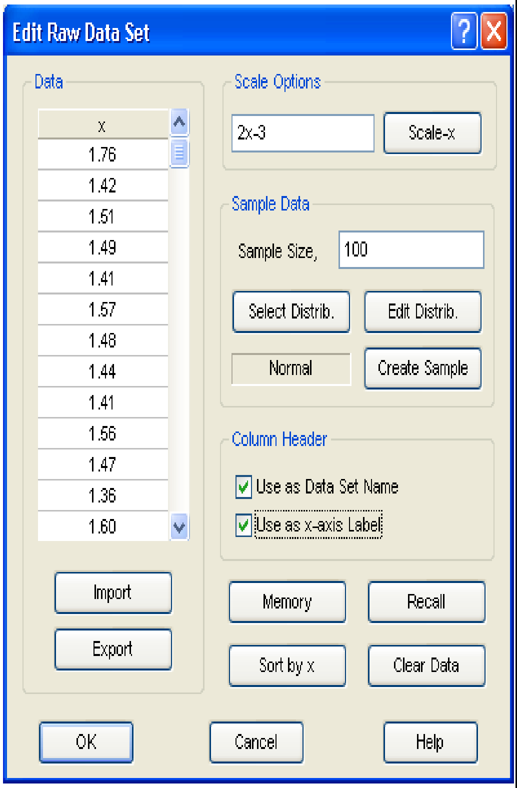
♦ USE (x,f) TABLE

This will enter classes and frequencies in column form. The 'x'column represents the start of each group, and the (optional) final value is the end of the last group.

These two columns can be pasted from a spreadsheet, and any text in the first row will become the column headers ("2-Dice scores" and "f" in the example). .

The headers can be edited (right-click option), and used as the data set name and x-axis label .





EDIT RAW DATA SET

♦ Either: **Enter** the data one at a time, separated by "Enter".

♦ Or: Click on the top cell and **paste a single column** of data from a spreadsheet. If the top row is text, this will be taken as the "column header".

♦ Or: use **Import - Export**

A single column of data can be imported from a saved CSV file (comma-separated value), which is compatible with Excel.

SCALE OPTIONS

Enter any f(x) formula here. Click "Scale-x" to scale the data.

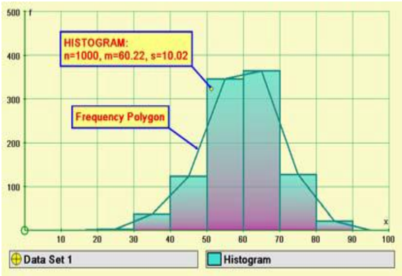
COLUMN HEADER

Use the header as the Data Set Name, and the x-axis label when plotted.

Use the right-click option to enter/edit the "column header".

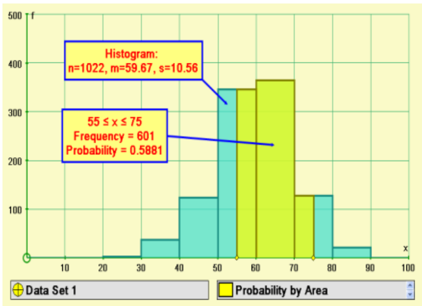
MANAGING THE RAW DATA

Use "**Memory**", "**Recall**", "**Clear Data**" and "**Sort by x**" as required.



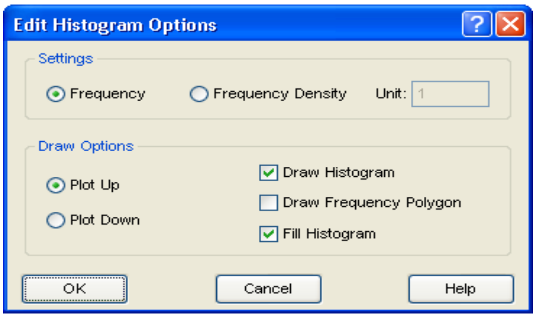
The Edit Raw Data Set dialogue box

- *can be grouped or remain ungrouped*



HISTOGRAM

A Histogram, plotting frequency or frequency density against ‘x’, can be created from any grouped data set (with or without underlying raw data).



Histogram “Probability by Area” calculation

In this example: the probability and frequency for the range $55 \leq x \leq 75$ are given. The yellow ‘diamonds’ can be dragged to vary the range dynamically.

5. RESULTS AND DISCUSSION

RESULTS

Pre-test. The data analyzed in this study were on students’ mathematical problem solving and mathematical connection ability after using dynamic software Autograph. Students’ activities and attitudes toward the teaching learning approach are also analyzed. The result of the pre-test was used to know the students’ prior ability with the purpose to group students into high and low groups. The results of the pre-test of the groups are shown in Table below.

The results showed that the average scores and standard deviation of the pretests on math problem solving and math connection are 74 (9.19) and 66 (10.81) with the lowest scores 55 and the highest 84 for math problem solving, and the lowest score of 42 and the highest score 92 for math connection.

Tabel 1. Pretest score of Math Problem Solving and Math Connection

Pretest	Group	N	Min	Max	Mean	Sd
PM	1 (upper)	17	75	84	80.94	3.07
	2 (lower)	17	55	77	66.12	6.65
	Total	34	55	84	74.00	9.19
KM	1 (upper)	17	92	76	80.0	5.35
	2 (lower)	17	42	75	62.24	9.46
	Total	34	42	92	66.00	10.81

Scores After Treatment . The results of analysis of the data on the second, the fourth and the sixth instructions on mathematical problem solving and mathematical connection are shown in Table 2 below.

Mathematical Problem Solving. Results of data analysis on student's problem solving ability in Table 2 shows that the average score of the upper level and the lower level students are increased. After 2, 4, and 6 times instructions the average scores on problem solving ability were increased from 82 to 84 and to 86 for upper level, while the average score on problem solving ability of students at the lower level are also increased from 70 to 75 and 79 on the 2nd, 4th, and 6th instruction.

Tabel 2. Data on Average Scores of Math Problem Solving and Math Connection

Instruction	Group	Math Problem Solving		Math Connection		
		Mean	Sd	Mean	Sd	N
2 times	Upper	82	3.25	76	10.16	17
	Lower	70	6.64	65	5.54	17
	Total	76	7.82	70	9.88	34
4 times	Upper	84	5.09	78	9.75	17
	Lower	75	6.95	69	7.07	17
	Total	80	7.47	74	9.51	34
6 times	Upper	86	4.22	80	7.25	17
	Lower	79	6.54	75	6.89	17
	Total	82	5.73	78	7.32	34

Hypotheses testing using test statistics parametric t-test was used to test the differences between the average scores on mathematical problem solving ability after 2, 4, 6 times of instructions with level of significant $\alpha = 0{,}05$ (two-tail test). The results showed that the use of Autograph keep increasing the students' math problem solving ability after 2, 4, and 6 times of instructions.

Mathematical Connection. Results of data analysis on student's mathematical connection in Table 2 shows that average score for the upper level and lower level students are increased. After 2, 4, and 6 times instructions the average scores was increased from 76, to 78, and to 80 for upper level, while the average score of students at the lower level are also increased from 65 to 69 and 75 on the 2nd, 4th, and 6th instruction. Hypotheses testing using test statistic parametric t-test was used to test the differences between the average scores on mathematical connection ability after 2, 4, 6 times of instructions with level of significant $\alpha = 0{,}05$ (two-tail test). The results showed that the use of Autograph keep increasing the students' mathematical connection.

6. DISCUSSION

The data collected from this study was analyzed to obtain an interpretation that can answer the research questions which described in the introductory section. The data included the score of the students in the tests, the observations and questionnaire results. From the results of all the tests, it can be concluded that the mean of students' mathematical problem solving in the treatment 1 which the implementation dynamic software just for presentation board for the teacher have no significantly difference with the mean of students' mathematical problem solving in treatment2 in which the implementation of the dynamic software Autograph was just used when the students work in pairing in the Think-Pair-Share setting. The significant difference occurred when the implementation of the dynamic software was involved in all of time during mathematical learning from treatment 1 to treatment 2 and treatment 3.

The effect of Treatment 3 was indicated by the students' mastery in learning. Before the mathematics learning by using the dynamic software Autograph, the students' mastery that passed the MCC (Minimum Competency Criteria) was only 44% or 13 among the 34 of students, and most of the students who passed the MCC were in the 'adequate' level. After the treatment 3 that is learning by using the dynamic software Autograph in all of times during the learning process, the students who passed the MCC was increase to 85,2 %, and most of the students were in the 'good' level. The enhancement in students mastery was about 41,2 % , and the number of the students who passed the MCC was twice of the number of students before.

From the results of the tests toward the students' mathematical connection, it could be concluded that the mean of students' mathematical connections in the treatment 1 in the implementation of the dynamic software just for presentation board for the teacher, there was no significant difference with the mean score of students' mathematical connections in treatment2 which the implementation of the dynamic software Autograph just use when the students are pairing in the Think-Pair-Share setting. The significant difference occurred when the implementation of the dynamic software was involved in all of time during mathematics learning.

The students' mathematical connections ability after the treatment 3 was greater than those of treatment 2, and greater than those of treatment 1 when the dynamic software autograph just used to presentation the subject matter by the teacher. The greatest effect was found for students' mathematical connections, with the implementation of the dynamic software in cooperative learning type think-pair-share when the use of the dynamic software autograph was involved in all times of mathematics learning.

The students' perception on the use of dynamic software Autograph related to the students' ability in Mathematical Problem Solving and Mathematical Connection indicated that most of the students answered Strongly Agree (SS) and Agree (S) for positive questions. In the other hand, answered were Disagree (TS) and Strongly Disagree (STS) for the negative question. The summary of the results are: (1) most of the students (88.62%) like to study mathematics; (2) most of the students (85%) want to learn mathematics seriously; (3) most of the students (88%) like the cooperative learning type Think-Pair-Share; (4) most of the students (93.21%) argued that the cooperative learning type Think-Pair-Share was useful in the mathematics learning; (5) most of the students like working with the Student's Worksheets and the manual book of Autograph; (6) most of the students like the dynamic software Autograph; (7) all of the students like to use the dynamic software Autograph in learning mathematics.

From the analysis of the observation sheets, it can be concluded that the student were in good activity during the mathematical learning with dynamic software Autograph (3.995 in scale of 5 or 79.95% in percentage). The lowest percentage was shown when the student asked to formulate thoughts and ideas and in writing their ideas as necessary to prepare for sharing with a partner was 73%. The highest percentage (88%) occurred when the students reading the Student's Worksheets in the core activity. During the introduction, the highest percentage (87%) was shown when the students listened about the aim of the subject of learning. The activity when the students listened to the motivation given by the teacher and answered the teacher's questions about the apperception have the same percentage, that is 78%. In the core activity, the lowest percentage was also the lowest percentage for these observations when the students formulating the thoughts and ideas, writing down ideas that is necessary to prepare for sharing with their partners. In the closing section, participation for making summary and conclusion was 87% and listening to the brief explanation for further subject was 75%.

7. CONCLUSIONS AND RECOMMENDATIONS

CONCLUSIONS

Based upon the findings derived from the data analysis of the investigation, the observation and questionnaire, it could be concluded that: (1) The cooperative learning type Think-Pair-Share with the use of the dynamic software Autograph enhanced the students' Mathematical

Problem Solving and Mathematical Connections; (2) The enhancements of the students' mathematical problem solving and mathematical connections with the use of dynamic software Autograph in cooperative learning type Think-pair-Share were shown by the students' mastery and the students' activity during learning processes; (3) With the use of dynamics software Autograph in cooperative learning type Think-Pair-Share, the students were more active and they did activities in better level; and (4) The students have a good perception about the use of the dynamic software Autograph in cooperative learning type Think-Pair-Share.

RECOMMENDATIONS

Based on the results of the study, it is suggested that (1) teacher can apply this learning model as an alternative in mathematics learning; (2) teacher has to carefully examined the components of a real-world mathematics instruction to assess their usefulness with other disciplines, as well as within mathematics; (3) the dynamic software Autograph can be used in mathematics instruction as a tool that helps students in learning and helps the teachers provide more active and enjoyable mathematics classroom environment (4) teachers need to be aware of the advantages and disadvantages of using this software in the subject-matter. Further study can be conducted with more populations and different mathematics topics.

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Improving the Students' Conceptual Mastery and Learning Trigonometry Achievement by Using an Applet Geogebra-Assisted Cooperative Learning Model

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Abstract: *This research is a classroom action research. It was aimed at finding out whether or not learning trigonometry using cooperative model with applet Geogebra exploration can improve students' conceptsual mastery and learning achievement. The subject of research was the eleventh grade of Engine A (Mesin A = MA) of SMK N 2 Wonosari (State Vocational High School of Wonosari). The class consists of 32 students. The data were collected by using a test and interview guide. Data of test and interview results were analiyzed and presented descriptively. There were two cycles in this study. Each cycle encompassed four stages. Those were planning, acting, observing and reflecting. The indicators in this study were (1) improving students' concepts mastery, (2) improving learning achievement that can be seen from the mean of the test scores in which at least 80% students should reach the minimum passing criteria (78). The result of research showed that after the two cycles of teaching and reflection, the students' scores were improved. It can be seen from the percentages of students' who could obtain the passing criteria. The improvement of mean score could also be seen in the test result in cycle I, and cycle II that were 21, 67, and 89 in prior condition. From the beginning up to the last cycle, it could be seen that both indicators of performance were achieved. So, the conclusion of research was that learning trigonometry by using cooperative model with applet GeoGebra could improve student' conceptsual mastery and learning achievement in the eleventh grade of MA of SMK N 2 Wonosari Gunungkidul in the academic year of 2014/2015.*

1. Introduction

The curriculum of 2013 emphasize in facilitating students to be active, interactive, and critical in mastering the mathematical concepts. The mindset should be implemented in the learning process based on the process standard of the curriculum. To achieve the objective of the learning process, the learning should be meaningful for the students. Based on Ausubel theory, a meaningful learning emphasizes more on cognitive rather than memorization. Meaningful learning could give unforgettable mastery that can become an asset to develop subsequent concepts which is also deal with problems in real world.

National Council of Teachers of Mathematics (NCTM, 2000:11) suggest that student's must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge. Meaningful learning should refer to process standard, by emphasizing on reasoning and sense making (NCTM, 2009:5). That means that every learning process should be related to the process standard that should be supported by infrastructures and learning media. A learning process will be less meaningful if the learning media emphasizes only on the procedural skills. The lesson will be easily forgotten by the students and potentially lead to the misconception and difficulty in trigonometry learning (Kilpatrick, Swafford, & Findell, 2001: 123).

Considering the result of observation and pretest, it can be seen that the students found difficulties trigonometry learning. The students' difficulties can be seen from the presence of errors in the students' concepts, principle, and procedure in solving the trigonometric task. Based on researches on learning by Weber (2005), Challenger (2009), and Orhun (2001), it can be seen that students develop the trigonometric function and conception separately (no connection between the contexts studied). The result of those studies also showed that students found difficulty and misconception in trigonometry subject. In his research, Demir (2012:1) stated that it is not easy for the students to develop an understanding (a conception) connecting the contexts in trigonometry, and conventional (lecturing/memorizing) method could not improve the student's learning difficulty.

In relation to the students' learning difficulty and learning outcome, Challenger (2009: 55) said that whatever the learning outcome, it could not be separated from their learning environment including the teaching method, student activity, curriculum, and learning media. Kultur et al. (2011: 123) suggested some ways to deal with the students' and the teachers' misconception, and to improve the meaningful learning to facilitate in-depth conceptual learning, to connect prerequisite knowledge to new knowledge and to use cognitive media for constructing abstract concepts.

Joolingen (1999: 389) stating that computer can support learning explicitly or can represent cognitive process. Joolingen (1999) explained about cognitive media that can be defined generally as an instrument designed to support cognitive process. Joolingen (1999) provided a cognitive tool likes software to visualize process and domain of chart, animation, and etc. Those instruments can help them too organize thinking process structure. Such the cognitive aid media or tools serve to support the achievement of objective in learning process.

Some studies on the use of computer program which deal with misconception and learning trigonometry difficulty have been conducted. Blackett & Tall (1991) conducted a research to find out the effect of learning by using software aid. The software aimed to help students to explore the relationship between numeric and visual representations of trigonometric comparison in right triangle. The result showed that the software gave positive effect on trigonometric learning. The similar conclusion was also found in Zengin et al. (2012: 187) that technology integration added positive effect in improving learning and understanding (conception). Meanwhile, Lotfi & Mafi (2012) produced COTACSI software which their experiments show that the use of COTACSI software gave positive effect on trigonometric learning. COTACSI software is an interactive multimedia that focused at : 1) perception of trigonometric cycle, 2) negative and positive angle, 3) perception of sin(.), cos(.), tan and cot, 4) perception of sin(.) and cos(.) diagrams, 5) perception of trigonometric simple equations and 6) comparison of trigonometric proportion of $(-\theta)$ and $(\pi - \theta)$.

Those studies concluded that multimedia can help make the learning process more effective, more interesting, and motivating. Then finally, the students could get high-quality experience. Computer-assisted learning trigonometry provides a change and opportunity to students to apply the trigonometric concepts and of interact directly with trigonometric ideas through technology in more active way. For that reason, to improve the learning trigonometry difficulty, this classroom action research integrated cooperative learning strategy and technology into conceptual learning. The technology was applet GeoGebra exploration that can be downloaded freely from GeoGebratube. This research was conducted in two cycles that was emphasizing on group discussion process and applet GeoGebra exploration. The objectives were to find out the process of the improvement of the conceptual mastery and to improve learning trigonometry achievement. By emphasizing on the conceptual mastery, the difficulty of trigonometry learning such as concepts, principle, and procedure error would expectedly be solved and would improve the student learning achievement.

2. Method

This research was a classroom action research method. The subjects of the study were the students receiving learning trigonometry in the odd semester, 32 students of the eleventh grade of MA graders of SMK N 2 Wonosari. The qualitative data were collected by using observation sheet and interview method guide, while the quantitative data was collected by using a test. The data obtained were processed by using descriptive analysis.

The research process was conducted in two cycles. Lesson plan for cycles II was revised based on the result of reflection on the previous cycle. Each cycle encompassed four stages: planning, acting, observing, and reflecting. The research indicators in this study were: (1) improving concepts mastery, (2) improving learning achievement indicated by the mean test score in which at least 80% students should reached the minimum passing criteria (78).

3. Result and Discussion

Considering the result of observation, pre-test, and interview, the research showed that learning trigonometry achievement had not achieved minimum passing criteria as shown in Table 3.1.

Table 3.1 Summary of Prior Data

Indicator	Prior Data
Mean	21
Highest score	55
Lowest score	10
Not passing (<78)	32
Percentage passing	0

In addition, it can be seen that the students experienced difficulty learning trigonometry including: degree concepts, radiant concepts, determining trigonometric function value in radiant, determining trigonometric function value in degree, determining trigonometric function in pi radiant, distinguishing function/non-function, recognizing domain and range as a real number set, identifying domain, identifying range, and identifying trigonometric function period.

To fix those problems, the author designed a technology-assisted cooperative learning, by giving the students an opportunity to discuss and explore applet GeoGebra in pairs to solve the problems on the students' activity sheets. This learning design is put into a plan. The plan covers the procedure of learning activity that is equipped with students' activity sheet and students' report form. The writer also prepared other research instruments such as the observation sheet, interview guide, research instrument validation sheet, blue-print, test, answer key, and interview guideline.

The difficulty understanding radiant and degree concepts was corrected by exploring applet GeoGebra as shown in Figure 3.1.

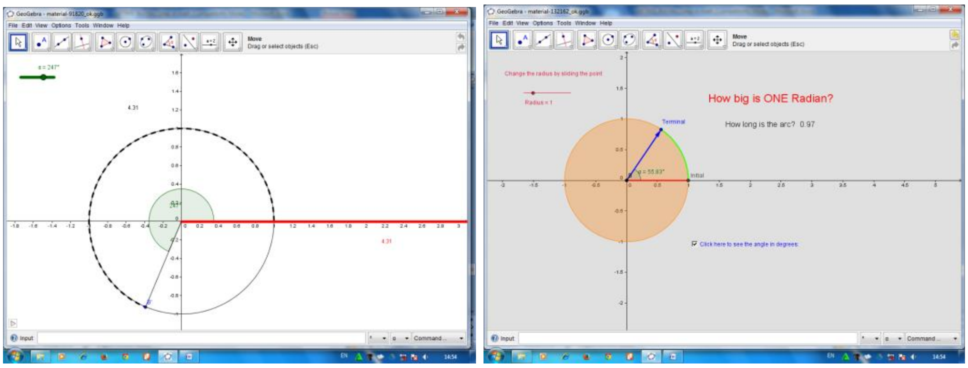

Figure 3.1 Applet GeoGebra of Radiant and Degree Concepts

In the implementation of cycle I, the writer was helped by an observer in conducting the teaching based on the prepared lesson plan. The lesson plan consisted of 4 meetings and 1 achievement test. The learning started with group formation and assignment based on the students' activity sheet. The students' task was exploring the applet GeoGebra and then writing the report, presenting, and discussing classically.

The result of observation shows that originally the students were weak and the class was less conducive. The students should take some times to understand the mechanism of using applet GeoGebra. The students claimed that they have never learnt mathematics using *applet GeoGebra*. Then, after the teacher explain it more deeply, the class became conducive and the activity could run well. The teacher gave instructions to operate and to explore *applet GeoGebra*. The result of observation showed that the students explored busily and actively. The presentation session showed that the students got some misconceptions and there was a fruitful discussion among groups.

Some students still weak in understanding degree and radiant concepts, and the procedure of converting them. The students said that it was because they were very rarely or never working on trigonometric problems in radiant system. Generally, trigonometric lesson were presented in degree system from the book, and limited to special angles only. The procedural error also occurred when the students were asked to determine cosines, for example $\cos 140^0 = \cos(180 - 40) = \cos(-40)$. The procedural error was revealed when the student wrote it inconsistently on the blackboard, so the final result was incorrect. The student said that he/she only memorizes the procedure of working on this type of question. The students also reported the difficulty of memorizing the conversion formula in various quadrants because it is very large in number and hard to the students.

The difficulty in determining trigonometric function in various quadrants was corrected by exploring *applet GeoGebra* as seen in the citation of *applet GeoGebra* in Figure 3.2.

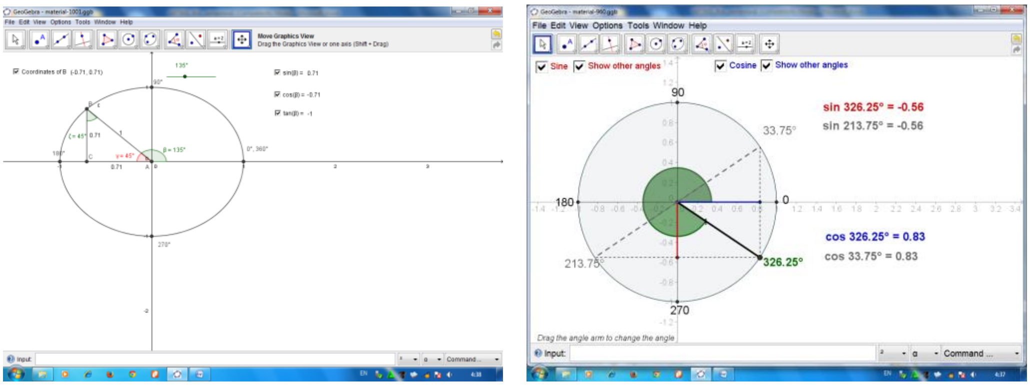

Figure 3.2 *Applet GeoGebra* of Trigonometric Function Value

The difficulty in recognizing trigonometric function chart property (*range*, *domain*, *amplitudo*, *etc*) was corrected by exploring *applet GeoGebra* apparent in the citation of *applet GeoGebra* in Figure 3.3.

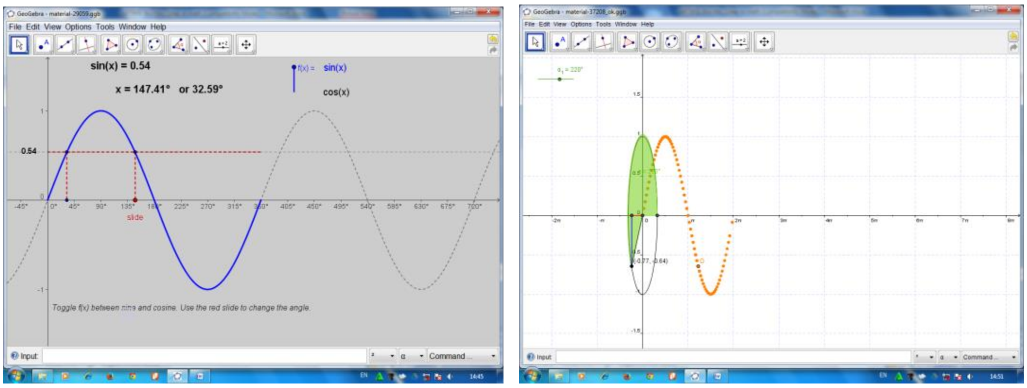

Figure 3.3 *Applet GeoGebra* of Trigonometric Function Chart Property

In the end of the session, the students and the teacher together made a conclusion including reflection, misconception verification, and the concepts confirmation. These activities were carried out in 4 meetings. In the fifth meeting, test of cycle I was conducted. The test consisted 10 questions related to the degree and radiant concepts, determining trigonometric function, and identifying trigonometric function chart property, with the result can be seen in Table 3.2.

Table 3.2 Description of Test Result for Cycle I

No.	Indicator	Prior Condition	Cycle I	Note
1	Lowest Score	10	35	Better
2	Highest Score	55	90	Better
3	Mean	21	67	Better
4	Passing	0%	28%	Better

There was an improvement in lowest, highest, and mean class scores, and the number of students who were passing or achieving the Minimum Passing Criteria (KKM = 78). However, the achievement of lowest and mean class scores was still far below the target. Only 28% students obtained the passing criteria. The improvement from prior condition before the action to cycle I test can be seen in Figure 3.4.

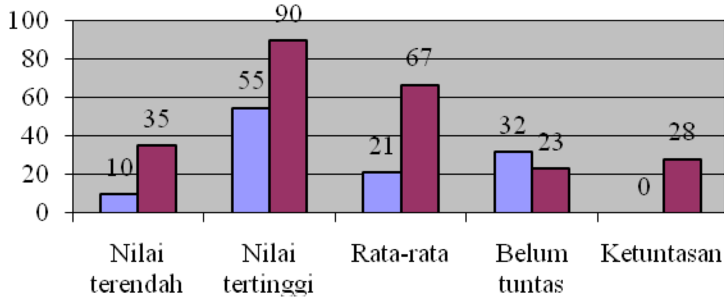

Figure 3.4 Chart of Learning Outcome Improvement in Cycle I

A more serious effort is required to improve the passing score in the subsequent cycle. In this cycle, reflection was made. In exploration assignment, the students were asked to keep working on and to discuss in group. In the presentation, the students were given an opportunity of expressing their opinion to generate discussion in order to reveal the misconceptions occurring in all students.

The improvement in cycle I was still low as a result of the less optimally presentation activity and the students not accustomed with analyzing the result of *applet GeoGebra* exploration. As the result of the less optimal presentation and discussion activities, the misconceptions of the students have not been yet revealed. While working on the test, many students still made misconception and procedural error.

In the implementation of Cycle II, the writer was helped by an observer in conducting the learning process based on the prepared lesson plan. This cycle consisted of 5 meetings and 1 achievement test. Learning started with group assignment in the students' activity sheet. The task was exploring *applet GeoGebra* and then reporting the result, presenting, and discussing classically.

The difficulty converting degree and radian was corrected by exploring *applet GeoGebra* as appeared in Figure 3.5.

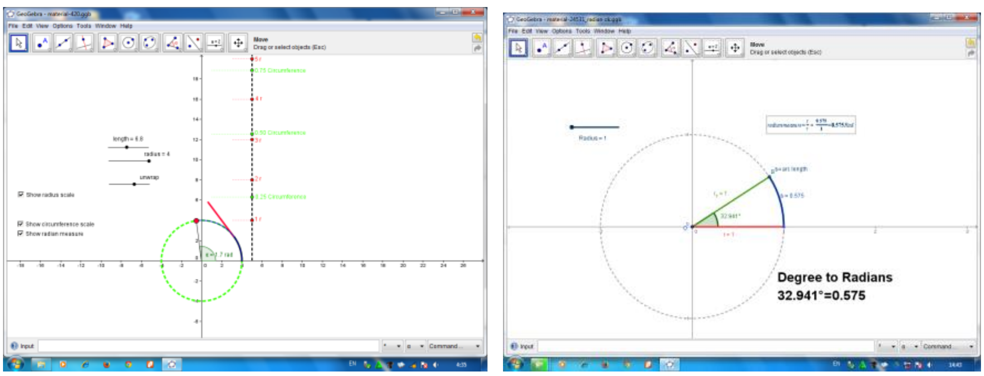

Figure 3.5 Applet GeoGebra of Radiant and Degree Conversion

The difficulty in identifying trigonometric function chart property was corrected by exploring applet GeoGebra as shown in Figure 3.6.

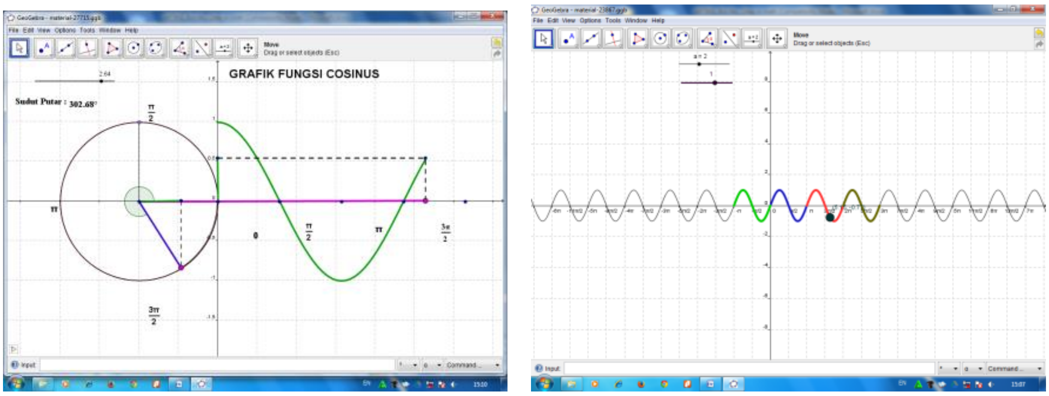

Figure 3.6 Applet GeoGebra of Radiant and Degree Conversion

The result of observation showed that during exploration process, the condition of students was more conducive and the discussion in group was more effective. In presentation session, fruitful discussion occurred with broader topic than what had happened in cycle I. In the end of the session, the students and the teacher together made a conclusion including reflection, misconception verification, and the concepts confirmation. The exploration activity was carried out in 1 meeting, and the presentation activity in 4 meetings. Then, in the sixth meeting, cycle II test was conducted. The test consisted of 10 questions related to degree and radiant concepts, trigonometric function value, trigonometric function. The results of the test were shown in the Table 3.3 below.

Table 3.3 Description of Test Result for Cycle II

No.	Indicator	Cycle I	Cycle II	Note
1	Lowest Score	35	70	Better
2	Highest Score	90	100	Better
3	Mean	67	89	Better
4	Passing	28%	88%	Better

There was an improvement in the achievement of highest, lowest, and mean class scores, and in the number of students passing or achieving the Minimum Passing Criteria (KKM = 78). The achievement of mean class score has passed KKM, but there was still a lowest score achieved below KKM, 70. Considering the result of reflection, it could be found that the students with 70 score experienced procedure error because they work on the questions inconsistently and carelessly. On the other hand, 88% students obtained the passing score. The improvement from cycle I to cycle II can be seen in the Figure 3.7 below.

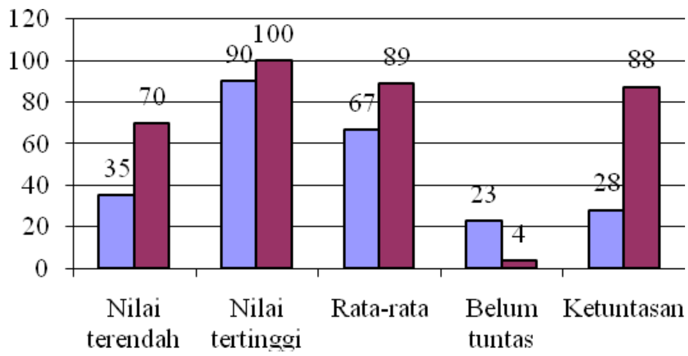

Figure 3.7 Chart of Improved Test Result for Cycle II

This research with two cycles applied a cooperative learning strategy emphasizing on concepts exploration by using applet GeoGebra with group discussion and presentation. In this type of learning strategy, the observation focuses on conception process and misconception correction, leading to improve learning achievement. The summary of achievement test result description from prior condition to cycle II is shown in the table 3.4 below.

Table 3.4 Description of Test Result Summary

No.	Indicator	Pre-test	Cycle I	Cycle II	Note
1	Lowest score	10	35	70	Better
2	Highest score	55	90	100	Better
3	Mean	21	67	89	Better
4	Passing	0 %	28%	88%	Better

Considering the data in Table 1, it can be seen that the improvement which is appeared in all indicators. The improvement from pre-test, cycle I to cycle II can be seen in Figure 3.8 below.

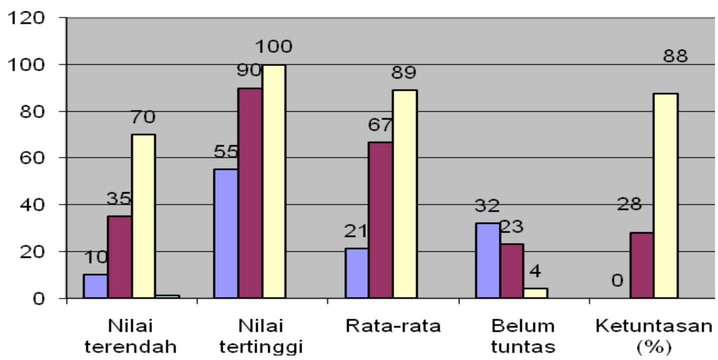

Figure 3.8 Chart of Improved Test Result Summary

In addition, for the increased score in concepts mastery can be observed in Table 3.5 below.

Table 3.5 Summary of Answer Type for Trigonometric Concepts mastery Test

Answer Type	Pre-test	Cycle I	Cycle II
Correct answer	15 %	64 %	86 %
Misconception	22 %	8 %	2 %
Procedure Error or no response	63 %	28 %	12 %

Based on the result shown in Table 5, it can be found that the mean number of students who answer correctly increases from 15% to 64% in cycle I, and to 86% in cycle II. The mean number of students who have misconception decreases from 22% to 8% in cycle I and to 2% in cycle II. Meanwhile, the mean number who has procedure error or no response decreases from 65% to 28% in cycle I and to 12% in cycle II.

Although some students still achieved score 70 (have not achieved yet the KKM 78), it was because of their carelessness rather than misconception. The findings in cycle II show that some students have misconception on pi value. The students could not successfully determine the value of trigonometric function in pi radiant because they were not accustomed to working on this type of question, and had misconception on pi. Some students had misconception by considering that $\pi = 180^0 = 3.14$. Thus, the students were confused and determined the value of trigonometric value lazily with angle size in pi radiant. It becomes an input for subsequent learning meaning that to learn trigonometric function and application, apperception should be conducted first about discovery and knowledge on pi.

Overall, all indicators of concepts mastery and achievement test result have achieved the programmed target. It means that this research was successful corresponding to the objective achieved.

4. Conclusion and Recommendation

Considering the result of Classroom Action Research on the eleventh grade of MA of SMK N 2 Wonosari by using cooperative learning model with *applet GeoGebra* helps the students and the following conclusions can be drawn. (1) The use of cooperative learning model with *applet GeoGebra* can improve the conceptual mastery of the eleventh grade of MA of SMK N 2 Wonosari, indicated by the increase of mean class score from 15% in prior condition to 64% in cycle I and to 86% in cycle II, in the term of the number of students who answered the conceptual mastery question correctly. In addition, it can also be seen from the decrease of mean class score from 85% in prior condition to 36% in cycle I and to 14% in cycle II in the term of the number of students who answer the concepts mastery question with misconception, procedural error, and or no response. (2) The use of cooperative learning model with applet GeoGebra can improve the learning achievement of the eleventh grade of MA of SMK N 2 Wonosari, indicated by the improvement of the mean class score from 21 with the passing of 0% in prior condition to 67 with the passing of 28% in cycle II and to 89 with the passing of 88% in cycle II. From prior process to the last cycle, it could be seen that both indicators of performance were achieved. So, the conclusion of research is that learning trigonometry using cooperative model with applet GeoGebra can improve conceptual mastery and learning achievement in the eleventh grade of MA of SMK N 2 Wonosari Gunungkidul in the school year of 2014/2015.

With the improved conceptual mastery and learning achievement of the students in learning trigonometry through cooperative learning with *applet GeoGebra*, this research is considered as a successful research. For that reason, there were some recommendations that can be given. (1) The teacher, in mathematic learning, should keep looking for various learning strategies that provide the students the opportunity of discussing and exploring concepts using technology or other cognitive aids. (2) In learning, the students should construct their own conception actively, creatively and innovatively in order to implement the knowledge they acquired into daily life. (3) The policy makers should be responsive, and provide adequate infrastructure to foster the teachers' and the students' creativity.

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Joyful and Meaningful Classroom in Mathematics Through Brainly.co.id

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Abstrak. *Permendikbud number 68 of curriculum 2013 states that one of the basic competencies at a junior high school mathematics expected of learners related to core competency-2 is a "show attitude logical, critical, analytical, consistent and conscientious, responsible, responsive, and does not easily give up ". Many variations are used teacher learning in helping students to achieve these outcomes,one of which is the role of communication technology in education that specifically influence the learning process. However, in the era of globalization that has entered into many sectors including education, until now still not many teachers who use communication media such as internet, electronic mail (e-mail), Face book, online discussions or conferences, bulletin boards, Twitter, Friendster, Myspace, Skype, chatrooms, mailing list and so on in the learning process. Along with the development of globalization through education, the information technology played an important role as a means to obtain the widest source of information relating to the subject matter being taught. The learning process is joyful and meaningful interaction between teachers and students is not only done through face-to-face relationships but also done using these media. Effective learning activities and fun requires a media that supports the absorption of as much information to deliver learners have the basic competencies. This paper provides an alternative example of learning math joyful and meaningful by using the internet through brainly.co.id, hopes to bridge the students in the basic competence on the second core competencies in the curriculum 2013.*

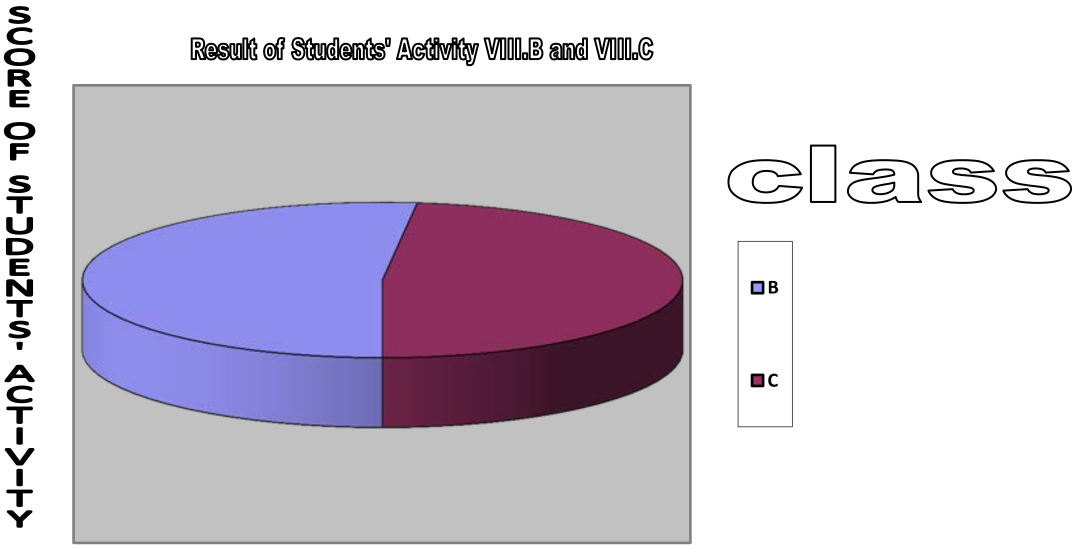
Keywords: Curriculum 2013, the Internet for learning, joyful and meaningful learning.

1. Background

The role of information technology is very important for human activity in the era of globalization at this time that has been entered into many sectors including education. Globalization through education needs innovatif teachers in mathematics class by using communication media such as internet, electronic mail (e-mail), Face book, online discussions or conferences, bulletin boards, Twitter, Friendster, Myspace, Skype, chatrooms, mailing list and so on. Teacher is a manager of the class and as change of agents should encourage the students learning in the real life contexts. Mathematics classroom needs technology as a tool to build conceptual knowledge and thinking mathematically than the tradiotionally displayed in textbooks. Mathematics teachers need innovation in teaching process to create the class fun, entertaining, interesting, joyful and meaningful learning by using the internet very appropriate in mathematics curriculum 2013 and can be linked to mathematics class. The internet has become popular and potential as a medium for learning in mathematics class joyful and meaningful but also motivate the students to think critically.

In this case the role of a supervisor as a teacher partner provides a major contribution to the performance and competence of teachers. This paper provides an alternative example of learning math joyful and meaningful by using the internet through brainly.co.id. hopes can help the students get the widest source of information relating to the subject matter being taught. In the basic competence on the second core competencies in Permendikbud number 68 of curriculum 2013 states that one of the basic competencies of a junior high school mathematics expected of learners related to core competency-2 is a "show attitude logical, critical, analytical, consistent and conscientious, responsible, responsive, and does not easily give up ". Many variations are used teacher learning in helping students to achieve these outcomes, one of which is the role of communication technology by internet activities for joyful and meaningful in education that specifically influence the mathematics learning is the target of a profesional supervisor and a profesional teacher to realize the importance of the quality of learning that is joyful and meaningful,

In order to build the teaching profession, these conditions require the teaching profession. Increased activity of the learner and teacher competence in State Junior High School 4 Metro City becomes the target of achieving writers, as with significantly increased teacher competence learners produce quality and dignified. Here presented an overview of the results of activities and VIII.C VIII.B grader at State Junior High School 4 City Metro based on initial observations of the impact of the performance and competence of teachers.



Specification:

A = 3.28 – 4.00 Highly Satisfactory Class VIII. B (2.81) Class VIII. C (2.80)
B = 2.78 – 3.27 Satisfactory C = 2.38 – 2.77 Less Satisfying

Diagram 1 The results of Student Activity Class VIII.B and VIII.C Before Using Internet Media

Diagram 1 shows that the activity and VIII.C VIII.B graders are among the range of B = 2.78 - 3:27 unsatisfactory category, things have been good this activity students need to be increased to achieve a very satisfactory value. In an effort to increase student activity, the teacher needs to develop and find solutions to a variety of approaches. Supervisory guidance to teachers, teachers to make efforts to implement technology-based learning variations in this case the internet through Brainly.co.id (Appendix 1 Student Activity Assessment Sheet).

1.1 Problems

The problem identified is the need to increase the activity of the learner in the learning process in the classroom and competence Mathematics teacher at State Junior High School 4 Metro.

1.2 Problem Formulation

The problem can be formulated as follows: How can increase the activity of students in Junior High School 4 Metro ?; How to improve the competence of teachers in the learning process in State Junior High School 4 Metro ?.

1.3 Problem Solving Approach

In addressing the above problems there are several ways that can be implemented, namely: Supervisors conduct training activities in order to improve the competence of learners and teachers in Secondary Schools 4 Metro; Supervisors conduct training so as to improve the learning process fun and meaningful in State Junior High School 4 Metro with internet-based learning through Brainly.co.id Moor and Zaskis (2000) "The interactive aspect of the internet holds the attention of the students much longer than a regular page of information such as is found in a textbook. This suggests that internet-based learning can motivate students to learn mathematics so that more meaningful and enjoyable.

1.4 Purpose

Coaching aims to: Increase activity learners through fun and meaningful learning; Improve the competence of teachers in the learning process fun and meaningful in State Junior High School 4 Metro with internet-based learning through Brainly.co.id

1.5 Benefits

Some of the benefits that can be derived from this paper as follows: To provide information to other supervisors to be able to improve the competence of teachers in elementary schools to use information technology (IT) as a means or effort in the learning process fun and meaningful; Institutions and teachers improve the quality of the learning process through the use of information technology (IT) to improve the activity of learners so as to create the learning process fun and meaningful.

2. Discussion / Analysis

2.1 Steps

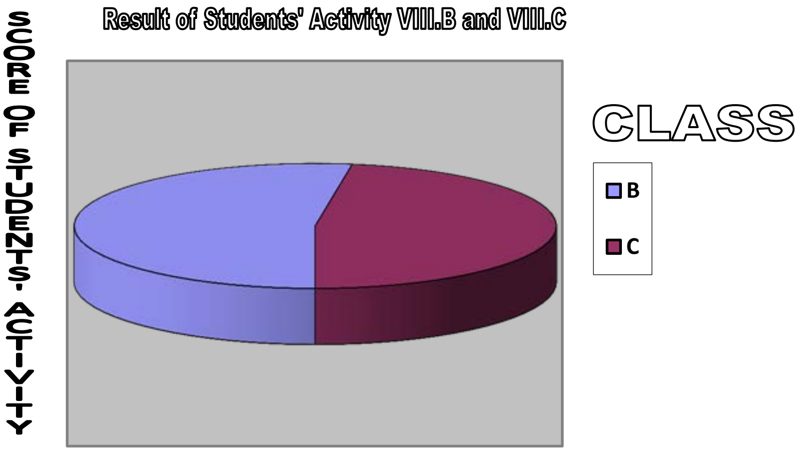
Many variations are used teacher learning in helping students to achieve competency, one role of communication technologies that influence the activity of the students in the learning process. Along with the development of globalization through education, the information technology played an important role as a means to obtain the widest source of information relating to the subject matter being taught. The learning process is fun and meaningful interaction between teachers and students is not only done through face-to-face relationships but also done using the media of information technology that supports the absorption of as much information with Internet-based learning through Brainly.co.id

2.2 Basis Theory

The development of information and communication technology (ICT) has an impact on education, especially in the learning process. According to Rosenberg (2001), with the widespread use of ICT there are five shifts in the learning process, namely: From training to performance; From the classroom to where and at any time; Of paper to the "on line" or channel; Physical facilities to network facilities; From time to time the cycle is real. Communication as a medium of education is done by using communication media such as telephone, computer, internet, e-mail, and so on. Interaction between teachers and students is not only done through face-to-face relationships but also done using these media (Rosenberg, 2001).

3. Results and Impact Achieved

Coaching by supervisors are not given instruction or order, but human relations atmosphere full of warmth, closeness and openness; so that teachers have a sense of security, and a willingness to accept the repair. The context of process improvement coaching is done as supervisory tasks. The teacher as a supervisor needs to get treatment target measured as prescriptive efforts towards improving the performance of teachers so that the teachers make learning improvement efforts, as suggested by the supervisor pembinanya in an effort to improve the quality of the learning process fun and meaningful to increase the activity of students and as an effort to increase performance. Object of research is the activity of learners VIII.B and VIII.C in Junior High School 4 Metro.



Specification:
A = 3.28 – 4.00 Highly Satisfactory
B = 2.78 – 3.27 Satisfactory
C = 2.38 – 2.77 Less Satisfying

VIII. B (3.28) VIII. C (2.90)

Diagram 2 Results of student activity Class VIII. B and VIII.C After Using Internet Media

Figure 2 shows that the activity VIII.B graders are among the range of A = 3.28 is very satisfactory category, things have been good this activity students need to be increased to achieve a very satisfactory value with a range that is until 4:00. Activity VIII.C graders are among the range of B = 2.90 satisfactory category. In an effort to increase student activity, the teacher needs to develop and find solutions to a variety of approaches. Efforts made by applying a variation of teachers that technology-based learning through the internet in this case Brainly.co.id (Appendix 2 Student Activity Assessment Sheet).

The role of information technology in human activity at this time was so great. Information technology has become the main facility and contributed greatly to the change - a fundamental change in the activity of learners. It is therefore important to increase the human resource capacity in this case the competence of teachers as the spearhead of change (agents of change) in education. Acquisition of data into learning activities are as follows:

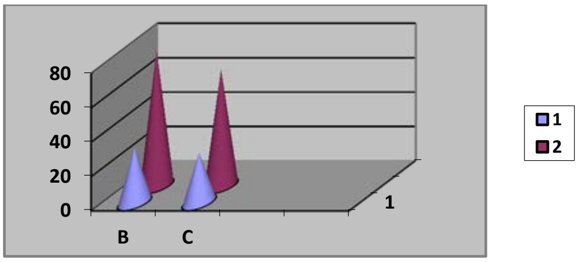


Diagram 3. Student Activity Percentage Score

Specification:
1 Student Activity 2.81 VIII.B be 3:28
2 Student Activity VIII.C 2.80 into 2.90

4. Conclusion and Recommendations

4. 1 Conclusion.

Taking into account the results that have been obtained from the regulatory guidance can be concluded that: There was an increase of activity in the learning process by using the internet through Brainly.co.id By using Internet-based learning model one through Brainly.co.id students can bridge the basic competence on the second core competencies in the curriculum, 2013; The learning process is fun and meaningful interaction between teachers and students is not only done through face-to-face relationships but also done by using Internet-based media in this case through the Brainly.co.id Effective learning activities and fun requires a media that supports the absorption of as much information to deliver learners have basic competencies in the curriculum 2013.

4.2.Suggestion

From the conclusion of this paper, it is suggested some of the following:

For Teachers: The need for improved competence by using technology in the learning process through a variety of media approaches technology (internet) to increase student activity; **For Supervisors:** The need for academic supervision to improve the competence of teachers in learning through a collaborative approach; **For Schools:** Need to motivate teachers to always communicate with the supervisor in the form of guidance related to the quality of learning in schools; **For Institution** Office of Educator training and workshops should be conducted on an ongoing basis of the concepts of supervision for school inspectors.

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To Have the 7 Grader Students Understand of Polygon Area by Using Open Ended Approach

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Abstract:*This is qualitative descriptive studies which types of classroom action research aims to describe learning that can make students understand polygon area by using open ended approach. This research was conducted at SMPN 2 Wagir kabupaten Malang in second semester of academic year 2012-2013. The data collected in this study were taken from observations during the learning process with an open ended approach underway. The data collected in this study were (1) the observation of learning activities, (2) open-ended tests, (3) students questionnaire, (4) interview, and (5) field notes.*

The learning steps using open-ended approach that can make students understand the polygon area are: (1) the teacher gives open-ended problems, (2) students solve open ended problems individually, (3) student groups discussion, and (4) a member of some groups presents their group discussion result in class discussion. Researcher (teacher) gave enough time to the students to work on problems individually or in groups so that students have more flexibility in resolving the problem. Researcher involves students actively in making conclusion during classroom discussion so that we can see the students' understanding clearly.

The results of this research show the following information: (1) the investigation of students work at the pre-action test and post-action test show that the concept errors and procedure errors happened to most students at first but gradually it was reduced in the second cycles, and students more understand polygon area, (2) the observations show that the learning activity run well in the first cycles and increased to the most in the second cycles, (3) the students' responses had been obtained through the questionnaire show that the number of students who enjoyed learning the polygon area using open ended approach increased from the first cycle to the second cycle, (4) the six students as interviews subject consisting of two students respectively from the lower, middle and upper groups, stated that they enjoyed learning process.

Keywords: polygon area, open ended approach.

1. Rationale

The understanding of polygon especially, polygon area is much beneficial to help solve problems in everyday life and to study the surface area and volume of polyhedrons in grade VIII. In fact, polygon learning at SMPN 2 Wagir kabupaten Malang was not optimum.

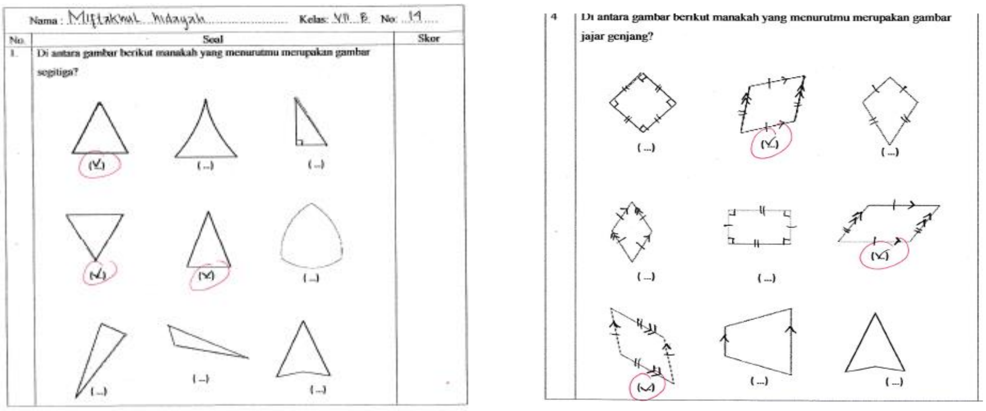
It is easy to find the models of polygon around us, such as tiles that are used to cover the floors, the walls of bathroom, etc. Geometric shapes of tiles diverse, including square, rectangular, hexagonal, and triangular and others. Tiles around us also vary in size. On Tuesday, April 2nd 2013 researcher visited a hardware store "Makmur Abadi" which is located at Jl. Genengan 133 Pakisaji County District of Malang Regency, and got examples of the various sizes of tiles, as seen in Figure 1 below

Figure 1: Various of tiles sizes

According to the mathematics syllabus, polygon has been introduced to the fifth graders, and it will be continued in the seven grade. Learning experiences in the seven grades in junior high school is a step to further mathematics (Hudojo, 1979: 107). Understanding of the material, especially the polygon area much beneficial to help solve problems in everyday life and to study the surface area and volume of polyhedrons in grade VIII. In fact, polygon learning at SMPN 2 Wagir kabupaten Malang was not optimum.

Pre action observation was done by the author on Tuesday, March 26th 2013 through a pre test. In the pre test the students identified polygon. From the first pre test obtained that the average scores were 68.71. Some students work of the pre test as seen in Figure 2 shows that there are students who do not understand about characteristics of triangles and parallelograms. According to the students through interviews conducted after the pre test, the students' answers to question number 4, about a parallelogram, the length of the sides that does not face may not be same, and the angles may not be right-angled, because it would be a rhombus when the sides facing have the same length, and whereas when the angles be right-angled it will be a rectangle.

Figure 2 : Some Students work



On Saturday April, 13th 2013 researcher continue to provide the second pre test to know students' ability to determine polygon area which combined of triangle, square, rectangle, parallelogram, trapezoid, and kite. Researcher obtained information that some of students difficulties are (1) do not understand how to separate the combined polygon into it parts, (2) do not know (forget) formula to calculate the area of polygon, and (3) know the formula but make procedures error. From the result of pre action observations, researcher concerned and want to attempt to have students understand polygon using learning approaches and choose the appropriate media.

In this study, researcher is looking for approaches that can help students to be able to understand the concept of polygon area and use in solving problems. Bush and Leinwand (2003: 22) states that according to the NCTM standards, in order to succeed in learning, students do not only have a clear understanding of the mathematical concepts, but they also need to be proficient with math skills, and more importantly they must be able to express their reason mathematically. Shimada and Becker (1997: 1) explain that open-ended approach starts learning with problems. Learning activities are carried out by using a lot of correct answers of the problems given to provide experience in finding something new in the process. This can be done by combining students' existing knowledge, skills or ways of thinking that have been studied previously.

Regulation of the Minister of National Education of the Republic of Indonesia Number 22 of 2006 on the Content Standards in appendix two states that the focus of mathematics instruction includes a closed problem with a single solution, an open problem with no single solution, and the problem in various ways completion. From the opinion of Bush and Leinwand, Becker and Shimada and Appendix Minister of National Education No. 22 of 2006, can be interpreted that in the learning process students need to be trained to be able to solve various problems with several of ways by using their knowledge, experience, and skill that has been owned by the students . A similar statement was expressed by Sukoriyanto (2001: 104) that in solving problems students should be given free way to express their ideas.

Nickson (in Hudojo, 1998: 6) says that mathematics learning is an attempt to help students to construct concepts or mathematical principles on their own through a process of internalization. According Hudojo (1998: 7) in constructivism view of mathematics learning have the following characteristics: (a) actively engage students in learning, students learn meaningful mathematics by working and thinking, (b) new information must be associated with the previous so that it blends with the students schemata, and (c) learning orientation are investigation and discovery which is basically problem solving. Reasoning that is considered important by S W. Bush and Leinwand S (2003: 21) in problem solving will be developed in this study when students solve problems.

Considering the important role of the polygon area in mathematics and in everyday life, the author thought that polygon problem solving skills need to be emphasized and start with activities which can help students understand it. Similarly, learning that focuses on open-ended tasks need to be applied. Therefore, the author conducted a study that is able to produce a description of learning that attempt the students to understand polygon area with open ended approach.

2. Method

This is qualitative descriptive studies which types of classroom action research aims to describe learning that can make students understand polygon area by using open ended approach. This study was conducted at SMPN 2 Wagir kabupaten Malang in second semester of academic year 2012-2013. This study begins with the initial observation by providing pre test to determine prerequisite knowledge on identifying triangle, square, rectangle, parallelogram, rhombus, kite, and trapezoid. The second pre tests on combined polygon area consist two of combination of triangle, square, rectangle, parallelogram, trapezoid, and kite. Results of pre test are used as a basis in grouping, and the selection of interview subjects. In this study, the researcher acted as a key instrument, because researcher who plan, implement, collect data, analyze data, make conclusions and make a report.

The data collected in this study were (1) the observation of learning activities, (2) open-ended tests, (3) students questionnaire, (4) interview, and (5) field notes**.** Technique of data collection in this study were: (1) observation of the learning activities undertaken during the learning process, (2) results of open-ended tests given at the end of an action, (3) the questionnaire of the student responses given at the end of the action, (4) the interviews conducted after all action has been completed, and (5) field notes made during the learning process underway.

Product of this study was description of polygon learning procedures with open ended approach. In this study, researcher prefers the learning process rather than the result of learning. The research design can be refined according to the situation during the study. With these characteristics, this study used a qualitative approach as stated by Moleong (2000: 6).

Success criteria for this study are (1) concept error or procedure errors that students did in the post test can be reduced or minimized, and is supported by the average of post test scores at least 75 of 100, (2) the observation of learning activities are in the good or excellent category, (3) students' responses obtained through the questionnaire show at least 75% students of class VII B stated enjoyed learning polygon area using open-ended approach, and (4) student responses obtained through interviews show at least five of the six interview subjects stated enjoyed learning polygon area using open-ended approach.

3. Research result

In the first cycle the average percentage score of learning activities observation given by both observers was 85.87%, the learning activities was in good criteria. The results of the final test classically show that 46.88% students reached score ≥ 75 with an average 62.58 results of these test have not met the criteria of success. Questionnaire of students responses indicate that 100% students of class VII B stating enjoy polygon learning process with open ended approach. The results of questionnaire responses have met the success criteria. Overall the first cycle has not met the criteria of success.

In the second cycle, the average percentage score of learning activities observation given by both observers was 93.77%, the learning activities was in very good criteria. The results of the final test classically show that 87.5% students reached score ≥ 75 with an average 82, 74 and it met the criteria of success. Questionnaire of students responses indicate that 96.87% students of class VII B stated enjoy polygon area learning process with open ended approach. The results of the interview conducted by the researcher to respectively two students from the lower, middle and upper group stated that they enjoy polygon learning process with open ended approach. Overall the second cycle has met the success criteria.

Based on the result of both cycles, this study obtained the following information: (1) the investigation of students work at the pre-action test and post-action test show that the concept errors and procedure errors happened to most students at first but gradually it was reduced in the second cycle, and students understand more on polygon area, (2) the observations show that the learning activity run well in the first cycle and increased to the most in the second cycle, (3) the students' responses had been obtained through the questionnaire show that the number of students who enjoyed learning the polygon area using open ended approach increased from the first cycle to the second cycle, (4) the six students as interviews subject consisting of two students respectively from the lower, middle and upper groups, stated that they enjoyed polygon area learning process using open ended approach.

4. Discussion of the research results

Based on the results of this study that has been described previously, we obtained information that the students of grade VII-B of SMP Negeri 2 Wagir have successfully understood the polygon area by using open-ended approach. It is based on four criteria that are: (1) observation of learning activities, (2) results of post action test of the cycle, (3) results of a questionnaire to determine students' responses on polygon learning using open ended approach that has been experienced, and (4) results of the interview to strengthen the questionnaire.

Observations of learning activities carried out during the learning undergo to observe the suitability of all the activities that teachers and students with the design as outlined in the lesson plan. This is as disclosed by Fraenkel (2012: 445) that the best way to answer the research questions is through observation of how people behave or how something looks. Researcher also may ask teachers about how their students behave in a class discussion, but more accurate information can be obtained through direct observation when learning undergo.

Post action test is needed to determine students' understanding of the material being studied in an action. Reduced concept and procedure errors that appear on the student's work may indicate that student's understanding of the material being studied is clearer than before. This is in accordance with that proposed by Arikunto (2010: 194) that in order to measure one's achievement after learning something done tests.

Questionnaire in this study was given after post action test. It helps to determine the students' responses to polygon area learning that has been experienced. This is as stated by Arikunto (2010: 194) that the questionnaire is a number of written questions used to obtain information of the respondents feeling. Act No. 20 of 2003 on National Education System in Article 40 explains that teachers and education personal are obliged to create a joyful learning. It is also described in the government regulation No. 19 of 2005 on national education standards of Article 19, paragraph 1, which stated that the process of learning in the educational unit organized as joyful learning. Based on these two rules, in this study student questionnaire responses are used as instruments to gather information about the student's responses after they experienced the learning, and reinforced through interviews.

Interviews conducted after all action is completed. Interviews were conducted in order to strengthen the questionnaire, and were also used to determine the students' understanding of polygon area. Interview guidelines was made to direct the interviewer about any aspects that need to be asked. Interview guidelines gives the interviewer a chance to think about how these aspects will be described concretely in the interrogative sentence, as well as adjust to the context during the interview. This stated by Fraenkel (2012: 452) that the data obtained will be systematically using interview guide.

5. Previous Study

Hayati (2012: 101) in his study of tesselasi said that the use of paper and colored pencils have limitations in doing tesselasi, because students have difficulty creating a model of the tiles in a short time. The use of expository method causes this tesselasi material less attractive for students. Mahlobo (2007: 1) used open-ended approach in his research. He gave actions with open-ended approach to the experiment school. He got the post test result better than it in the control schools.. Though he gave the same pre-test and post-test, and the same schedule of activities in both schools. Imprasita (2006: 103) said based on the results of a study of Khon Kaen University Education Faculty’s students in Thailand who carry out teaching practice in the school for one semester using an open-ended approach that got valuable experience in learning mathematics is not just teaching to finish the material, but more important is emphasizing student learning, original ideas and attitudes towards learning mathematics that can improve one's ability. Bove (2009: 47) in his study concluded that learning will only be successful if we pay attention to the student’s unique context individually, so they have to build their own understanding of things related to learning. In line with the results of research conducted by the Hayati, Mahlobo, and Imprasitha, the results of this study indicate that learning with open-ended approach can improve students understanding on polygon area.

6. Conclusion

In this study can be concluded that learning steps using open-ended approach that can make students understand the polygon area are: (1) the teacher gives open-ended problems, (2) students solve open ended problems individually, (3) student groups discussion, and (4) a member of some groups presents their group discussion result in class discussion. Researcher (teacher) gave enough time to the students to work on problems individually or in groups so that students have more flexibility in resolving the problem. Researcher involves students actively in making conclusion during classroom discussion so that we can see the students' understanding clearly.

Suggestions

Some suggestions for research, writing or more about activities of learning with open-ended approach are as follows:

1. The use of time during the learning process needs to be controlled in order to obtain more optimal learning outcomes.
2. In the classes who have never used an open-ended approach, teachers need to give motivation to the students in order to make them actively involved in learning.
3. Asking questions and expressing opinions in a discussion needed to be growth as students' habit. If needed the teacher can give a reward to the students who gave a good questions or opinions then the discussion will be more active.
4. The use of open-ended approach by the author has given good results in learning of polygon area; therefore, this approach can be used as an alternative approach to other teachers in mathematics learning.

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Learning the Root Form Numbers Meaningfully and Joyfully Through Golden Snail Learning Strategy in Senior High School

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Abstract: *This research aims to determine mastery of the concept of the root form numbers and attitudes of students towards math after being given treatment application of interactive learning, inspirational, fun, challenging, and motivating through the golden snail learning strategy. The study design used in this research is descriptive qualitative design. Subject in this study were students in grade X at State Senior High School Number 6 of South Bengkulu and mathematics teacher. Data collection was done with student's work documents, learning log and interview from the students and the teachers. From the research and triangulation discussion knew that the application of golden snail learning strategy to the mathematics teaching and learning gave success to them in understanding about root form numbers well. The application of golden snail learning strategy was very influential on the results of the root form number concept mastery and from the analysis of students learning log and interview indicated that 100% of students to be positive in mathematics. This study concludes that the model of learning with the application the golden snail learning strategy on the subject matter of the root form number well and worthy to be applied. Trough the application of golden snail learning strategy made teaching and learning mathematics became meaningfully and joyfully.*

Key words: Root Form Number, Golden Snail Learning Strategies, meaningfully and joyfully.

1. Introduction

Nowadays the writer's observation learning of mathematics in high school is a growing realization by teachers and education personnel that important aspects related context, and the humanist aspects of affective aspects of learning mathematics. Learning mathematics is not only a measure of cognitive aspects of learners alone but must also be able to measure aspects of psychomotor and affective aspects, although for cognitive aspects of learning are not given directly in the classroom.

In the last three years, the majority of new students accepted into the high school in South Bengkulu district had math scores on national exams with average tends to decrease. Impression or a negative image towards mathematics learners who still feels thick touted to be one of the causes as well. In general, the results of interviews that have been done, they were overshadowed by the confidence is low and assume that the mathematics courses in high school was very difficult, abstract, boring and difficult to apply to activities of daily living have taken possession of their minds. Other cause in the majority of non-piloting school curriculum project in 2013 and partly in the school curriculum in 2013 piloting project implementation approaches, methods, or strategies and learning of mathematics educators who use, while still centered on the teacher looks less active learners during the teaching and learning process takes place so learning becomes less meaningful and boring. Teachers convey information to the lecture method, giving examples and answer questions, provide exercises to be done in class and at the end of the learning activities provide homework assignment in the form of questions, as well as on some matter that is considered difficult by students discussed at the next meeting. This incident occurred monotonically done by the teacher. Shown in this learning process is more emphasized for students working on exercises, procedural as well as using the formula, less students are given the opportunity to find their own way and learning strategies and less exploration. As a result the level of less advanced learners think and if they are given the questions were slightly or moderately different from the questions that have been drilled and implementation issues that may be related to the environment or daily lives, they find it difficult to understand and determine the solution, whereas it is very important for them to know the benefits of mathematics being studied for their everyday lives. According to Graham (1984: 2) in every problem solving activity there is some degree of inventiveness and discovery required and solving mathematical problems, both inductive and deductive reasoning are used.

Based on the above problems requires a new learning strategy that refers to the patterns of change and improvement system for the learning of mathematics involving all aspects of the affective aspects of learning mathematics, cognitive and psychomotor learning in the classroom. Therefore, the authors are interested in discussing and assessing the learning of mathematics at the concept of root form number meaningfully and joyfully through golden snail learning strategy in high school.

2. What is the Golden Snail Learning Strategy?

Golden Snail learning strategy is a planned learning concepts strategy and practical techniques based on the number of roots to draw the pattern by drawing a certain right triangle on the field or the geometry of Euclid flat, combined with the expansion of the Pythagorean theorem sketched pattern is formed such that the golden snail picture done by learners, individually or collectively through the guidance of teachers. By sketching a picture that is formed for learners to determine the characteristics and patterns of root form numbers as subset from irrational number and further them can also determine the values of the approximation or estimate the root form numbers formed. Learning golden snail is also a proof that there is a linkage (intertwining) the linkages between geometry Euclid (geometry of a flat), the Pythagorean theorem, the number of roots (which are combined in a cluster number irrasional or Irrational numbers) and fractional ranks. Irrational numbers is the number that can not be expressed as a ratio or comparison of an integer such as "z" with natural number, let's say: "a" so that the irrational number cannot be written $\frac{z}{a}$. While the root form number is the root of a number is the result not of rational number (Budi, 2011: 8).

Golden snail learning strategy emphasizes learning can provide opportunities for learners to be able to better train and hone skills of thinking, think it make sense and dig all the potential that he could position themselves as objects in interactive learning, inspiring, fun, challenging, and motivating to actively participate and provide enough room for initiative, creativity, and self-reliance 'talents, interests, and physical and psychological development of all combine and impact on the dominance of subject matter concepts learned.

3. How are the Mathematics Teaching and Learning Process through Golden Snail Learning Strategy?

Teaching and learning process of mathematics through Golden Snail Learning Strategy with the details it consist of: (1) the opening activity; (2) the main activity; and (3) the closing activity.

On the opening of mathematics teaching and learning process reflect on preliminary activities: (1) The teacher provides information that the subject matter to be discussed is concept root form number as member of irrational number, and the teacher reminiscent of previous learning about the Pythagorean theorem states that in a right-angled triangle, broad square on the hypotenuse or the hypotenuse equals the number of broad squares on the other two, The integers that represent the sides of a right-angled triangle as an examples how to determine Pythagorean numbers (Kisacanin, 2002: 75) ; (2) Learning approach that will be done is through golden snail learning strategy; (3) Teacher recalls the sense rational number is a number that can be expressed as a ratio (ratio) of integers with the natural numbers (Loedji, 2007). Suppose z is an integer, and natural numbers a, rational number q, it can be stated that $q = \frac{z}{a}$. z position as the numerator and the denominator by the position of z and a requirement that has no fellowship factors other than 1, and a real number which is not a rational number is called an irrational number. The number $\sqrt{2}$ is an example of an irrational number (Hart, 1988:15-16); and (4) teacher showed some images downloaded from Google images on the internet like this while asking the students what the picture is showing and ask them to represent them.

Figure 1: "Image Presentation Golden Snail and Application"

The two images at the top is a picture of the golden snail, and one image at the bottom is other types of snails with the golden snail and the IMAX Theater in Jakarta, Indonesia.

On the main activity it consists of exploration, elaboration, and confirmation. On the exploration on learning are: (1) the teacher said that the following activities are painting sketch golden snail but linked to the subject matter covered today. Teacher raises such questions, namely: (a) "How do you obtain the numbers $\sqrt{2}$, $\sqrt{3}$, $\sqrt{4}$, $\sqrt{5}$, $\sqrt{6}$, $\sqrt{7}$ and so on? And (b) which of these numbers those are rational and irrational numbers? Explain your answer! Teacher asks students in groups that work on the problems. The teacher asks the students to draw a right triangle to represent golden snail mathematics for the first step as follows. It can consider the isosceles right triangle, each of whose sides has length 1, by the Pythagorean Theorem (Gross & Miller, 1971: 85), it followed that the length of the hypotenuse of this triangle was $\sqrt{2}$.

As the show on the figure 2 below.

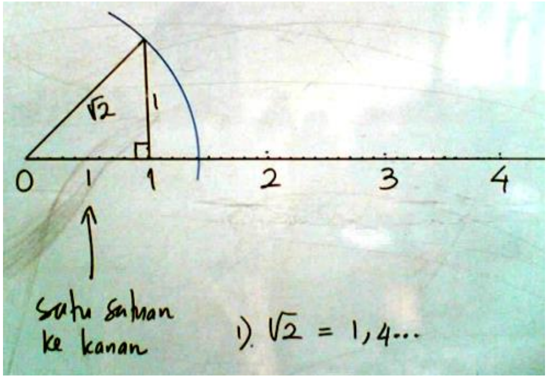

Figure 2: " A Sketch Drawing Process I of Golden Snail Mathematics"

(2) The teacher asks a student to continue drawing on top followed by the other students to follow suit by creating in quarto paper that has been provided either individually or in groups, as follows.

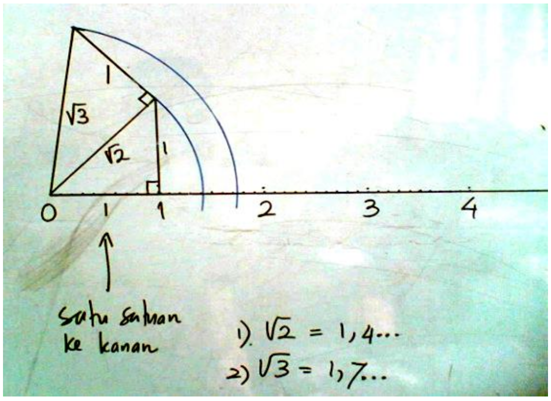

Figure 3: " A Sketch Drawing Process II of Golden Snail Mathematics"

(3) In the same way the previous step figure 4 is obtained as follows.

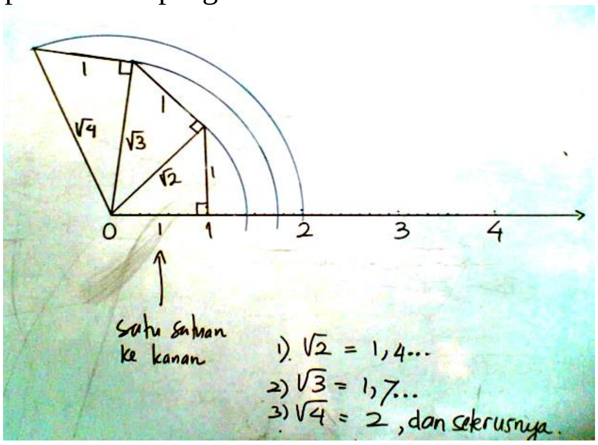

Figure 4: " A Sketch Drawing Process III of Golden Snail Mathematics"

(4) In the same way the previous step of the obtained figure 5 as follows.

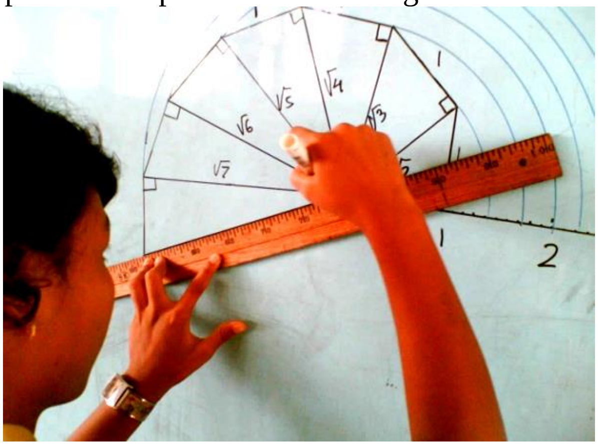

Figure 5: " A Sketch Drawing Process IV of Golden Snail Mathematics "

(5) Picture forwarded again to obtain figure 6 as follows.

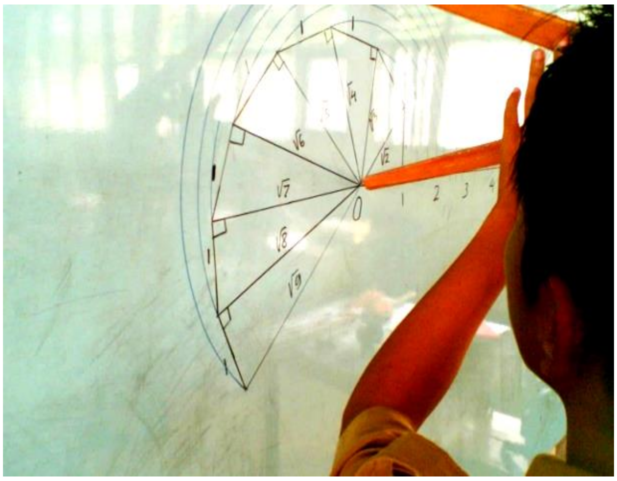

Figure 6: " A Sketch Drawing Process V of Golden Snail Mathematics"

(6) Next figure 7 obtained as follows.

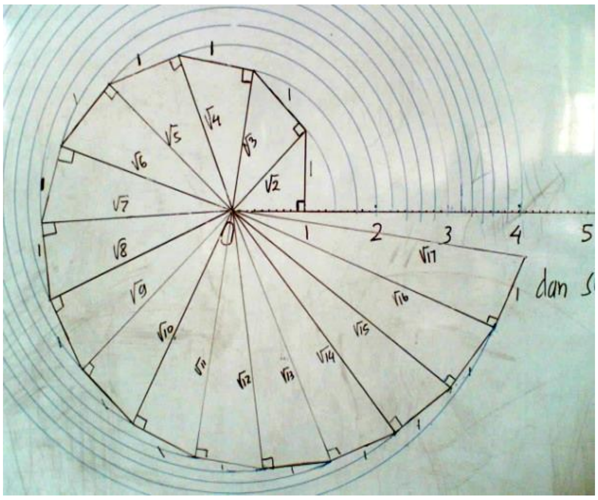

Figure 7: " A Sketch Drawing Process VI of Golden Snail Mathematics"

On the elaboration on learning are: (1) The teacher asks the students to present their work in the form of the golden snail sketches that have been done in the classroom, while providing explanations of mathematical interpretation of the sketch that has been produced.; and (2) Teachers asserted that the images have been produced similar to the golden snail, the more triangles are described in the paper as a medium quarto plane in Euclidean geometry it will be more apparent sketch formed the golden snail and the more numbers- rational numbers and the number of the root or irrational number is formed. The students answer is hoped: $\sqrt{4} = 2$, $\sqrt{9} = 3$, $\sqrt{16} = 4$ and so on → the rational numbers and $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, $\sqrt{6}$, $\sqrt{7}$ and so on → the root form numbers which subset irrational numbers.

On the confirmation on learning are: The teacher asks again informally as follows: (1) "Can you make the estimated value of $\sqrt{2}$, $\sqrt{3}$, $\sqrt{4}$, $\sqrt{5}$, $\sqrt{6}$, $\sqrt{7}$ through the golden snail sketches are you painting? "Try you specify each value estimates of the numbers are! (2) How are the area three right-angled triangles the first sketch that has formed the golden snail? And (3) How are the area seven triangles the first on the golden snail sketch that has been formed. The students answer is hoped: If the area of a triangle is equal to half the base multiplied by height multiplied by the triangle, then the area the first of three right-angled triangle is the first right-angled triangle + the second right-angled triangle + the third right-angled triangle, the mathematical expression is written $L = L_I + L_{II} + L_{III}$ so $L = (\frac{1}{2} x1x1) + (\frac{1}{2} x1x\sqrt{2}) + (\frac{1}{2} x1x\sqrt{3}) \Leftrightarrow L = (\frac{1}{2} + \frac{1}{2}\sqrt{2} + \frac{1}{2}\sqrt{3})$ that means $L = \frac{1}{2}(1 + \sqrt{2} + \sqrt{3})$ the unit area. The area the first of seven right-angled triangles is the first right-angled triangle I + the second right-angled triangle + the third right-angled triangle + … + the seven right-angled triangles, the mathematical expression is written

$L = L_I + L_2 + L_3 + L_{IV} + L_V + L_{VI} + L_{VII}$ so $L = ((\frac{1}{2}(1+\sqrt{2}+\sqrt{3}) + \frac{1}{2}(2+\sqrt{5}+\sqrt{6}+\sqrt{7})$ that means $L = \frac{1}{2}(1+2+\sqrt{2}+\sqrt{3}+\sqrt{5}+\sqrt{6}+\sqrt{7})$ the unit area.

On the closing activity, teacher leads the students to draw a conclusion, that the root form numbers consists of: $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, $\sqrt{6}$, $\sqrt{7}$ and so on. With the formal notation irrational numbers are numbers that can not be expressed as the ratio of integers, let's say "z" with the natural number let's say "a" such that irrational numbers cannot be written as $\frac{z}{a}$. While the root form number is the root of a number that the result is not a rational number so that from this understanding of the root number is also called irrational numbers because the root form number is a subset irrational number. Then the teacher gives assignments as homework let's say: make a sketch drawing back the golden snail individually and give a task: How is the total area of the right triangle on the golden snail sketch that has been formed it? The students answer is hoped: Area of all right-angled triangle is certainly $L_{\Delta total} = L_I + L_{II} + L_{III} + \ldots$ so $L_{\Delta total} = \frac{1}{2}\{(1+2+3+4+\ldots)+(\sqrt{2}+\sqrt{3}+\sqrt{5}+\sqrt{6}+\ldots)\}$, from this sense more students are expected to understand that the total amount of area of a right triangle is formed in the golden snail sketch it is equal to half multiplied by the number of natural numbers and certain irrational numbers.

4. The Result of Research and Discussion

4.1 The Result of Research

The learning outcomes assessment could be presented through the result of work's students as follow.

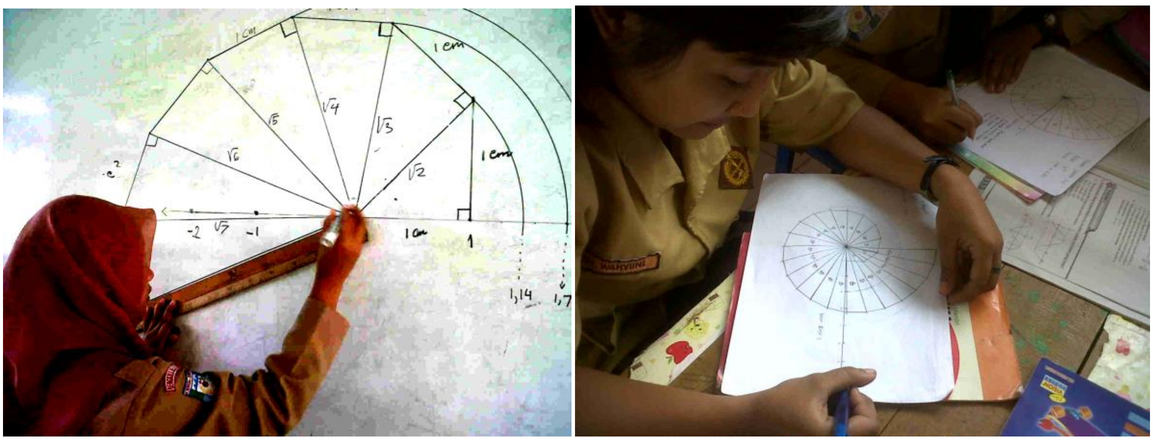

Figure 8: "The Student's Activity in Learning Process I"

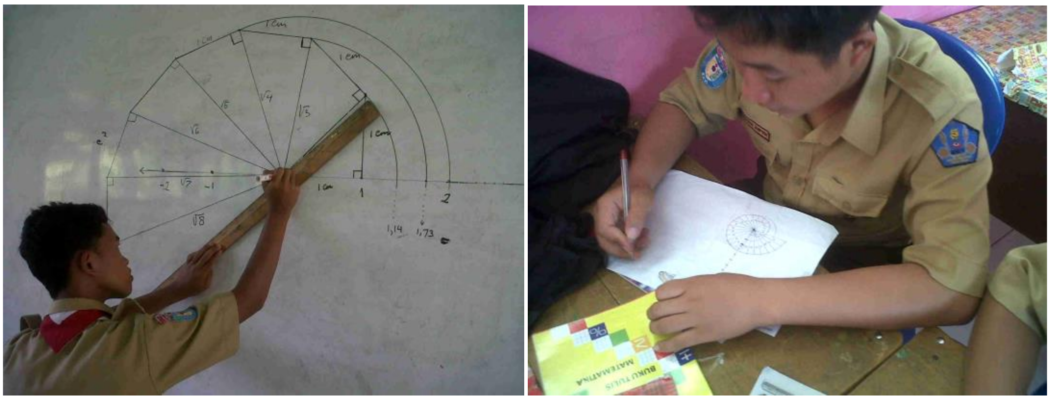

Figure 9: "The Student’s Activity in Learning Process II"

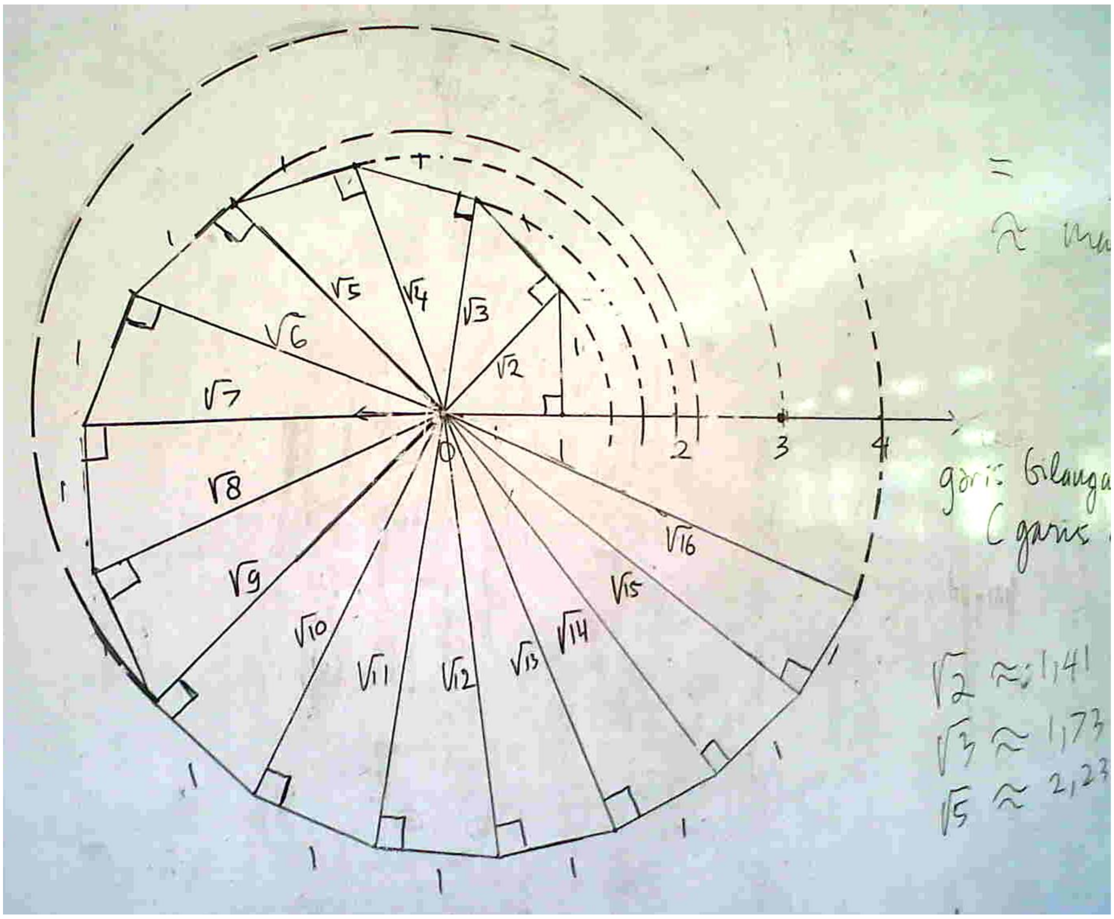

Figure 9: "The Student’s Sketch Golden Snail Mathematics and Estimation Value of Root Form Numbers at the White Board"

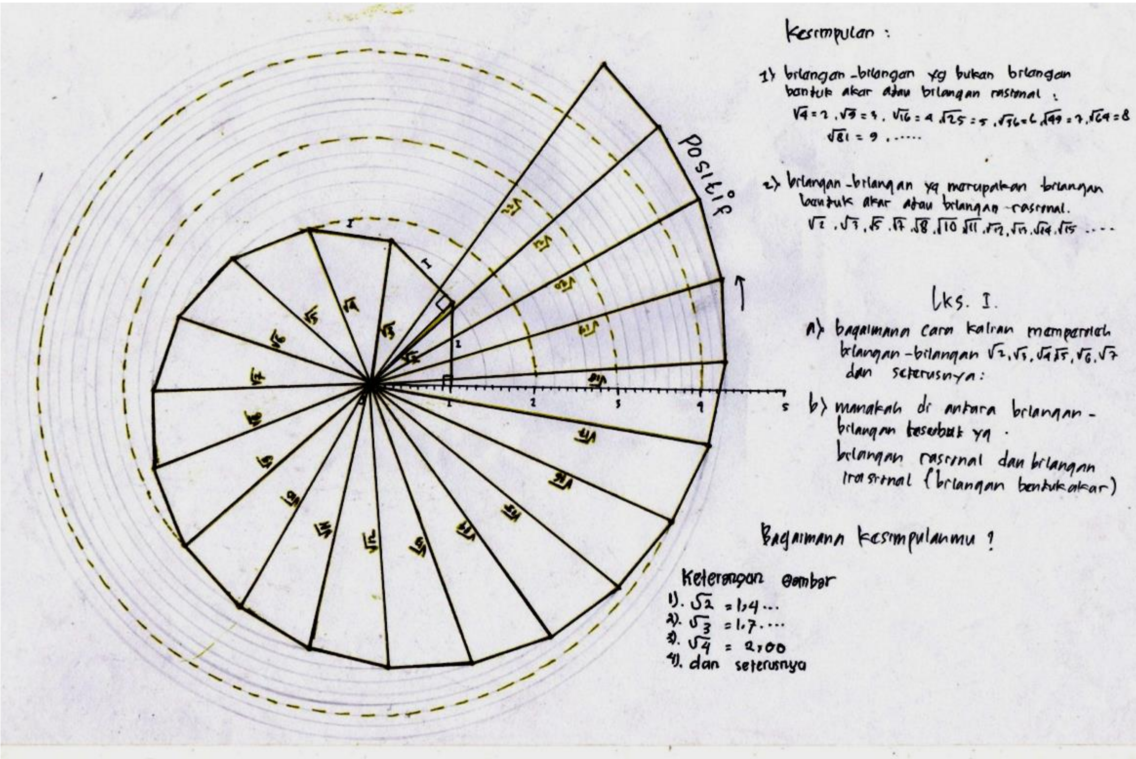

Figure 10: "The Student's Sketch Drawing of Golden Snail Mathematics on the Student Worksheet"

Now when given the problems that the images through mathematical snails have established that we would count the entire wide-angled triangle pattern generated to stop? Now when given the problems that the images through mathematical snails above have established that we would count the entire wide-angled triangle pattern generated to stop? Of course our answer is infinitely many sketch a right triangle is obtained. If we look at the right triangle I in the picture above mathematical snail until right triangle to infinity then we will get a broad overall right triangle ($L_{\Delta total}$) is like form:

$$\frac{1}{2}+\frac{1}{2}\sqrt{2}+\frac{1}{2}\sqrt{3}+1+\frac{1}{2}\sqrt{5}+\frac{1}{2}\sqrt{6}+\frac{1}{2}\sqrt{7}+\frac{1}{2}\sqrt{8}+\frac{3}{2}+\frac{1}{2}\sqrt{10}+\frac{1}{2}\sqrt{11}+\frac{1}{2}\sqrt{12}+\ldots=$$

$$\frac{1}{2}\left(1+\sqrt{2}+\sqrt{3}+2+\sqrt{5}+\sqrt{6}+\sqrt{7}+\sqrt{8}+3+\sqrt{10}+\sqrt{11}+\sqrt{12}+\ldots\right)$$

$$=\frac{1}{2}\left(1+2+3+\sqrt{2}+\sqrt{3}+\sqrt{5}+\sqrt{6}+\sqrt{7}+\sqrt{8}+\sqrt{10}+\sqrt{11}+\sqrt{12}+\ldots\right)$$

$$=\frac{1}{2}\left((1+2+3+\ldots)+(\sqrt{2}+\sqrt{3}+\sqrt{5}+\sqrt{6}+\sqrt{7}+\sqrt{8}+\sqrt{10}+\sqrt{11}+\sqrt{12}+\ldots)\right)$$, such that:

$$L_{\Delta total}=\frac{1}{2}\left((1+2+3+\ldots)+(\sqrt{2}+\sqrt{3}+\sqrt{5}+\sqrt{6}+\sqrt{7}+\sqrt{8}+\sqrt{10}+\sqrt{11}+\sqrt{12}+\ldots)\right).$$ From here obtained that line number: (1, 2, 3, ...) is the natural numbers and $\left(\sqrt{2},\sqrt{3},\sqrt{5},\sqrt{6},\sqrt{7},\sqrt{8},\sqrt{10},\sqrt{11},\sqrt{12},\ldots\right)$ are irrational numbers. From here we will recall the discovery of the Theaetetus about 415-369 BC (Sumardyono, 2003) that the root of a natural number is irrational numbers if and only if the original number is not a number quadratic form. It is filled with that 2, 3, 5, 6, 7, 8, 10, 11, 12, ... are the original numbers are

not a quadratic form of numbers such that the roots of these numbers are irrational numbers. For further explanation that root form number is the root of a number is the result not of the meaning of rational number to the number of roots referred to as the irrational number.

4.2 Discussion

The application of golden snail learning strategy to the mathematics teaching and learning gave success to them in understanding about root form numbers well. The application of golden snail learning strategy in teaching and learning mathematics at grade X in Senior High School State 6 South Bengkulu was very influential on the results of the root form number concept mastery. This is shown by the results of student learning are very good. All students (100%) can learn with good success. They managed to reach the minimum completeness criteria established in this study. From the analysis of students learning log and interview indicated that 100% of students to be positive in mathematics. This study concludes that the model of learning with the application the golden snail learning strategy on the subject matter of the root form number well and worthy to be applied. Trough the application of golden snail learning strategy made teaching and learning mathematics became meaningfully and joyfully. Through the application of the golden snail learning strategy students can explore different forms to find the root form number. They also can determine the values estimated from the root form number of the real line through a predetermined and snails mathematical representation that they have described.

Furthermore in this study also found evidence in the form of analysis of the numbers of certain irrational prove that students explore a number of the root is an irrational number or not. For example $\sqrt{7}$ is the irrational number. The proof is assuming $\sqrt{7}$ is a rational number (using indirect evidence or by way of contradiction) that can be written as $\sqrt{7} = \frac{z}{a}$ with $z \in B$ and $a \in A$ where B is an integer and A is natural number; z and a have no common factors other than 1 and write (z, a) = 1. This means that $7 = \frac{z^2}{a^2} \Leftrightarrow 7a^2 = z^2 \Leftrightarrow 7|z$ and means that z is a multiple of 7, because z is multiple of 7 then $7a^2 = 49w \Leftrightarrow a^2 = 7w$ with $w \in B$. This means that 7 can divide a or in other words that a is multiple of 7 to be written $7|a$. Since $7|a$ and $7|z$ then (z, a) = 7 then this is a contradiction with the supposition that $\sqrt{7}$ is the irrational number (Budi, 2005). Thus if we want to prove that $\sqrt{7}$ + 4 is a irrational number, It's sufficient to be assumed that $\sqrt{7}$ + 4 is a rational number $\frac{z}{a}$ so $\sqrt{7} + 4 = \frac{z}{a} \Leftrightarrow \sqrt{7} = \frac{z}{a} - 4$ it just means that $\frac{z}{a} - 4$ rational number, and note that the number of rational obscurantist at reduction. This is a contradiction because $\sqrt{7}$ is the irrational number so the assumption that $\sqrt{7}$ + 4 is the rational number was incorrect. So concluded that $\sqrt{7}$ + 4 is the irrational number. Furthermore, if we are going to prove that $3\sqrt{2}$ is irrational and has been known that $\sqrt{2}$ is irrational number then simply assumed that $3\sqrt{2}$ is also a rational number so that we can write that $3\sqrt{2} = \frac{z}{a}$. Then by multiplying both sides by $\frac{1}{3}$, it's means $\frac{1}{3}\left(3\sqrt{2}\right)=\frac{1}{3}\left(\frac{z}{a}\right) \Leftrightarrow \sqrt{2} = \frac{z}{3a}$. Since $\frac{1}{3}\left(\frac{z}{a}\right)$ is the multiplication of two irrational numbers and is closed on multiplication then $\frac{z}{3a}$ is a rational number that $\sqrt{2}$ is also a rational number. This is certainly a contradiction or conflict with the assumptions above, it means $3\sqrt{2}$ is a rational number is incorrect. It was concluded that $3\sqrt{2}$ is irrational number. Some examples of irrational numbers are $\sqrt{5}, \sqrt{11}, \pi, \log 5, \cos 45^0$, as an example decimal form (.28288288828888288888 ...), (0.303003000300003 ...), the form of the number of such examples $\sqrt[3]{5}, \sqrt[7]{13}$, and so on.

5. **Conclusion**

Mathematics learning strategy using the Application of golden snail learning strategy on the subject matter of the root form number well and worthy to be applied in State Senior High School 6 of South Bengkulu as could be seen that the study of students in the root form numbers concept was very good once applied learning through the application golden snail learning strategy and could be presented teaching and learning process are meaningful and joyful well. Through the analysis of result work's sheet student's document and learning log through golden snail learning strategy also affects to the achievement of the achievement learning outcomes and positive attitude or good impression to mathematics of students. Development of learning materials through a root form number of the golden snail learning strategy is very accurate to instill the concept of the root form number of a subset of the irrational number to learners. Based on the results of the research done and the interview turned out to be a very good student response and conclude that mathematical learning strategy adopted golden snail learning strategy is very interesting and they realize that mathematics is an integral part of mathematics and mathematics itself was so connected with nature around us. Trough the application of golden snail learning strategy made teaching and learning mathematics became meaningfully and joyfully.

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The Hypothetical Learning Trajectory on Addition in Mathematics GASING

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Abstract: *The purpose of this study is to look at the role of learning addition operation in Mathematics GASING (Math GASING) in helping students understand and masterthe addition concept from the informal level (concrete) into formal level. The research method used is a design research with a preliminary design, teaching experiments, and retrospective analysis stages. This study describes how the Math GASING made a real contribution for students to understandthe concept of addition. The whole strategy and model that students discover, describe, and discuss as the construction or contribution show how students can use to help their initial understanding of the addition concept. The stages in the learning trajectory have an important role in understanding the addition concept from informal level to formal level and also make the study of mathematics more easy, fun, and enjoyable.*

1. Introduction

Professional teachers, as the product of reform in education, must have higher educationand be able to innovate in teaching and learning (Prahmana R. , 2013). So, every prospective teacher should be prepared to become a professional teacher to equip himself with a high education and knowledge of the learning and teaching process, for example using a spreadsheet, such as Microsoft Excel, to provide educators with a creative tool for the study and teaching of mathematics, mathematical modeling, and mathematical visualization where Excel's graphics can create eye-catching animated graphic displays and inject more fun into the study of mathematics (Arganbright, 2007). On the other hand, prospective teachers who come from rural areas have a little access to get a decent education and information as requirements to become professional teachers. Surya College of Education has a responsibility to help them toget a great education to become professional teachers, mathematics teachers. In addition, Surya and Moss have created and applied a learning innovation in mathematics education named Mathematics GASING (Math GASING). This learning has been applied to student from Papua, which began with the introduction of numbers and number operations and produced many Olympic champions both nationally and internationally (Surya & M.Moss, Mathematics Education in Rural Indonesia, 2012).

Furthermore, learning number operations at the primary school is important for learning other subjects. Because learning number operations help them to understand notation, symbols, and other forms to represent (reference numbers), so it can support the students' thinking and understanding to solve problems (National Council of teachers of Mathematics (NCTM), 2000). The results of several previous studies show that students have a difficulty to understand the number operation concept (see (Prahmana R. , 2013) and (Prahmana, Zulkardi, & Hartono, 2012)). It's supported by the results of rural area students, namely Ambon, Serui, and Sorong Selatan, classroom observations toward to learning number operations conducted by researchers in pre-test. Students are more likely to be introduced by the use of the formula without involving the concept itself and learning number operations separate the concrete situation of learning. Addition operation is a number operation which first must be mastered before student learns another number operations starting from the introduction of a number (Reys, Suydam, Lindquist, & Smith, 1984). This underlies the researcher to try designing addition operation learning in Math GASING which always starts from the concrete (informal level) to the abstract (formal level) for matriculation prospective teachers students at Surya College of EducationTangerang derived from Ambon, Serui, Yapen, and South Sorong, Papua.

2. Math GASING

Surya and Moss stated that GASING has four basic premises (Surya & M.Moss, Mathematics Education in Rural Indonesia, 2012). First, there is no such thing as a child that cannot learn mathematics, only children that have not had the opportunity to learn mathematics in a fun and meaningful way. Second, mathematics is based on patterns and these patterns make math understandable. Third, a visual context to mathematical concepts should come before the symbolic notation. The last, mathematics is not memorization, but knowing basic facts comes easily with a conceptual and visual understanding. Memorization of basic mathematics facts is easy if it is based on conceptual learning and visual representations. Additionally, Shanty and Wijaya describe Math GASING as the learning process that make students learning easy (*Gampang*, in Bahasa, red), fun (*ASIk*, in Bahasa, red), and enjoyable (*menyenaNGkan*, in Bahasa, red) (Shanty & Wijaya, 2012). Easy means the students are introduced to mathematical logic that is easy to learn and to remember. Exciting means the students have motivation which comes from by them to learn mathematics (intrinsic factor). Fun is more in the direction of outside influences such as visual aids and games (extrinsic factor). On the other hand, Prahmana had been conducted research for division topic in Math GASING, where the learning process begins with the activities share sweets fairly, then move into the process of how each student gets distributed sweets after a fair amount of candy (concrete), ranging from division without remainder to division with remainder, and ends with the completion of division operation in Math GASING (abstract) (Prahmana R. , 2013). Math GASING shows how to change a concrete sample into an abstract symbol so the students will be able to read a mathematical pattern, thus gain the conclusion by themselves.

2.1. Addition Operations in Math GASING

Math GASING, as one of the innovations in learning mathematics, offers a critical point in its learning process. When studying a topic in Math GASING, there is a critical point that we must pass that is called GASING’s critical point. After reaching this critical point, students will not have difficulty anymore working on the problems in that topic (Surya, Petunjuk Guru: Dasar-Dasar Pintar Berhitung GASING, 2011). The critical point in learning addition is the addition of two numbers between 1 and 10 with a sum less than 20. In other words, when a student has mastered the addition of two numbers between 1 and 10 with a sum less than 20, the student can learn a variety of addition operation problems more easily.

The Hypothetical Learning Trajectory (HLT) in this study had several learning goals expected to be reached by the students. To reach the goals formulated, researcher designs a sequence of instructional learning for learning addition in Math GASING on the following diagram (see Figure 1).

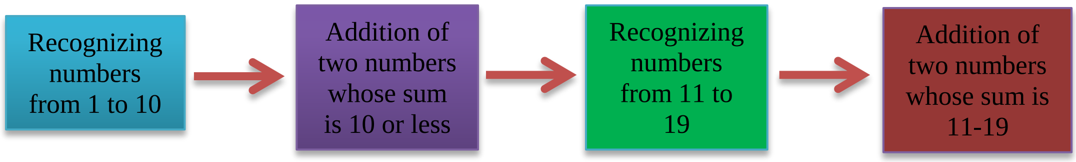

Figure 1.the HLT of Learning Addition in Math GASING

The explanation of Figure 1is as follows:

1. Students are recognizing numbers from 1 to 10 by using their fingers. For example, the teacher introduces the notation of number "1" by showing 1 index finger or 1 thumb or 1 middle finger or 1 ring finger or 1 little finger; the notation of number "2" by showing 1 index finger and 1 thumb or 2 index fingers or 2 thumbs, and various other variations; the notation of number "3", "4", until "10" by using various other variations from their finger.

2. Students learn the addition of two numbers whose sum is 10 or less by using their fingers. For example,the teacher shows 3 fingers on the right hand and said "these are three fingers", and then shows 2 fingers on the left hand and said "these are two fingers". After that, she combined the fingers on both hands together and said "these are five fingers", so "three plus two is equal to five". Lastly, students learned how to write in abstract symbols: 3 + 2 = 5. The teacher shows all various combinations of addition from 2 to 10.

3. Students are recognizing numbers from 11 to 19 by using "number cards". For example, "black cards" are tens and "white cards" are units. The teacher showed 1 black card and said "this is ten", and then showed 1 white card and said "this is one". After that, she combined the two cards that consists of 1 black card and white card, and said "this is eleven". This way the students are able to imagine that eleven consists of ten and one. The teacher shows all numbers from 11-19 by using black cards and white cards.

4. Students learn the addition of two numbers whose sum is 11-19 by using "number card". The learning process consists of some addition types namely 10+, 9+, 8+, 7+, and 6+ and starts from 10+ type where student add the number 10 with numbers from 1 to 9, to 6+ type (see Table 1). They also learn about the commutative of addition according to Table 1.

Table 1. Addition of Two Numbers Whose Sum is between 11 and 19

10 +	9 +	8 +	7 +	6 +
10 + 1 =				
10 + 2 =	9 + 2 =			
10 + 3 =	9 + 3 =	8 + 3 =		
10 + 4 =	9 + 4 =	8 + 4 =	7 + 4 =	
10 + 5 =	9 + 5 =	8 + 5 =	7 + 5 =	6 + 5 =
10 + 6 =	9 + 6 =	8 + 6 =	7 + 6 =	6 + 6 =
10 + 7 =	9 + 7 =	8 + 7 =	7 + 7 =	
10 + 8 =	9 + 8 =	8 + 8 =		
10 + 9 =	9 + 9 =			

2.2. Research Question

Based on a few things mentioned in the introduction above, researcher formulates a research question in this study, as follows:

"What is the student learning trajectory of learning addition in Math GASING, which evolved from informal level to formal level for rural are a student at Surya College of Education?"

3. Methods

This study uses a design research as research method, which is an appropriate way to answer the research questions and achieve the research objectives that start from preliminary design, teaching experiments, and retrospective analysis (Prahmana, Zulkardi, & Hartono, 2012). Design research is methodology that has five characteristics, which is interventionist nature, process oriented, reflective component, cyclic character, and theory oriented (Akker, K., & Nieveen, 2006). To implementation, design research is a cyclical process of thought experiment and instruction experiments (Gravemeijer, Developing Realistic Mathematics Education, 1994). There are two important aspects related to design research. They are the Hypothetical Learning Trajectory (HLT) and Local Instruction Theory (LIT).

According to Freudenthal in Gravemeijer and Eerde (see (Gravmeijer & D.V, 2004)), students are given the opportunity to build and develop their ideas and thoughts when constructing the mathematics. Teachers can select appropriate learning activities as a basis to stimulate students to think and act when constructing the mathematics. Gravemeijer states that the HLT consists of three components, namely (1) the purpose of mathematics teaching for students, (2) learning activities, devices or media used in the learning process, and (3) a conjecture of understanding the process of learning how to learn and strategies students that arise and thrive when learning activities are done in class (Gravemeijer, Local Instructional Theories as Means of Support for Teacher in Reform Mathematics Education, 2004).

For the data, researchers have collected research data derived from multiple sources, to get a visualization of the students' mastery of basic concepts of addition operations, namely video recording, documentation (learning activities photo), and the written data (the results of students' answers and observation sheet). Furthermore, the data were analyzed retrospectively with HLT as a guide. In addition, these studies have been completed in two days in the first semester of academic year 2013/2014 with the subjects being 11 matriculation prospective teacher students at Surya College of EducationTangerang from Ambon, Serui, Yapen, and South Sorong, Papua, and also a teaching assistant who acted as a model teacher. This study consists of three steps done repeatedly until the discovery of a new theory that a revision of the theory of learning is tested (see in Figure 2).

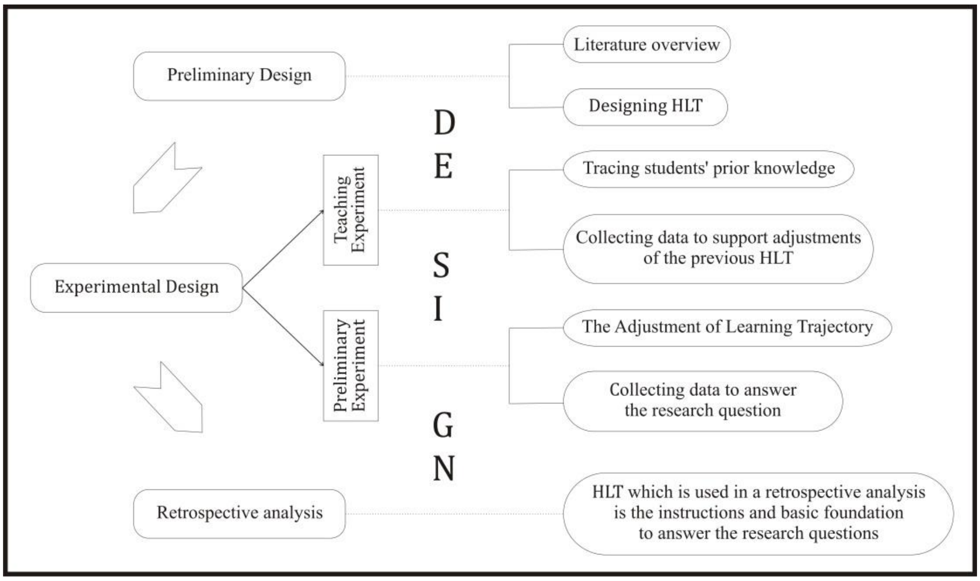

Figure 2. Phase of the Design Research (see (Prahmana, Zulkardi, & Hartono, 2012))

4. Results

The learning activities start from recognizing numbers between 1 and 10 using students' fingers to introduce the concept of number in concrete levels asa sum of their fingers. Furthermore, Students learn the addition of two numbers whose sum is 10 or less by using their fingers. Lastly, students are recognizing numbers from 11 to 19 by using "number cards" and learning the addition of two numbers whose sum is 11-19 by using "number cards" that consist of black cards as tens and white cards as units. At the end of the second meeting, students do mental arithmetic activities namely *mencongak* as one of assessment process in this learning activities and exercise by using student evaluation sheet. As a result, students were able to master the addition operation in Math GASING seen from the results of the final evaluation and were pleased to learn Math GASING can be seen from the comments of students who wish to abandon the old way of learning mathematics. The results of this study indicate that learning design of addition in Math GASING has a very important role as the starting point and improve students' motivation in learning addition.For more details, researchers will discuss the results of this study, which is divided into three stages that are called preliminary design, teaching experiments, and retrospective analysis.

4.1. Preliminary Design

At this stage, the researcher is beginning to implement the idea of addition operation in Math GASING by reviewing the literature, conducting observations in matriculation class, and designing hypothetical learning trajectory (HLT), as shown in Figure 2. A set of activities for learning addition operation in Math GASING has been designed based on learning trajectory and the thinking process of students hypothesized. The instruction set of activities has been divided into four activities that have been completed in two meetings, starting from recognizing numbers between 1 and 10 using students' fingers as the concrete form, learning the addition of two numbers whose sum is 10 or less process by using their fingers, recognizing numbers from 11 to 19 by using "number card", learn the addition of two numbers whose sum is 11-19 by using "number cards" that consist of black cards as tens and white cards as units, doing a variety of fun activities that make students happy in the learning process, and ending with the evaluation process.

4.2. Teaching Experiment

In the teaching experiment, the researcher tests the learning activities that have been designed in the preliminary design stage. When the teacher models have started to see students do not get excited, then the teacher models provide educational games that make learning activities fun, because it is one of the characteristics on Math GASING. There are four activities in this stage. First, the teacher introduces the notation of number 1 until 10 by showing her finger and student recognizes it by using their fingers as the concrete form using various other variations from their fingers (see in Figure 3).

Figure 3. StudentsRecognize Numbers between 1 and 10 Using Their Fingers

Second, students learn the addition of two numbers whose sum is 10 or less by using their fingers. A student showed four fingers on the right hand and said "this is four", and then showed two fingers on the left hand and said "this is two". After that, she combined the fingers on both hands together and said "this is six", so "four plus two is equal to six". Lastly, students write in abstract symbols: $4 + 2 = 6$. The teacher showed all various combinations of addition from 2 to 10 (see in Figure 4).

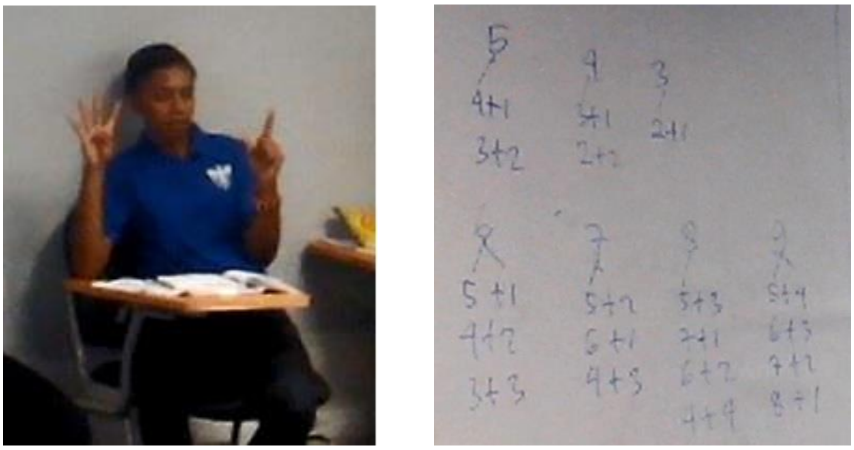

Figure 4. Students Learn Addition of Two Numbers (Left) and All Various Combinations (Right)

Third, students are recognizing numbers from 11 to 19 by using “number cards” that consists of a “black card” as ten and a “white card” as a unit. A student showed one black card and said, “This is ten”, and then showed one white card and said, “This is one”. After that, she combined the two cards of a black card and a white card, and said, “This is eleven”. The teacher showed all numbers from 11-19 by using black cards and white cards (see in Figure 5).

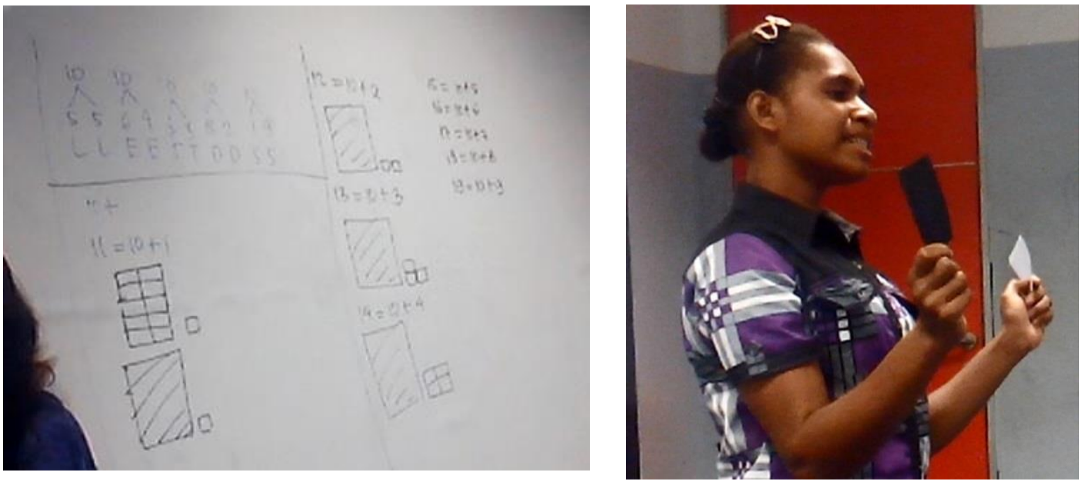

Figure 5.the Concrete Form of 10+ Type (Left) and Students Presented the Number Card (Right)

Lastly, students learn the addition of two numbers whose sum is 11-19 by using “number card”. The learning process consists of some addition types namely 10+, 9+, 8+, 7+, and 6+ and starts from 10+ type where students add the number 10 with numbers from 1 to 9, to 6+ type (see in Table 1). They also learn about the commutative of addition according to Table 1. Students count nine white cards plus three white cards, and then count all white cards that she gets. After that, she switches 10 white card with one black card and now she gets one black card and two white cards, and said, “This is twelve” (see in Figure 6).

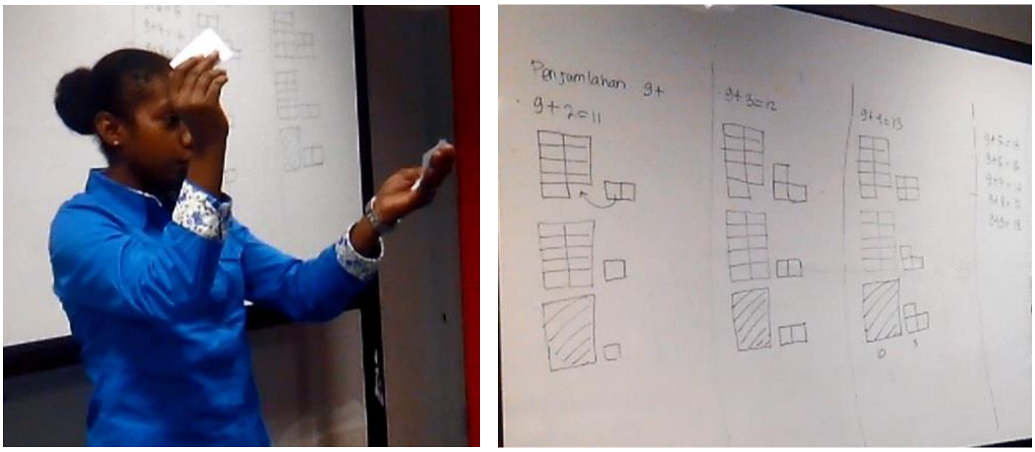

Figure 6. Students Learn Addition of Two Numbers (Left) and All Various Combinations 9+ (Right)

The learning process in this study ends with two forms of evaluation. First, students are given aspontaneous problem in front of class and answers on the whiteboard. Second, students are given a worksheet that consists of many questions about addition and should be able to finish it within a few minutes(see in Figure 7). The results of the evaluation process are quite amazing that all students get satisfactory results and are able to explain it either concretely or abstractly.

Figure 7.the First Evaluation Form (Left) and the Second Evaluation Form (Right)

4.3. Retrospective Analysis

The addition process in Math GASING is different from the addition process in mathematics in general. As a result, all activities which have been designed can be used to answer the research question above. The activities are as follows:

1. The learning trajectory which has been modeled in Figure 1 are the activities undertaken in this study to guide students to master addition operation. So that, the researcher designed an activity using students' fingers to recognize numbers from 1 to 10 and to learn by using their fingers too. The goal is that students are able to imagine the concrete form of number notation between 1 and 10 and the addition of two numbers whose sum is 10 or less. Fingers are the greatest learning tool to introduce the concrete form of numbers between 1 and 10 and that addition. Next, to introduce a number greater than 10

and its sum, the researcher uses a number card starting from units, tens, hundreds, etc. by using different colors such as a white card for a unit, a black card for tens, an orange card for hundreds, etc.

2. Furthermore, from these activities, teachers guide students toward the concept of number notation and addition using their fingers and number cards. As a result, when students have mastered the addition process of two numbers whose sum is between 11 and 19, they were able to complete the various forms of addition operations more easily, using the addition process in Math GASING. So that, they can do *mencongak* to solve all addition problems given.The addition process in Math GASING starts from "front addition" where addition process is from left to right and "scratch system" for the addition of two numbers whose sum is between 10 and 19. For more details, see Figure 8.

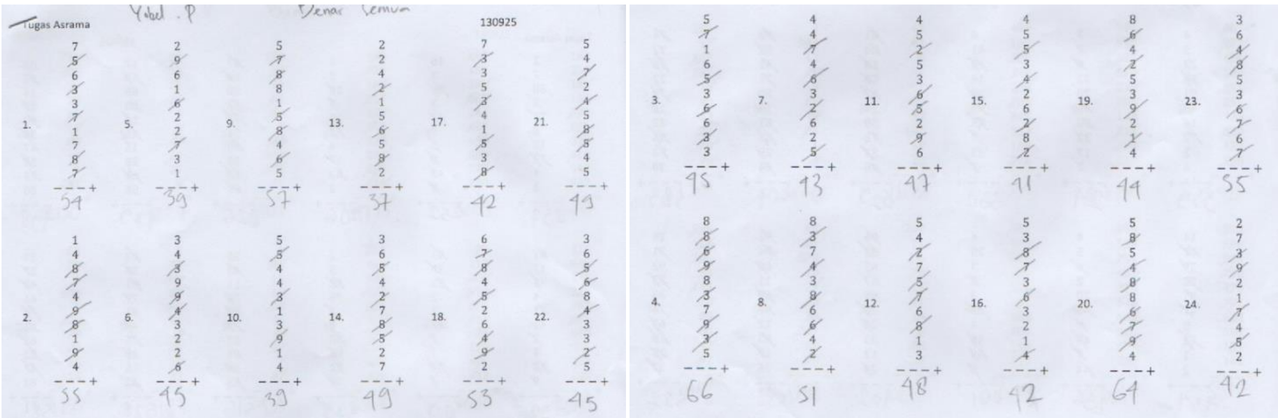

Figure 8. Yobel's Answer Sheet using "Scratch System"

3. In Figure 8, Yobel's answer sheet shows the addition process in Math GASING where every addition of two numbers whose sum is more than or equal to 10, he "scratchs" on that number and so on until finished and then counted the number of scratches made and wrote it followingthe last number that he counted. For example, based on problem 4, 8 + 8 is equal to 16. It means he must scratch on "8", and then 6 + 6 is equal to 12. It means he must scratch again on "6", and then2 + 9 is equal to 11. It means he must scratch on "9", and then1 + 8 is equal to 9. It means he doesn't scratch on "8", and continues until finished. Lastly, he got 6 scratches and "6" as the last number that he count. So, he can write "66" as the result of that problem.On the other hand, Figure 9 shows that Rosita can solve 100 addition problems in only 7 minutes and gets one mistake using "front addition". It is apparent that addition process in Math GASING is much more effective than the usual process of addition, when students have mastered the addition of two numbers whose sum is between 11 and 19.

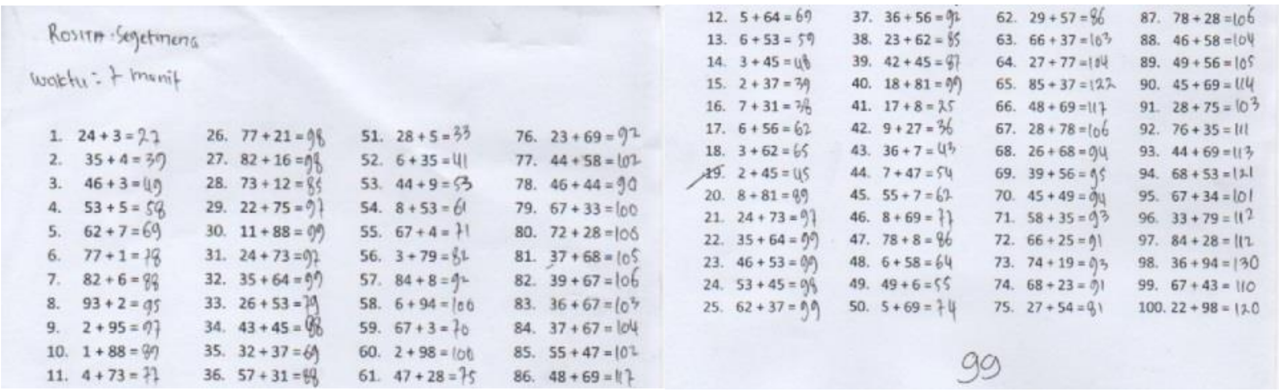

Figure 9. Rosita's answer sheet

4. Based on all the activities above, it can be seen that the students have gone through the process of activity based on experience using their fingers and "number card", moving toward a more formal, the understanding of formal level from the critical point, and then reached into the formal level desired as the ultimate goal of this learning activity.

5. In the design of this study, the researcher used the learning steps of addition operation in Math GASING as shown in Figure 1. When the activity takes place, the dialogue is very good in the process of introducing the basic concepts of addition operations. In the dialogue, it seems that students feel learning addition operation in Math GASING looks so easy and so much fun. As a result, the learning process can guide students in understanding addition operations. It can also be seen from the student evaluation of learning the addition process given by the teacher to evaluate student understanding. As a result, students seemed to be able to apply addition operation process in solving each problem is given in terms of evaluation (see in Figure 10). Therefore, it can be seen that learning addition operation in Math GASING can be used to raise students' understanding of integer addition operations, or in other words, the design of this study can be used as the starting point of learning addition operations.

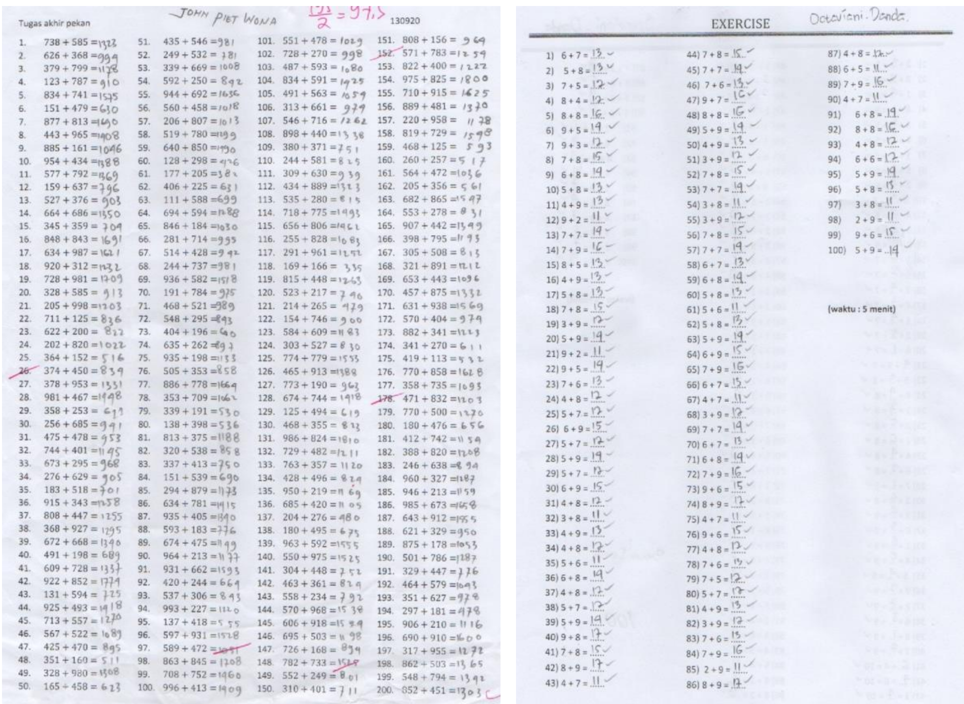

Figure 10. Some Students'Answer Sheets

5. Conclusion

Based on the result of this research that has been described above, the researcher can conclude that the learning of addition operation in Math GASING has a very important role as the starting point and improves students' motivation in learning addition operation. In addition, the activities have been designed in such way thatstudents find the concept of addition operation starting from recognizing number between 1 and 20 to calculating addition operation of two numbers whose sum is between 11 and 19, which is the critical point of addition operation in Math GASING. This process begins with the activities recognizing the numbersbetween 1 and 19 using their fingers and number cards (black and white card), and then moving into the process of how to calculate two numbers addition operation of two numbers whose sum is between 11 and 19. Lastly, each student can do *mencongak* for any given addition problem and resolve many addition questions very quickly and precisely where is both of this are one of assessment forms in Math GASING.

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Supporting Students' Understanding of Area Measurement Through Verknippen Applet

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***Abstract**: In this paper, we will present a case study of the possibility to use applet in teaching and learning of area measurement. The researcher investigated how an applet can support students' understanding of the concept of conservation of area. Data collected through video registration, students' work and interview. Students have involved in six meetings of learning of area measurement. In some meetings, students worked on hands-on activities. For instance, through cut and paste activities, students would acquire the concept of conservation of area. They witness that if there is no part thrown away when cutting and pasting a figure, the area will not change. However, when it comes to reshape a figure into another one (rectangle), students would do trial and error. Some students would face difficulties reshaping the figures into a rectangle. They could not undo their cutting when they failed in making a rectangle. To some extent, hands-on activities will constraint students. The use of applet which provides the same activities will help students to be more creative in reshaping a figure.The school has no internet facilities to access the applet. To see the possibilities of an applet to support students understanding, after some weeks, we invited students to play with applet. We recorded students' activities in working with the applet. Students were enthusiastic to try when they could not solve the problems. After the session with applet, students worked on alike-problems on paper. They successfully dealt with the problems. It reveals that students prefer to use applet because they can try again when they fail. It shows that students could understand how to measure areas of irregular figures.*

1. Introduction

This paper is a continuation of our study of area measurement. Area is an amount of two-dimensional surface that is contained within a boundary and thatcan be quantified in some manner (see Baruto&Nason, 1996). Reynolds & Wheatley (1996)area measurement assumes that: 1) asuitable two-dimensional region is chosen as a unit, 2) congruent regions have equal areas, 3)regions do not overlap, and 4) the area of the union of two regions is the sum of their areas.The design learning sequences in the previous study apply all the concepts (see Yunianto, 2014). However, the researcher started the lesson by focusing on the comparing areas to ignite overlapping strategy. Afterward, students will cut and pasted if they are provided with cutting tools.

We research on supporting students understanding of area measurement through reallotment activities. Students participated in designed activities in six meetings. The activities were designed and developed within realistic mathematics education (RME) tenets. Reallotment means an activity of redistributing or reallocating or rearranging something (Holt, Rinehart, & Winston, 2003).In area measurement, it embeds the concepts of conservation of area in which the area of a figure is preserved when it is reshaped. Some studies have shown that understanding theconservation of area is important and fundamental before students learn area measurement (Kordaki, 2003). To master this concept, students can experience activities of cutting, pasting and rearranging a figure into a new one with an equal area (see Kordaki, 2003). Therefore, reallotment activities in the designed lessons involved cutting, pasting and reshaping into a rectangle. We planned to integrate ICT in the teaching and learning in the designed lessons. Considering that the school's facilities are not supported with the internet access, the researcher omitted the use of applet in the initial designed activities. Therefore, in the six meetings, the applet was not used but only involving the hands-on activities.

We found that students could attain their understanding of the concepts of area measurement through hands on activities (Yunianto, 2014). Students could understand the meaning of area; the region inside within a boundary. They witnessed that if no parts are wasted in the cutting and pasting, then the area would not change. Reshaping into a rectangle would help students to measure areas of others quadrilaterals or even triangles. Dealing with cutting and pasting to reshape a figure into a rectangle, some students had difficulties once they failed in making a rectangle from the figures. Students would not easily undo what they have made because they have also glued it. Some researches show that ICT or any applets can support students' understanding and improve their motivation. There are three crucial factors for the success of digital technology in mathematics education including the design of the digital tool, the role of the teacher and the educational context (Drijvers, 2013)The researcher chose the verkinnipen applet because its design for the tasks is so powerful to support students' understanding of the concept of conservation of area. Conservation of Area and its Measurement (C.AR.ME)microworldhad been designed and used to support students' understanding of conservation of area (Kordaki, 2003).In this paper, we raise a research question: to what extent verkinnipen applet could help students learn concept of conservation area.

2. Method

The purpose of this study is to investigate whether verknippen applet can be used to support students' understanding the conservation of area. The applet can be accessed in:http://www.fi.uu.nl/toepassingen/00457/serie1.html. It has been two weeks since the last meeting ended. We selected four students from a classroom used previously in the six meetings teaching and learning of area measurement to participate in this study. These students are considered as medium and high performance students based on the classroom teacher and observations. Data collected through video registration, screen recording and students' work.

3. Result and Discussion

Since the researcher only used one laptop to conduct this study, in turn, students worked on the applet.While students were working on the applet, Camtasia recorded the screen. All students' steps can be captured and saved into a video data. This recording can be used as an assessment because how students solved the tasks can be observed. Through this recording, we might see students' different strategies or even the same strategies. After a student finished with the tasks on the applet, then he or she worked on a paper based tasks. The applet has five series of task in each consists of nine to ten tasks. If they successfully made a rectangle the button became green. The researcher sometimes interviewed students informally while they were working on the tasks.

Students various strategies It is obvious that working with verkippen applet allows students to try out and experience the tasks many times. They could cut and paste many times in order to reshape a figure into a rectangle. When they made mistakes and got difficulties, they can start over again the task. The following is the display of the applet with Dutch language. It was not a big problem for students with the language. The researcher let students work with the applet first and ask them to make a rectangle from figures. The applet asks students whether students can make a rectangle from the shown figure. After students understood what they should do, in turn, students did the task.

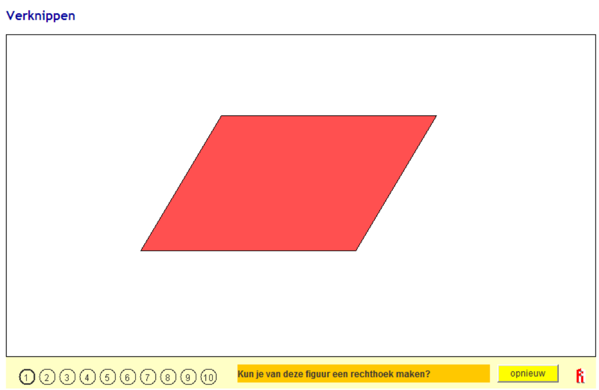


Figure 3.1 The task in Verknippen applet serie1

The following figures are captured from the screen recording.

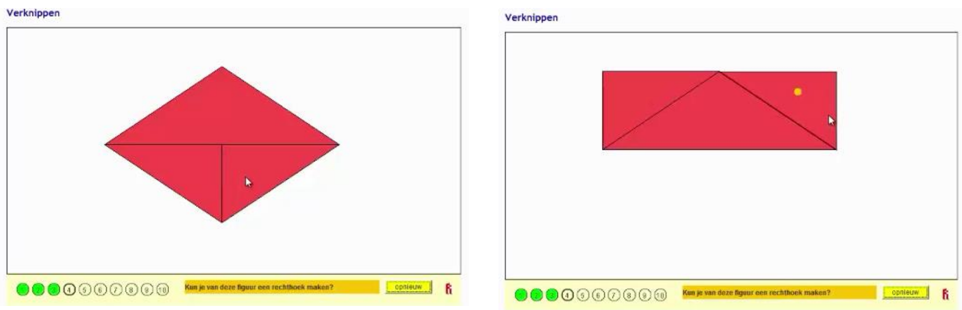


Figure 3.2 Student's work on the rhombus

It can be seen that this student divided the rhombus into two parts (upper and lower). He then divided the lower part into two parts. Finally, he moved the last two parts into the upper part and made a rectangle. Another student did differently as seen in figure 3. In fact, it is not so much different but students might do what they think easier to do.

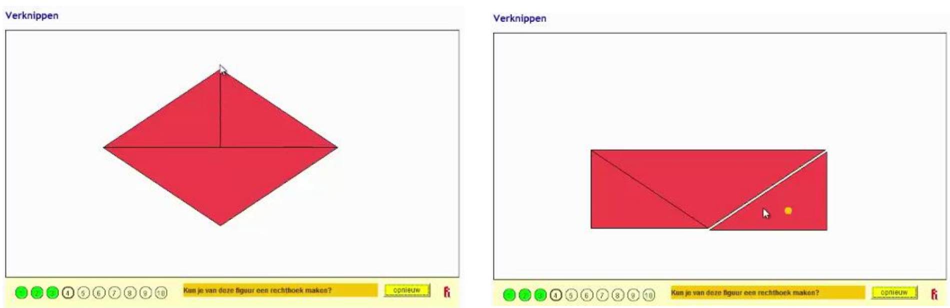


Figure 3.3 Student's work on rhombus

In doing task 7, students also had different ways of reshaping the irregular figure (figure 4). This task is more difficult compared to previous ones.

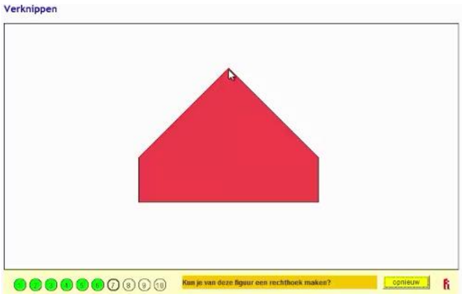


Figure 3.4 Task 7 in the verknippen applet

Students had to think how to reshape this figure into a rectangle. In previous tasks, students could easily see parts will be cut and moved. In this task, students tried out by firstly dividing the figure equally into two parts. They arranged the parts to see the possibilities of creating a rectangle. Some students made a vertical rectangle, and others made it horizontally.

Figure 3.5 Students' rectangle on task 7

One student who is considered as a high performance student asked for more challenging tasks. Then the researcher provided other tasks in the applet. Task 1 to task 4, she confidently made it. When it came to task 5, she faced troubles. She could not make it easily into a rectangle. She tried many times but failed. Her friends who were sitting next to her tried to help.

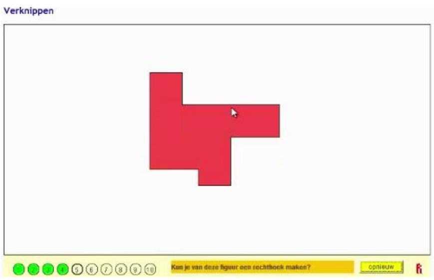


Figure 3.6 Task 5 in verknippen serie 2

The following figures show how she failed to make a rectangle.

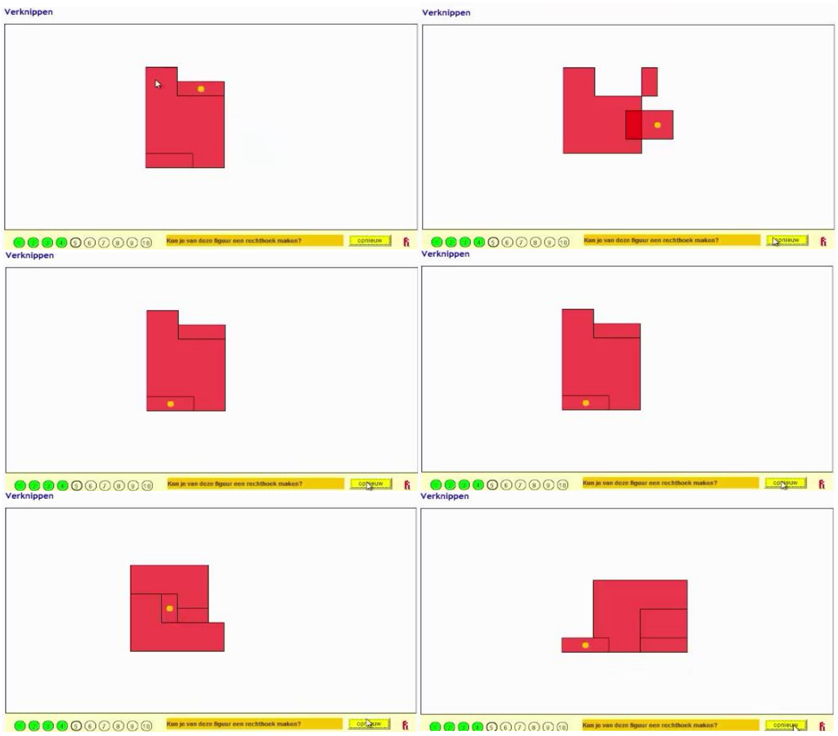


Figure 3.7 Student could not make a rectangle

More than six times she failed to reshape the figure into a rectangle. One of her friend asked for trying it out. She gave him a try. Interestingly, he could not make it in his first trial. However, she remembered that his friend was close to make it. He knew that she should have made it. He finally made it and felt happy.

Students' motivation In an informal interview, the researcher asked students about their opinion related to the activities in the applet.

R : Which one is better, the cut and paste activity or playing this applet?
S_D : This one (the applet) If we made mistakes, we can start it over again
R : So, which one is better?
S_A&S_D : This applet indeed
S_A : I could start it over

From this segment, students argued that the applet is better because they could start over and do the tasks provided in the applet. Compared to cutting and pasting paper where students could not redo what they have cut. When they have reshaped the figured improperly, then they would give up since they could not do it again. The flexibility of the applet proved that the feature helps students encourage them to have more strategies.

Another segment caught in the recording was the conversation among students while one student working with the applet. The researcher and the persuaded a student to give up but she did not want to give up.

R	:	**Just give up!**
S_C	:	**No, I will not**
S_A,B,D	:	(Singing a song)
S_C	:	Stop singing
S_A	:	(Still singing)
S_C	:	I cannot divide into two (still working on the task)
S_A,B,D	:	(keep singing)
S_C	:	I cut too many parts so that I cannot cut it again, don't I?
S_B	:	(keep singing)
S_C	:	Ah…(cannot concentrate),(starting over the task), hahaha (laughing)
S_D	:	Cut this part, ok?
S_C	:	Ok
S_B	:	Cut also this part
S_C	:	This part?
S_D	:	Yes
S_C	:	Then, what should I do?
S_B	:	Rotate that part! Cut the below part into two, or even three!
S_A	:	If you divide it into three then there is no part to be cut later
S_C	:	(Keep trying)
S_D	:	**Just give up!**
S_C	:	**No way, if I give up I will be curious later**

It is obvious from the bolt sentences that this student was not easily to give up. She kept trying to solve the task. The task is more difficult than the tasks in serie 1. Therefore, it is indeed not easy to reshape the figure into a rectangle. The task also made this student curious if she did not finish it. Thus, we can say that this applet can motivate students because the tasks challenge students. For the smarter students, they can try out other series of the applet. Therefore, they smarter students still have opportunities to try more difficult tasks in order to maintain their motivation.

Students' understanding of the concept After students finished the tasks in the applet they worked on a worksheet. The following figures show how they answered the questions.

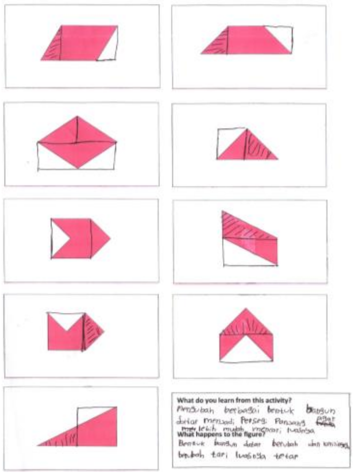
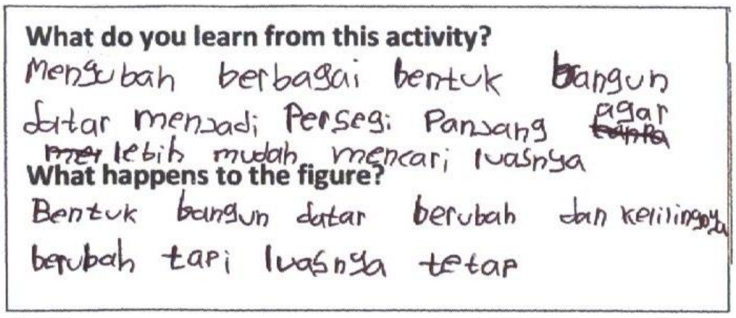


Figure 3.8 Student's answer on the worksheet



What do you learn from this activity?
Mengubah berbagai bentuk bangun datar menjadi Persegi Panjang agar ~~supaya~~ ~~mer~~ lebih mudah mencari luasnya
What happens to the figure?
Bentuk bangun datar berubah dan kelilingnya berubah tapi luasnya tetap

What do you learn from this activity?
Reshaping various figures into a rectangle in order to measure their areas easier
What happens to the figure?
The figure and its perimeter change but the area remains still.

Figure 3.9 Student's explanation about the activity

Figure 8 and figure 9 is one of student's work. Even though the tasks on the paper pencil workseet were simple, students could manage to make a rectangle from its initial figure by drawing it without cutting. Most students answered that the activity is about reshaping into a rectangle to measure the area easier. They also stated that the shape and its perimeter change and area remain still. Therefore, it can be said that students got the point of what they did in the activity. They could understand that reshaping a figure into a rectangle will help them to measure its area. They also could understand the concept of conservation of area.

This study has its limitation in which only a few students participating this study and cannot be generalized to a real classroom setting. Therefore the role of the teacher did not appear in this setting. However, it is important to notice that students were motivated in learning and they could grasp the mathematical concept from the activity they did in the applet.

4. Conclusion

The answer to the research question is the tasks in verknippen applet let students have more strategies in reshaping into a rectangle. Students could started over the tasks and try them again when they faced difficulties. This is one of the strength of the use of this applet that students were motivated to do the tasks more than once. Different series of the tasks also provide students with more challenging tasks. To see the progress from the tasks in the applet to the paper pencil task, it can be seen that students could reshape the figure into a rectangle by just making drawing and arrows on it. To some extent, students move from real cutting and pasting in the applet into drawing the sketch. In drawing, students need to determine the shape of a rectangle they are going to make. Therefore, should think more rather than just try to cut and paste. Students also could understand that reshaping into a rectangle will not change the area of its initial figure. In addition, the applet can be used if learning designs support students' development of the concept.

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