PROCEEDINGS

1st International Symposium on Mathematics Education Innovation

18-19 November 2011, Yogyakarta, Indonesia

Connecting Practice and Research: Working Towards Mathematics Literacy

organized by:

Southeast Asian Ministers of Education Organization (SEAMEO)
Regional Centre for Quality Improvement of Teachers and Education Personnel (QITEP) in Mathematics

In collaboration with
Holistic Assessment Team- Mathematics and Mathematics Education,
National Institute of Education (NIE) – Nanyang Technology University (NTU), Singapore

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Editors

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Director's Address

Teaching and learning is a complex system, not only a matter of transferring knowledge from teachers to students. A good teacher should be able to create a conducive environment as for the students to learn joyfully and meaningfully. Hence, innovation in education is essential for teachers, educators and whoever concerns in education. To be innovative in mathematics education, doing such research is very crucial. The number and quality of the research reflect that teachers and education personnel are always eager to develop their knowledge, challenge the existing theories and seek for the improvement of the theories in the field. The result of the research done should be applied and connected to the teaching and learning practices.

Regarding the mentioned fact, the 'First International Symposium on Mathematics Education Innovation – Connecting Practice and Research: Working toward Mathematics Literacy' finds its momentum. This symposium will be held by SEAMEO QITEP in Mathematics today and tomorrow. The aim of the symposium is to provide a great opportunity for mathematics educators and researchers to review issues, exchange of ideas and share experiences especially on innovation in mathematics education at all levels.

Mathematics teachers and education personnel are here today to share their interest and idea as well as experiences in mathematics education innovation. Further, it is expected that they can work hand in hand to conduct a joint research to produce a better and more qualified research result in the future. It is our sincere hope that this symposium will not stop tomorrow after the closing ceremony but continuous in the future with a cooperative relationship among us to plan and carry out some innovative research in mathematics education.

I would like to take this opportunity to address my sincere gratitude to the esteem keynote as well as plenary speakers, University of Western Sydney (UWS) Australia, National Institute of Education (NIE) Singapore, Gadjah Mada University (UGM), Bandung Institute of Technology (ITB), Indonesian University of Education (UPI) and our host institution PPPPTK Matematika (Centre for Development and Empowerment of Mathematics Teacher and Education Personnel) for their endless supports to enable this symposium happens and successful. My special thanks goes to our paper presenters from universities and schools in Indonesia, Singapore and The Philippines as well as all participants for their invaluable participation in this program.

On behalf of SEAMEO QITEP in Mathematics, I warmly welcome all the participants in this International Symposium on Mathematics Education Innovation. We welcome any inputs and criticisms for the improvement of the program next time. Thank you and wish you have a fruitful conference.

Prof. Subanar, Ph.D.
Director of SEAMEO QITEP in Mathematics

The Head of PPPPTK Matematika's Address

The International Symposium on Mathematics Education Innovation that we have today is the first international symposium held by SEAMEO QITEP in Mathematics. It marks the serious effort of SEAMEO QITEP in Mathematics to continuously realise its vision and mission to improve the quality of mathematics education within the Southeast Asia region. As the Centre within SEAMEO that focuses on mathematics education, QITEP in Mathematics invites teachers and education personnel in Indonesia and other countries within SEAMEO to take part in this symposium, share idea and knowledge in mathematics education so that they can develop the theories in related field. In this way, we expect to be able to contribute to the development of mathematics education in the region.

This first international symposium is held because of the continuous support from some other institutions without which this symposium could not be realised as it is today. Our expression of gratitude goes to UWS Australia, NIE Singapore, UGM, ITB and UPI Indonesia. They sent their experts to share their expertise in this symposium. Let me also thank the paper presenters and participants from universities and schools within SEAMEO member countries who had taken part in carrying out this program. My appreciation is dedicated to our beloved committee for the sincere hard working to manage this symposium.

More abstracts came to the committee, but due to the space limitation only 26 papers will be presented in this conference. The papers are written and presented by mathematics teachers, lecturers, and experts in mathematics education. We do hope more academic papers can be presented in the next time.

The theme of the symposium today is Connecting Practice and Research: Working toward Mathematics Literacy. It is hoped that the 26 papers will be presented and discussed can enlarge the knowledge of mathematics teachers and education personnel to be more innovative in their teaching and learning, and encourage them to continuously conduct research in the field. As a country with very large number of teachers, Indonesia may provide a good source of data any studies conducted.

Last but not least, as the Acting Head of PPPPTK Matematika, Centre for Development and Empowerment of Mathematics Teachers and Education Personnel, I would like to welcome all of you to the conference and hope that all participants can benefit much from this conference.

Dra. Ganung Anggraeni, M.Pd.
Acting Head of PPPPTK Matematika,

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KEYNOTE PAPERS

What might happen to school mathematics in 2013?

Prof. Lee Peng Yee

THE YEAR 2007

What was new in the O level mathematics syllabus 2007? Perhaps the change was mainly in the teaching approach and not so much in the content knowledge. Among the changes, we put more emphasis on the process and we made an attempt to put mathematics in context.

In fact, there is nothing new about the process and the context. Chinese did it over 2000 years ago. Look at the classics "Nine Chapters of Arithmetic" and many other classics in mathematics in ancient China. These books normally contained a collection of problems. The problems in these books were always presented in context. The problems were always solved by means of a process. Naturally, there was no formula. If the process itself produced an answer, there was no need for a formula. We re-discover the approach in modern time. In fact, there is nothing new about it.

There is also nothing new about the process and the context as far as Singapore is concerned. Years ago, more precisely 50 years ago, Euclidean geometry and Newton's mechanics were part of school mathematics at the time. In Euclidean geometry, we prove theorems. To prove a theorem, we have to go through the process. In mechanics, we construct models. To construct a model, we have to put it in context. So there is nothing new about it.

We lost Euclidean geometry gradually over the years starting from the days of Math Reforms in the 60s in the west and in the 70s in Singapore. As a consequence of Math Reforms, mathematics became pure mathematics. Gradually, mechanics was replaced by statistics. Henceforth we lost two rich, indeed very rich, areas for learning mathematics and for setting exam questions. We do not know what we have until we have lost it. Now we are trying very hard to recover what we have lost. In other words, we want to teach mathematics with emphasis on the process and pose mathematical problems in real-life or pseudo real-life context. In fact, we had it all along in Euclidean geometry and mechanics and then we lost it

In education, there are very few new ideas. People simply re-cycle old ideas and give them new names.

WHAT HOW WHEN AND WHY

Let us discuss the what, how, when and why of this so-called new approach to mathematics teaching. In more words, we ask the following questions:

What is it?
How do we do it?
When and where do we do it?
Why do we want to do it?

We shall elaborate in what follows.

What is it?

We want to teach knowledge and we also want to teach the use of knowledge. We want our students to be able to answer the problems in TIMSS and also the problems in PISA. It is said that TIMSS tests the content of mathematics, whereas PISA is more on applications. In other words, we want our students to learn mathematics and also to learn how to apply mathematics.

Put it in practice, we want to set open problems and expect our students to be able to answer them. The key word here is open. To solve such problems, students have to think differently and not relying only on recall.

How do we do it?

We do it by asking open questions. If modelling provides a good way to pose open problems, then use it. Though called by different names such as performance tasks etc, they serve the same purpose as modelling. Note that problems in modelling are always posed in context.

We often use rubrics to mark performance tasks. There should be a distinction between rubrics used for research and rubrics used for classroom assessment. We may not want to go for the full rubrics. An abridged version is more than enough.

When and where do we do it?

The usual comment is that we have no time. We are not expected to do modelling tasks everyday. Maybe have it at least once a year or at most once a term, assuming one year has four terms. Asking questions is a way of life, a habit, and an art. Suppose we say the area of a quadrilateral of sides a, b , c, d is $\left(\frac{a+c}{2}\right)\left(\frac{b+d}{2}\right)$. Ask not whether the formula is good or bad. Ask how good it is.

Why do we want to do it?

We teach mathematics and we should also teach mathematics for understanding. For understanding, we must look at the process. As I believe, this is known.

However we still need formulas and algebra. It is often through formulas that we make mathematics simple and make it easy to apply. Do not condemn formulas. We need both processes and formulas.

THE YEAR 2013

What is new in the O level mathematics syllabus 2013? There is at least one new element in the syllabus, that is, *learning experiences*. It is called knowledge requirements in the Swedish mathematics syllabus (2011). Roughly speaking, we make explicit what we want our students to experience during the learning process. For example, when we teach quadratic equations, we want our students to see that the graph of a quadratic function is nothing but a projectile. We can call it an experience, and we make it explicit. Further, we can use quadratic functions to find the maximum or minimum point. This is an application. It is another experience, and we make explicit about it. In a way, this is a natural consequence of putting emphasis on the process. Now we move one step further to spell out the specific experiences to be emphasized in the learning process.

We are not the only nation introducing this idea in the syllabus. Sweden is doing it. So are the United States in Common Core State Standards (CCSS 2010) and Chile, South America, in Content and Pedagogical Standards for Secondary School Teacher Education in Mathematics (Draft 2011). There is no national mathematics syllabus in the United States. CCSS is the nearest to it. Both the United States and Chile did not call it by a name. But they made it explicit in their syllabuses (standards). However there is one difference. In the 70s, we imported the Math Reforms from the west. This time we did it independently. So were other countries, at least three of them, other than Singapore. As I said above, this is a natural consequence.

You will find all the details of the syllabus 2013 on the website when announced officially. It is a more extensive document than the syllabus 2007. We want to change, but we cannot change overnight. This is only the beginning. We must do it in steps. Probably, this is the only way that we may succeed. This is not fire-fighting, and it should not be fire-fighting. We are training our students in schools for the work place different from our own. Hence we must do it differently.

IN THE CLASSROOM

A reform, if it is a true reform, can move only as fast as teachers can move. Suppose we, as teachers, believe in the proposed change. Suppose we want to move forward. What are we supposed to do in the classroom?

I have no doubt that there will be training programme. Indeed, it has already started. What I am saying here is something which I think might happen in 2013. Let me quote an email that I sent to my students. Here is the email.

Why is the answer important?
Suppose you build a house. When completed, the house collapsed. Do you still pay the contractor because the process was correct?

Why is the process important?
You may do the wrong thing and still get the right result. I am not sure you will be lucky again next time. If you did it correctly in the process then I have reason to believe that you do not have to depend on luck.

Why is the presentation important?
If you can say it well, you can get through one door. I mean what you say will reach the person behind the door. If you can write well, you can get through at least three doors. I mean what you write will reach the boss of the boss of your boss. Only good presentation travels.

End of the email. In summary, we may wish to do the following in the classroom.

- Make sure mathematics we teach is correct.
- Make explicit the experiences in our learning process.
- Pay more attention to presentation of mathematics.

By all means, teach for examination. There is nothing wrong to teach for examination, but not for examination alone. The examination will not change in the short term. In time to come, it will change. It will be too late for students to catch up if we do not start changing our teaching now.

We must accept the fact that though we may give the same amount to all students, students may not receive the same amount at their end. Since we apply differentiated syllabuses, differentiated teaching, therefore we must also accept differentiated learning.

As I said it elsewhere, we keep changing and changing, we reach a point that we have nowhere to copy from and we have to find our own solution to our unique problem. We, I mean curriculum designers, teacher trainers, and teachers, have to do it together to find a way of moving forward, a way that works for us. For example, we may have to build up jointly the resources to be used in the classroom.

In conclusion, we must teach mathematics differently. We want our students to be able to solve problems beyond the textbooks. Perhaps teaching out of syllabus will no longer be an exception. However we may not want to do that all the time. One important thing to remember is: students must learn how to follow rules first before learning how to break them.

References

Lee P. Y. (2006). What is new in the new O level mathematics syllabus? A written record of a talk given to Singapore school teachers on 20 June 2006.

Lee P. Y. (2007). Will the O level exam test what students know or will it test whether students know how to apply what they know? A written record of a talk given to Singapore school teachers on 18 July 2007.

[Some of the materials above have been given in a talk to Singapore teachers on 08 September 2011]

Who Are The Real Super Heroes?

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Abstract

Hollywood recently released a film titled 'waiting for superman'. A young boy was imprisoned within a system and classroom that not only did not stimulate his learning but actively destroyed his motivation and engagement with the educational process. The film implied the task of fixing the problem was so great that only Superman who is a super hero could fix it. So what is a super hero? It is someone with extraordinary powers beyond the reach of most mortals. In this paper I will propose that most mathematics and science teachers are super heroes who combat the spread of darkness and ignorance of mathematics and science. I will present evidence to prove my proposal that most mild mannered mathematics and science teachers are really super heroes in disguise. Mathematics and science teachers have super powers. They understand and value mathematics and science, something that is beyond the vast majority of humans. What gives these superheroes their power? It is their mathematics and science pedagogical knowledge. Not only can they do mathematics and science using their procedural proficiency, but using their super powers they can construct a learning environment whereby their students develop conceptual knowledge and deep learning. They use the latest developments in technology to assist their battle with the forces of darkness and innumeracy. In the process of proving the proposal, this talk will range across the latest developments in mathematics and science where more has been invented in the last 50 years than in all the proceeding years. While Hollywood may be waiting for Superman, the real super heroes are every day engaged in the battle to reveal to their students the power and the beauty of mathematics and science that can transform their lives.

Introduction

Hollywood recently released a film titled "Waiting for Superman". The film documented the story of a young boy imprisoned within a dysfunctional school system and classroom that failed to stimulate his learning and actively destroyed his motivation to learn and his engagement with the educational process. The film implied that the task of fixing the problem was so great that only Superman, a super hero could fix it. The history of movies produced in Hollywood reveals a range of super heroes such as Superman, Spiderman, Batman and Wonder Woman. These heroes share a number of common traits such as: (1) What do super heroes do? Usually super heroes battle the forces of ignorance and darkness which threaten human civilisation. (2) What is special about super heroes? All super heroes have super powers that other humans do not have or abilities that are beyond those of ordinary humans which help them overcome the forces of darkness. (3) How do you identify super heroes? Most super heroes on the surface are difficult to identify and are usually mild mannered, plain looking, conservatively dressed and do not stand out in a crowd. Yet all superheroes have special costumes (superman's cape), places (such as bat cave) or devices (Spiderman's web) that assist them in their battles.

Can mathematics or science teachers be super heroes? One premise of the documentary, "Waiting for Superman," was that the weaknesses of public education were due to an invasion in classrooms of uncaring and incompetent teachers and in the United States of America there have been attacks on teachers and their unions by politicians and others. The teachers are portrayed as forces of darkness rather than super heroes. This paper will disagree with this premise and it will claim that most mathematics or science teachers are super heroes. It will present evidence using the structure of the three questions listed above to justify this claim.

Super Heroes Battle the Forces of Darkness

All super heroes battle the forces of evil and darkness which threaten human civilisation. So what are these forces of evil and darkness that are faced by mathematics or science teachers? I will dwell on only three of the many that I could list. It is not meant to be an exhaustive or comprehensive treatment but rather a brief mention as deeper treatments are available elsewhere. The three selected are: firstly there is the crippling after effects of behaviourism on the teaching and learning process in school classrooms; secondly there is the problem of large scale examinations and their use by authorities as a measure of quality assurance; and thirdly there is the collision of the lack of mathematics and science knowledge in the general population with the explosion of new mathematics and science that underpins modern technological innovations.

The Darkness of Behaviourism

Behaviourism as a philosophical tradition has had a large influence upon mathematics education, particularly the western tradition and to a lesser extent in science education. Skinner's (1953) theory that by using cause and effect, then behaviour could be manipulated by conditioning; Bloom's (1956) *Taxonomy of Educational objectives*; and Gagne's (1967) work on learning hierarchies were highly influential upon mathematics and science teaching and teacher training programmes. Some of the negative aspects of this movement were the use of behavioural objectives, outcomes based education, mastery learning, programmed learning, an over emphasis on skills drill and practice, and a focus on large scale skills based testing (often multiple choice questions) as opposed to testing understanding and the application of knowledge. It was common to hear that the essence of teaching was contained in the saying "a long journey consists of many small steps", and it was assumed that a child could master any skill as it just depended on the teacher making the steps small enough. However, some students proved this assumption incorrect and became known as slow learners. Yet the truth was that no matter how much time was devoted to practising by these students, they would not learn until the approach was changed.

The behaviourist pedagogical approaches became problematic and there was a need to change (Clements, 2003). The first aspect of reducing a task to small steps requires the teacher to reduce a student’s role by 'emptying' the task of much of its cognitive challenge for the students (Brousseau, 1984). A task is broken into a number of smaller steps. If the student answered each step, then the teacher tended to believe that the student had learnt what had just been taught, and assumed that the student would construct the whole from the parts. In that sense, the students were presumed to have learnt what they were expected to learn from the original question. Cognitively challenging questions were removed from the classroom and replaced by bite-size portions. When teachers adopted this style, in an attempt to help students tackle higher-level mathematics and science tasks, they denied their students the opportunity to formulate and apply strategies of their own (Clements, 2004). The students failed if given unseen or novel problems because there was no one to 'cut them up' or tell them which tool to use. Skemp (1976) in his work on instrumental and relational understanding highlighted why this approach failed. From his research studies he showed that if A, B and C are steps in a learning hierarchy and the teacher instructs students on how to go from A to B and then from B to C then the students usually failed to acquire a holistic understanding. Thus they mostly did not see how A, B and C were related, nor could they return from C to A.

A clear local example comes from the Chang Mai district Thailand, where Vaiyavutjamai (2004) investigated why so many students failed to learn the material covered in mathematics lesson. She focussed on the questions asked by the experienced teachers during 16 lessons of six Form 3 algebra classes. The sample involved 4 teachers from two government schools and a total of 231 students (across the three streams of high, medium and low). She reported the prevalence of the 'cognitive empting' process across the 16 lessons. A high level question was followed by a sequence of low level questions by the teacher to give structure to students' thinking. Low level questions required very brief answers and were usually chorused by the class. Detailed examples of this emptying process can be found elsewhere (see Clements, 2004; Vaiyavutjamai, 2004). The research showed that after having participated in the Form 3 lessons on linear equations and inequations, many of the 231 students were still struggling to cope with the elementary questions. The retention data showed that in regards to a long term view, the approach failed except for the high stream students. As one teacher taught all three streams, it appears that this result has more to do with the students than the method.

What is surprising is that both students and teachers were happy with this approach because they had become accustomed to this way of teaching, even though the results were poor. Brousseau's (1984) research into didactical contracts helps explain why behaviourist pedagogies are resistant to change. The didactical contract encompasses the conscious and subconscious beliefs, behaviours and relationships that guide and control what teachers and students do within mathematics and science lessons. His research shows that once teachers and students become accustomed to an approach then they resist any change. Teachers and students develop sets of ingrained actions that arise from, yet simultaneously determine, didactical contracts. The teachers and their students have reciprocal expectancies, and their actions tend to become economical in the sense that they are guided by expectations of what can and cannot be done in 'normal' lessons (Jaworski & Gellert, 2003). Although these expectations generate common classroom practices, it is usually the case that neither the teacher nor the students subject the expectations to reflection or scrutiny. These findings have been replicated in Brunei (Lim, 2000), and New Zealand (NZ) schools. Barton (2003) maintained that in NZ classroom settings, didactical contracts influence teachers’ aims, methods, behaviours, content covered, and choice of procedures for assessing learning.

So the darkness of behaviourism influences many mathematics and science classrooms, through the process of cognitive emptying and unchallenged didactical classroom contracts. Who is there to fight this dark influence upon the teaching and learning of our children?

The Darkness of External Examinations

Assessment was designed to provide useful, timely and appropriate information that was fair and equitable and helpful in making plans for improving the classroom or system’s learning and teaching cycle. Assessment data could consist of the mathematical understanding of an individual student, the achievement of a group or a class, or the overall achievement of a system. Assessment data was collected at different points in the learning and teaching cycle. However, more recently in some countries there has been a developing confusion where teachers have been asked to meet the principles of assessment listed above, yet are told that externally imposed testing will be used to rate the effectiveness or quality of schools or teachers, and used in the distribution of resources. Usually this external testing is merely a measure of how many facts can be stuffed into the students' short term memory, to be regurgitated on a multiple choice examination and then promptly forgotten. The information gained from these examinations is not usually helpful in improving the teaching and learning process.

The use of standardized test results are such a misleading indicator of teaching or learning that successful efforts to raise scores can actually lower the quality of students' education. For example, in Atlanta the large scale testing regime is under attack. The pressure upon teachers and educational authorities has lead to cheating and even fraud (Torres, 2011).

There are many other things wrong with large scale testing such as: "The US tests have been criticised for narrowing the curriculum to reading and maths and multiple-choice formats... 'We have learnt about the potential negative effects of very narrow tests, particularly when they are put in a high-stakes context,' said Professor Darling-Hammond" (Patty, 2011, p.1). However, this section will confine itself to briefly discussing one. It concerns the faulty assumption (linked to behaviourism) that in the process of the mastery of skills, the students would come to an understanding. So if students show a high degree of mastery on a test then they have a good understanding of the underlying mathematical concept. While this may happen with some students, there were many students where this assumption proved false. The work of Erlwanger (1975) showed that elementary American students who passed mastery tests were unable to apply the mathematics and developed a mechanistic view of mathematics. The eminent Dutch mathematician Freudenthal (1979) attacked the concept of mastery learning. Researchers Ellerton and Olson (2005) conducted a study of 83 Grades 7 and 8 North American students completing a test comprising items from Illinois Standards Achievement Tests. Their findings indicated a 35% mismatch with students who gave correct answers with little or no understanding and others who gave incorrect answers but possessed some understanding.

Surely large scale testing cannot be all bad, what of the apparent success of some countries on predominantly skills based international comparison tests? There are some countries with outstanding performances from their Confucian-heritage students on international comparative studies (e.g., on TIMSS or PISA). While these countries have achieved high results, the authorities are concerned with the poor attitudes and engagement of their students towards mathematics and the small number who choose to continue studying mathematics at university. Zhao gave a keynote address at an East Asian education forum and claimed: "The East Asian students suffer, actually. There is psychological stress, there is a lot of direction, a lack of social experiences and therefore emotional development, he said. The concern about consequences of the approach - typified by self-styled 'Tiger Mum' Amy Chua - spreads beyond the suffering of individual students. East Asian educators are not at all happy with what they have achieved; they look at what they have not achieved. They look at the children's lack of confidence, for example, creativity, entrepreneurial spirit and imagination," (Stevenson, 2011, p. 1).

Australia is also feeling the negative effects of external testing through the National Assessment Program - Literacy and Numeracy (NAPLAN) which examines students in Years 3, 5, 7 and 9. "We're seeing a great deal of stress, anxiety, and concern among kids who are being kept in at lunch, sitting practice tests on the weekends, and are under increasing pressure to perform because the teachers and schools have so much riding on the children's performance" (O'Keefe, 2011, p. 8).

Society and educational authorities expect teachers to resist these negative influences upon education and to remain knowledgeable of recent philosophies, their resultant pedagogies, and to integrate them into their classroom practice. Is this not a task for a super hero? Surely it would take someone with super human strength and patience to weather the demands of external testing and concentrate on assessing their students in ways that go beyond basic recall to include diagnostic procedures, investigations, problem solving, creativity and the ability to generalize principles and apply them to novel problems.

The Darkness of Ignorance

A third darkness results from the combination of two influences that clash with each other to produce confusion, dissatisfaction and disempowerment. The first influence results from the speed of technological change which is underpinned by developments in mathematics and science. Most members of the general population are either unaware or do not understand the mathematics and science that is used. How many know that there has been more mathematics invented in the last 50 years than all the preceding years of human knowledge, or that a mathematical monster forms the aerial of their mobile phone? The second influence results from the poor numeracy skills of the general population. Skills learned in schools many years ago are forgotten and technological devices are relied upon to fill the void.

The resulting confusion from the clash of these two influences impacts upon schools. For example, one development has been the questioning of the amount of time students spend becoming highly proficient with computational algorithms when it is claimed that more time should be devoted to mathematical investigations enabled by the technology (see Wolfram, 2010). The mathematics and science curriculum are expected to reflect modern developments (such as fractal geometry) and teachers are required to prepare their students for a future world. Surely this is a task for a super hero?

Having briefly established some of the dark forces attacking the quality of school mathematics and science classrooms which the super heroes need to daily confront, the next section considers what powers super heroes use in their battles.

Super Heroes Have Super Powers

All super heroes have super powers that other humans do not have which help them in their quest to overcome ignorance and stupidity. The super powers that mathematics and science teachers have to help them win their battle are many and I will limit this discussion to three. The superpowers I will briefly describe are: mathematical pedagogical content knowledge; the integration of Information Communication Technologies (ICTs) into the classroom; and, the use of diagnostic assessment to uncover rich data to direct the teaching and learning process in mathematics and science classrooms.

Pedagogical Content Knowledge

Teaching is a process of continual striving for excellence, a quest for the perfect lesson and the understanding that it can never be achieved. There is always something, upon reflection, that could be improved to meet the individual needs of the students. It is the process of reflection, professional learning and experience that contribute to the gradual accumulation of pedagogical knowledge and power. It is the teachers' mathematical or scientific pedagogical content knowledge, the special knowledge that teachers have, which gives them the power to construct a learning environment whereby their students develop conceptual knowledge and deep learning.

Teachers' mathematical pedagogical content knowledge is an area of considerable research. While it is beyond the scope of this paper to give this area the treatment it deserves, it is necessary to make some brief points. The initial work of Shulman (1986) and colleagues proposed that a basis of mathematics teacher professional knowledge would contain: (i) mathematics content knowledge both substantive and syntactic; (ii) general pedagogical knowledge that included generic principles of classroom management; (iii) mathematics curriculum knowledge including materials and programmes; (iv) mathematical pedagogical content knowledge that included forms of representation, concepts, useful analogies, examples and demonstrations;(v) knowledge of learners; (vi) knowledge of educational contexts, communities and cultures; and (vii) knowledge of educational purposes. Shulman's work stimulated the growth of further research and other frameworks, such as Hill, Ball and Shilling (2008) which focused on conceptualising the domain of effective teachers' unique knowledge of students' mathematical ideas and thinking (see figure 1).

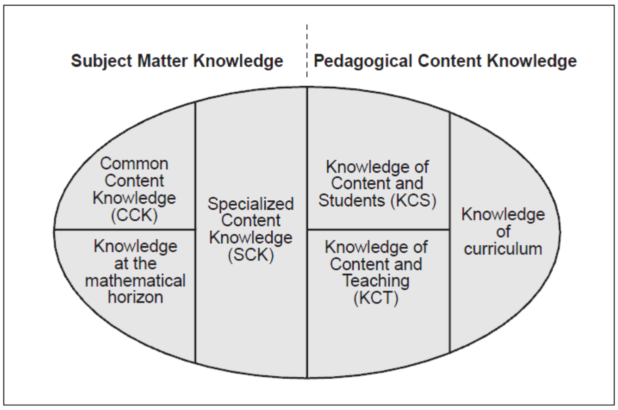

*Figure1.*Domain map for mathematical knowledge for teaching (from Hill, Ball, & Schilling, 2008, p. 377).

As teachers gain experience and develop their pedagogical content knowledge they move beyond the traditional approaches so greatly criticised by researchers such as Clements (2004) who is scathing of these arguing "that traditional 'teacher-telling' approaches to teaching mathematics are so ingrained in the cultures of school mathematics programs that students and teachers alike believe, mistakenly, that such methods are maximally useful in assisting students to learn mathematics" (p. 1). His research showed a prevalence and popularity for a 'teacher telling approach' that follows the lesson structure of: teacher review; teacher models examples; student seatwork where they practice similar examples to those modelled by the teacher. When he analysed the classroom discourse patterns, the data revealed that teachers asked questions that were of a low cognitive level and very few high level questions.

In developing their pedagogical content knowledge many teachers adopt a mathematics or science curriculum that focuses on real-life problems that still exposes students to the abstract tools of mathematics or science, especially the manipulation of unknown quantities. They incorporate mathematical investigations as they are fundamental both to the study of mathematics itself and to an understanding of the ways in which mathematics can be used to extend knowledge and solve problems in many fields. Teachers recognise there is a world of difference between teaching "pure" mathematics or science, with no context, and teaching relevant problems that will lead students to appreciate how a mathematical or scientific formula models and clarifies a real-world situation. In this way super heroes reveal the power and majesty of mathematics and science as ways of making sense of the world. It is the teachers' deep passion for their discipline and appreciation and concern for their students that drives them to seek to improve their teaching strategies with the goal of improved student learning.

Integration of Information Communication Technologies

The second super power I wish to briefly mention which is allied to the first is the integration of ICTs into the classroom. An examination of the research literature, it could be argued, produces five broad categories or metaphors of teacher response (White, 2004). These metaphors describe how teachers tend to view ICTs as either: a demon; a servant; an idol; a partner; or, a liberator. In terms of this paper I would argue that teachers who may belong to the first category are casualties who need gentle care and encouragement, those in category two and three are developing their powers, and those in the last two are the powerful super heroes.

ICT as demon: The evidence for this approach is observable in those teachers who actively oppose and subvert any attempt to integrate ICT into the curriculum. They are either afraid or unwilling to learn and so conduct a campaign of active or passive resistance. If compelled by the authorities, they will do the minimum and often the result leads to surface integration and sometimes to an inappropriate use of ICT. Some teachers' resistance has resulted from the frustration of being sent to a professional development program on the use of a software package only to return to school where the software is not available.

ICT as servant: These teachers are accepting of ICT but adopt a conservative position towards the technologies being used. The technology is new yet the pedagogy remains much the same as in the past. ICT thus is a tool for enhancing students' learning outcomes within the existing curriculum and using existing learning processes.

ICT as idol: This approach promotes ICT as a tool for use across the curriculum where the emphasis is upon the development of ICT-related skills, knowledge, processes and attitudes. It is more focused upon teaching about computers rather than with computers. There are many examples of professional development programs that give teachers intensive experience of software packages but fail to assist teachers with the teaching and learning implications. The difficult task of using the ICT in the classroom is given very surface treatment. Thus teachers then struggle to integrate what they have learnt.

ICT as partner: There are teachers who have seriously attempted integrating ICT into their classrooms. These classrooms are where students are actively engaged in gathering data, aggregating their data with those gathered by other students, and making meaning of their results. Here, ICTs are integral to the pedagogy that will change not only *how* students learn but *what* they learn. It means the use of ICTs in the teaching of mathematics or science moves beyond pointing to how ICTs can support, improve, and provide new ways of teaching to how ICTs change the way mathematics or science is expected to be performed.

ICT as liberator: This is a radical approach where integration is a component of the reforms that will alter the organisation and structure of schooling itself. There is over one hundred virtual schools already existing in the U.S.A. as evidence of this trend.

The link between the first and second super power is obvious, with the second being a powerful addition to the first. The second power is important in allowing students to develop and understanding of the concepts before developing their procedural proficiency.

Diagnostic Assessment Procedures

The earlier remarks concerning the usefulness of assessment to the teaching and learning process within the classroom are particularly true for diagnostic assessment' Even external examinations may have some small value. "While we generally accept the usefulness of diagnostic assessments, both internal and external to an individual class or school at all levels of schooling... When external assessments are conducted we seek to emphasise their diagnostic applications, even though many tests are of limited value, particularly at student level" (Alegounarias, 2011, p. 10). Diagnostic tests give teachers the data to find targets to aim at with their super powers.

While there is a wealth of mathematical diagnostic procedures available to the classroom teacher I will only briefly mention two. The first is an innovative online resource called SMART (Specific Mathematics Assessments that Reveal Thinking - Stacey et.al., 2009). Based on the Victorian Mathematics Developmental Continuum (Stacey et al., 2006), the site offers a set of online tests covering most topics commonly taught in Victoria; Years 7 to 9. Teachers choose one of the available smart tests appropriate to their class. Students are given a password so they can attempt the test in class or at home. Responses are marked online, and teachers receive the patterns of results electronically analysed with diagnosis when requested online. This feedback includes a summary of the findings, along with information on the common misconceptions in the topic and relevant links to the syllabus (SMART, 2010).

The second concerns word problems, usually written textbook problems and the Newman's Error Analysis (NEA) procedure which focuses upon the mathematical and literacy aspects of problem solving. NEA was originally designed to assist teachers diagnose the nature of the difficulties experienced by students working with mathematical word problems, but it has developed further super powers by teachers involving pedagogical and classroom problem solving strategies. Teachers have a framework to determine where misunderstandings occur and where to target effective teaching strategies to overcome them. Moreover, NEA provides a nice link between literacy and numeracy.

Newman (1977, 1983) maintained that when a person attempted to answer a standard, written, mathematics word problem then that person had to be able to overcome a number of successive levels: Level 1 Reading (or Decoding), 2 Comprehension, 3 Transformation, 4 Process Skills, and 5 Encoding (see Figure 2 for the interview prompts). Along the way, it was always possible to make a careless error and there were students who gave incorrect answers because they were not motivated to answer to their level of ability. NEA is popular among teachers because it is a simple diagnostic procedure that resonates with their experiences in the classroom. Studies typically reported approximately 70 per cent of errors made by Year 7 students were at the Comprehension and Transformation levels. These researchers also found that Reading errors accounted for less than 5 per cent of initial errors, and the same was true for Process Skills errors being mostly associated with standard numerical operations (Ellerton & Clements, 1996).

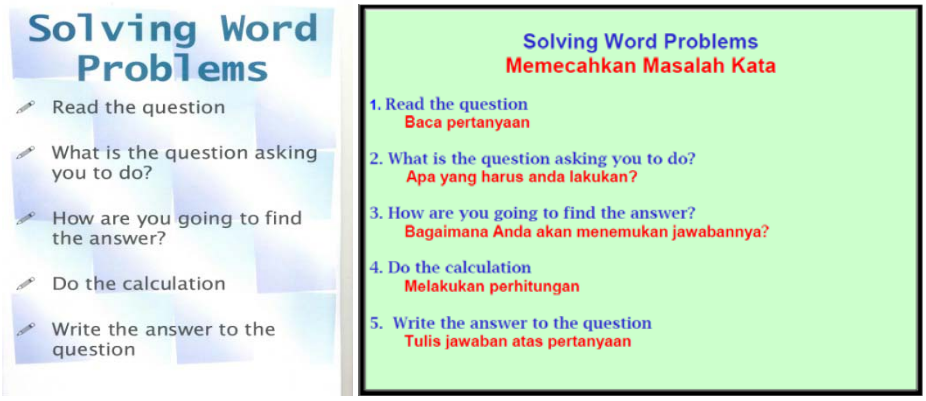

Figure 2. Two problem solving classroom posters (English & Indonesian).

Unfortunately there is not space to include the pedagogical strategies in this paper but they are available elsewhere (see White 2009; 2011). Having identified the need for super heroes and some of their super powers with which they fight the forces of darkness and ignorance, it is now time to consider how to identify them.

Identifying Super Heroes

In the movie world, most super heroes are mild mannered, ordinary looking, who do not stand out in a crowd. This is true in real life, as most mathematics and science teachers appear to be just ordinary humans. Yet all super heroes have special costumes, places, attributes or devices which make them special. Where will I find and how will I identify a super hero? Again there are so many that I could mention but space limitations restrict me to mentioning only one identifying trait: mathematics or science teacher super heroes gather at professional conferences, meetings or associations. These mild mannered, ordinary looking super heroes will be found participating in research and professional conferences, or as active members of professional teaching organisations, or collaborating in professional teacher learning activities. For example, you will find them all around you in the audience of conferences run by Southeast Asian Ministers of Education Organisation (SEAMEO) Regional Centre for Quality Improvement of Teachers and Educational Personnel (QITEP) and the PPPPTK (P4TK) Yogyakarta Indonesia.

Conclusion

Schools are collaborative enterprises and the mathematics and science teaching quality and school performance depends upon whether the institutional systems provide support for mathematics and science teachers' efforts. Mathematics and science teachers are key contributors to improving education and every effort should be made to bring teachers together to help each other become more effective professionals. Thus the formation of QITEP is a wonderful organisation for the encouragement and development of super heroes. QITEP will become an amazing mathematics and science centre in the world. Institutions like QITEP are not to be found in other parts of the world (except RECSAM Malaysia). The fact that so many teachers attend a symposium run by QITEP in the search for knowledge and pedagogy (ie. super powers) is proof they are indeed super heroes.

There is a story of a business man complaining about education to a mathematics teacher. He asked, "You're a teacher, so be honest. What do you make?" The teacher had a reputation for honesty replied, "You want to know what I make? Well, I make kids work harder than they ever thought they could. I make a small achievement feel like a medal of honour. I make kids sit through 40 minutes of class time when their parents can't make them sit for 5 minutes without television. You want to know what I make? I make kids wonder. I make them question. I make them apologize and mean it. I make them have respect and take responsibility for their actions. I teach them how to write and then I make them write. I make them read, and use their brains to reason. I make them see the wonder, the beauty, and the power of mathematics and science and use them to make sense of their world. I make my classroom a place where all my students feel safe. Finally, I make them understand that if they use the gifts they, were given, work hard, and follow their hearts, they can succeed in life." Only a super hero could do all this and more.

During this conference I hope that you all take this opportunity to develop your super powers as there are many interesting and dynamic presenters. I conclude this paper by thanking all you hard working mathematics and science teachers for the super human efforts you make for your students. In my eyes you are all super heroes.

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Variation of tasks as a strategy to enhance students' learning of algebra

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Abstract

Algebra challenges students and teachers alike. Students find algebra abstract. In particular the structural conception of algebra confounds students. Why should the task (a) simplify $x^2 + 3x + 2$ be any different from (b) solve $x^2 + 3x + 2 = 0$? Despite their best efforts, teachers find that many students continue to solve task (a) when there is no reason for doing so. Research shows that these two tasks require students to have constructed different meanings of structure and meanings of letters for such algebraic objects. What does it mean to simplify and what does it mean to solve? What meanings do letters have in each of these cases? In this talk, I wish to share how simple yet innovative strategies could be used to help students discern one set of tasks from another. These strategies, underpinned by the theory of variation (Marton & Tsui, 2004) were tested out by teachers in Singapore. Their work showed that students improved in their performance with various types of algebra tasks. Students' improvements were reflected in terms of their capacity to justify their choices.

Introduction

It is a fact that algebra is a difficult subject for students to master. For novice learners, their initial encounter with this most mystifying area of mathematics is fraught with confusion. Those conventions acceptable in arithmetic may not necessarily be acceptable in algebra. For example, while it is the norm to write the sum of a whole number and a fraction such as $3 + \frac{1}{8}$, as $3\frac{1}{8}$, this convention cannot be applied to the addition of two letters $a + b$. The sum of this expression is left suspended as a process and its sum can only be determined when the specific values of the letters are provided. Hence $a + b \neq ab$. This example illustrates students' dilemma when to regard an expression as a solution, what Davis (1975) described as a process-product dilemma and that letters are variables, i.e. they represent any numerical value at any given time (Kuchemann, 1981, Usiskin, 1988). It could be said that novice learners of algebra find structural relationships inherent in many algebraic objects most confusing. It is not uncommon to find students erroneously solving quadratic expressions when they were asked to simplify. Erroneous solution in Figure 1 is common amongst novice learners of algebra.

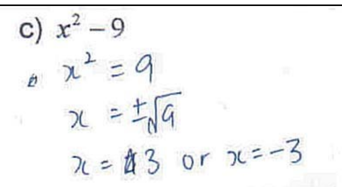

Figure 1: Instead of factorizing the quadratic expression, this student chose to solve it instead.

Also it is not uncommon for students to make the errors listed in Figure 2 when factorising quadratic expressions. Although these students did apply the correct processes but these processes did not result in the correct equivalent forms of $2(x+1)^2$.

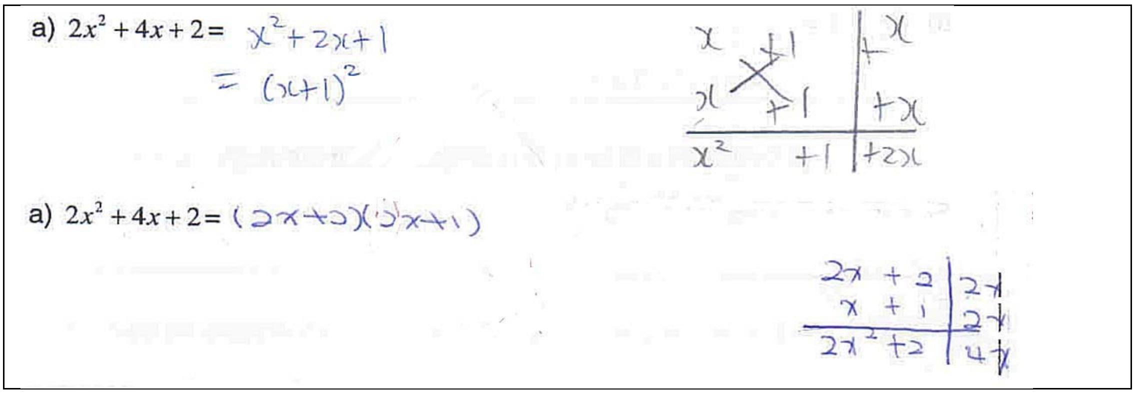

Figure 2: The solution at the top is partly correct but the common factor of 2 is not considered a factor. The solution at the bottom is wrong but the process on the left is correct.

The above examples focused on the symbolic manipulation and transformation aspects of algebra. Although these are not the core of algebra they constitute important skills required to solve problems. Advocates of intensive-computer learning environment which may include using CAS and other handheld technology argue that acquisition of such skills is no longer that crucial as technology could easily perform the processes and thus should not form the core aspect of learning of algebra. Although it may be true that the sophistication of the computers are such that they are able to takeover many of the symbolic manipulative tasks, nevertheless it is equally important for novice learners to master the basics of symbolic manipulation and transformation. I often suspect that such advocates are themselves masters of symbolic manipulation and transformation and when computers are not available, they would be able to apply their knowledge successfully. While it is true that symbolic manipulation and transformation are not the sum total of algebra, such skills are necessary and contribute to the success of problem solving. Also the capacity to carry out the symbolic manipulation and transformation draws from students more knowledge than just the necessary keystrokes. Fey (1984, p. 28) pointed out this difference very clearly.

> As procedural operations are increasingly mechanized (by computers), there remains an important task of conceptualization and planning. Problems must still be identified and cast in mathematical form; the proper analyses must be structured and the results of computer-assisted calculations must be properly tested and interpreted. To perform this fundamental role, individuals must have a sound understanding of the scope and structure of available mathematical methods.

Although teachers have taught students the necessary symbolic and manipulation skills, students continue to make errors with such basic skills. Some of the errors include but are not limited to the following.

- Instead of stopping after they have found the factors, many students continued to solve for the unknown values.
- Many students do not know why equations which are contradictions will have no solutions.

- Students are unaware that there are many types of equations.
- Students are confused by the different form of algebraic expressions and hence how to factorise them.

Why do students have this problem? It could be that because teachers may have taught students to be proficient at symbolic manipulation skills and procedures, the many related and similar mathematical objects may cloud students' capacity to differentiate one set of mathematical objects from another. Lack of clarity on the part of students could mean they are unable to differentiate what they do with one set of mathematical object cannot be applied to another almost similar task. Once they have learned new procedures they are unable to inhibit what they have learned to do with one object to another very similar set of objects. Thus such students may benefit from activities which require them to *reflect* on the similarities and differences that may exist among almost identical objects. Engaging students with such activities may help reduce their impulsiveness and hence improve their performance with such tasks.

In this paper I discuss how different Singapore teachers used sorting activities to help sensitize students to the different expectations of each set of tasks and to improve their performance with these tasks. What are sorting activities? This is will be discussed in the first section of this paper. Why should sorting activity help students improve their knowledge of what they have learned? A theoretical framework is offered to explain why engaging students in sorting activity may contribute to improved performance with basic algebraic skills and procedures. Who conducted these sorting activities? What pedagogical approach did these teachers use? What did students learn from the sorting activities?

What are sorting activities?

Sorting activities build upon the theory of variation (Marton & Tsui, 2004). In sorting activities, students are presented different tasks that share certain similar structural properties but are also different in other aspects. To encourage reflection students are asked to use these questions to help them help focus on what are the common aspects of the tasks and what are the differences between the tasks.

- What remains the same?
- What changes?
- What can you do with each object but not with the other?

Because it is not possible to discuss all the different activities, for this paper I will focus only on how to improve students' capacity to differentiate between algebraic equations and expressions. In Figure 3, two *almost* identical mathematical objects are presented for students' consideration: A is a quadratic expression and B, a quadratic equation. Although B is an equation, the expression to the left of the equal sign is identical to the expression in A. Although A and B may look alike, there are overt and subtle differences between them. Hence it is the responsibility of the teacher to sensitise students to the variations between these objects. The variations are to address the following learning points:

What are the overt differences? In this case the overt differences could be structural differences: expressions versus equations. Expressions can be factorised but equations can be factorised and solved.

What are the subtle differences? The subtle differences are located within the meanings associated with the letters. In the case of expressions, the resulting equation is an identity which is true for all values of x. Unlike expressions, however, equations are only true for specific values of x. Hence such equations are known as conditional equations.

A:	B:
$x^2 + 5x + 6$	$x^2 + 5x + 6 = 0$

Figure 3: Comparing and contrasting two different objects: What remains the same? What changes? Identifying the different processes associated with one but not the other. What can you do with one but not with the other?

Comparing and contrasting two different objects: The two questions What remains the same? and What changes? focus students' attention on the similarities and differences between two mathematical objects.

What remains the same? This question focused students' attention to the fact that in B the quadratic expression to the left of the equal sign is identical to that of A.

What changes? B is a quadratic equation but A is a quadratic expression.

Identifying the different processes associated with one but not the other.
What can you do with each object but not with the other? Expression A can be factorised and can be expressed in the form of linear factors. The factorised form of A is an identity and is true for all values of x: $x^2 + 5x + 6 = (x+3)(x+2)$

The quadratic expression in B can be factorised and it can be solved by first expressing the quadratic expressions as a product of two linear factors.

$$: x^2 + 5x + 6 = 0$$
$$(x+3)(x+2) = 0$$
$$x = -3 \; or \; x = -2$$

Hence B is a conditional equation only true for the two values of $x = -3 \; or \; x = -2$.

How do sorting activities help sensitise students to differences? A Theoretical framework

Research shows that working memory and literacy are two important contributing factors to solving algebraic word problems (Lee, Ng, Ng & Lim, 2004) and mathematical problem solving in general (Bull & Scerif, 2001; Gathercole, et al. 2008). Working memory can be defined as an individual's mental workspace where relevant information is held and to be acted upon. Baddeley and Hitch's (1974) tri-partite working memory model comprising the three components, central executive, phonological loop and visual-spatial sketchpad is used in this discussion. Miyake et al. (2000) building upon work related to working memory (Baddeley, 1996, Baddeley and Hitch, 1974) fractionated the central executive into three interrelated executive components: Shifting, updating and inhibition. Shifting refers to the capacity to alternate between multiple tasks, operations or mental representations. Updating refers to the capacity to evaluate information and appropriately edit it with more relevant information. The capacity to resist automatic but inappropriate responses describes the last of the three executive functions: inhibition. Literacy refers to students' capacity to decode information presented to them orally.

When confronted with algebraic objects with similar properties, there could be a working memory overload and novice learners of algebra may be unable to shift, update and inhibit their responses to these items. For example at the end of the introductory course to beginning algebra, students would have learnt how to factorise and solve a variety of equations. Because they have so much related information within them, when asked to factorise an expression, they may be unable to shift their attention between different representations and the related operations. They may have difficulty updating the requirement of the task and evaluate what is required of them. Finally they are unable to resist inappropriate responses to the selected task. Hence instead of stopping after they have found the factors to related expressions, these students continue to solve the expression although that was not the demand of the task.

Using sorting activities with students who have completed the introductory algebra course may help sensitise them to the different demands of related yet different algebraic objects, e.g. factorizing versus solving. With sorting activities, two alike tasks are presented for students to compare and contrast. Guided by these questions: How are the two tasks alike? How are they different? What can you do with one but not the other?, students are asked to compare and contrast these two tasks. Engaging students in such tasks encourages them to reflect on the different demands of each task. When asked to compare and contrast two tasks, students need to *shift* their attention between the representations of the two tasks, *update* with the information that equations are to be solved and *inhibit* the tendency to want to solve expressions. Rather factors are to be found for expressions instead. When they practise with many but different such mathematical objects, they rehearse the mathematical concepts and procedures. With repetition they commit these ideas to long term memory. By repeating such activities, such habits of working become part of the habits of mind and become part of their mathematical disposition. By rehearsing the language used to discuss these mathematical objects and the related processes students' capacity to use mathematical language is enhanced.

Sorting activities can be conducted with students working on their own or in groups of two. It is best to get students to work in pairs first. The need to justify their answers to their significant other encourages students to speak aloud their thoughts. Why would they categorise a certain task as an expression and another an equation? If one member of a pair is unable to start, the significant other could serve as a model of how to articulate their thoughts. This social cultural mode of learning (Vygotskiĭ, 1978) means that the other member of the pair learns from the significant other. The public talk could then become private speech when students are asked to work independently. Unless there is a monitoring system which ensures everyone is actively engaged in the task, it is not advisable to work with too big a group.

In summary the objective of sorting activities is to improve novice students of algebra performance with basic skills by (i) encouraging students to be more reflective, (ii) strengthening the executive components of the central executive of the working memory, (iii) providing students the opportunities to engage in mathematical talk and the act of listening to mathematical talk, (Mason, Burton & Stacey, and finally and most important of all, (iv) cultivating a mathematical habit of mind in problem solving. Although the examples cited in this paper are related to letter symbolic manipulation and transformation, sorting activities have found be to very effective in other areas of mathematics. These include sensitizing students to presenting better solutions to the same problem, helping students to avoid common mistakes in solving problems, etc. The diagram in Figure 4 sums up the framework. The flowchart for the sorting task can be found in Appendix B.

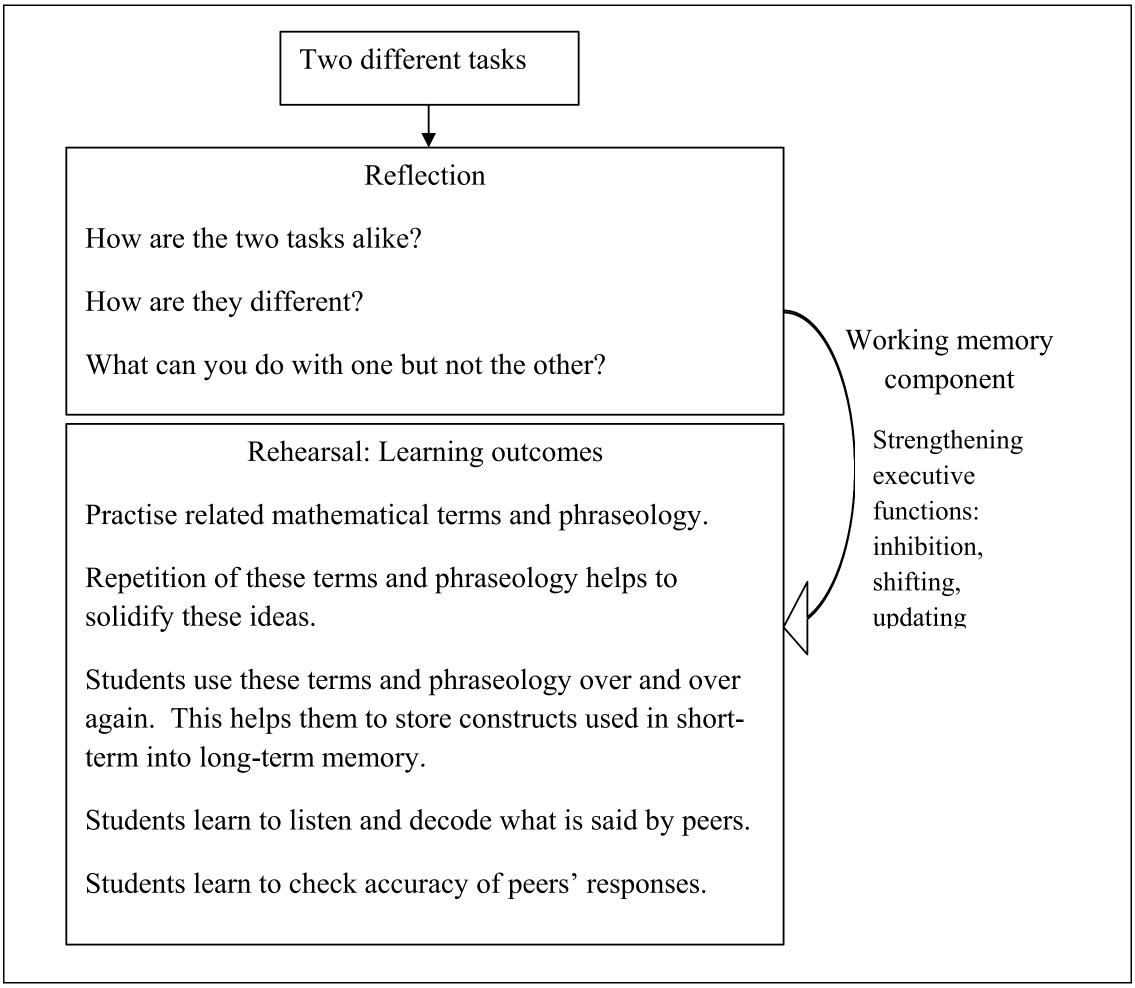

Figure 4: Framework delineating the various processes underpinning sorting activities.

Testing the efficacy of sorting activities

Aliza Main (2006) was the first teacher to use sorting activity as part of her master's dissertation where students were asked to sort and categorise algebraic expressions of varying complexity according to their level of difficulties and the procedures needed to factorise these expressions. The students who participated in this activity explained that the sorting activity sensitized them to the nature of algebraic expressions (linear versus quadratic expressions) and the various procedures needed to factorise them (factorization by common factors involving two terms, factorization by common factors involving three terms, factorization by grouping, factorization of trinomials in the form of $ax^2 + bx + c$, and factorization of binomials in the form of difference of two squares). Inspired by Main's work, a group of teachers conducted these sorting activities. These teachers were enrolled in the Algebra and the Teaching of Algebra module which was part of the Master of Education Programme offered by the Mathematics and Mathematics Education (MME) academic group of National

Institute of Education, Nanyang Technological University of which I was the tutor. As part of the assessment, they were required to demonstrate how they could improve performance in areas which their students consistently found difficult. Although many teachers tried the sorting activity, only the work of Amutha d/o Annathurai (2010), Chua (2010) and Teo (2010) are referenced here and because of space I will cite only Chua's (2010) work. The post-test instrument is presented in the appendix.

Method

Each teacher identified a problematic area and then designed sorting activities with which they could engage their students. The sorting activity could be based on those that others have done or they could construct sorting tasks that addressed their specific needs. First teachers constructed an instrument to evaluate the knowledge held by their students. An intervention in the form of the sorting activity was conducted. Teachers showed students how to do the sorting activity. Students then worked in groups of 2 to 4 on sets of sorting tasks. After conducting the intervention activity a post-test was carried out to evaluate the efficacy of the sorting task. Also students were asked to provide feedback on what they thought of the sorting activity. The flowchart in Figure 5 provides the design of the study.

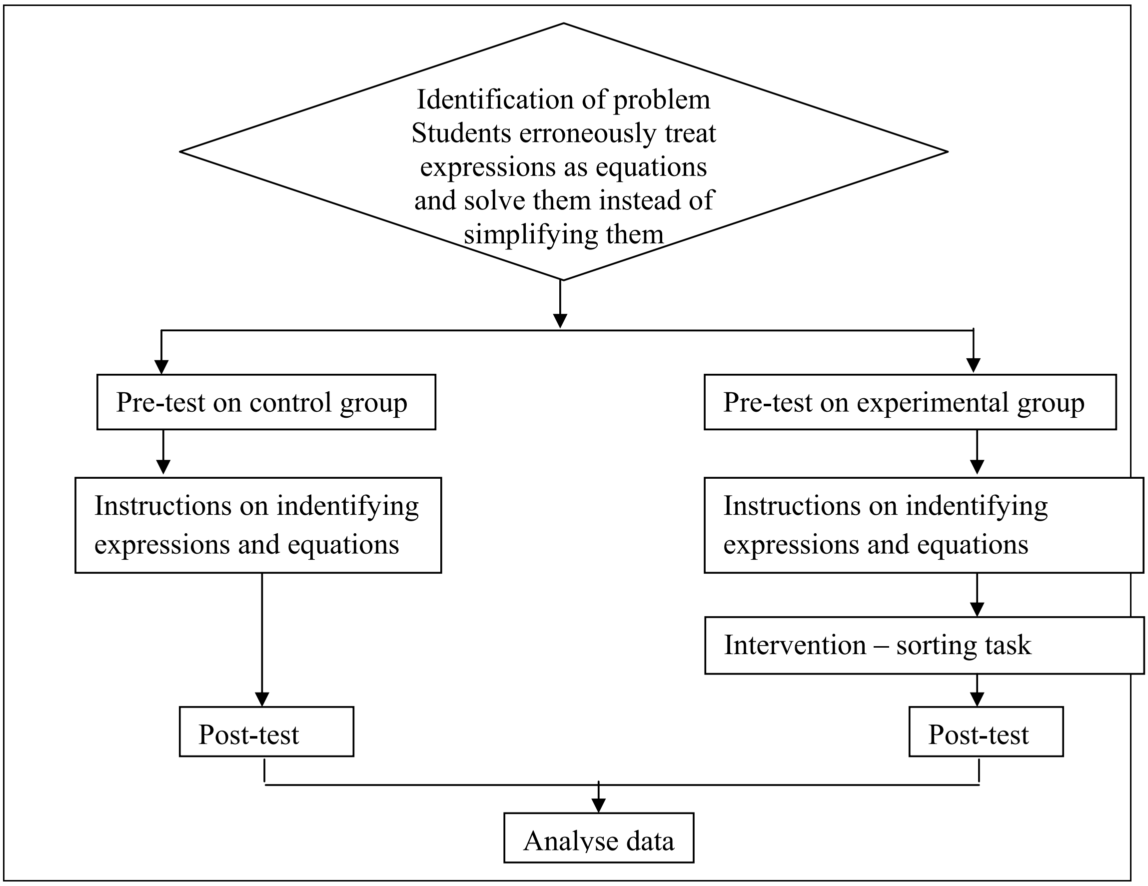

Figure 5: The flowchart provides the design underpinning the intervention studies.

Teachers compared teaching methods. With the control group, teachers used conventional teaching method to review the lessons on factorization and solving of equations while the sorting activity was tried out with the experimental group. Teachers presented the sorting activities to encourage students to reflect upon what differentiates different mathematical objects and hence what they can do with one object but not with another. Pre- and post-tests were carried out to evaluate the efficacy of the teaching methods. Student of similar ability were randomly assigned to each class.

Instrument

The instrument in Appendix A was the post-test given to the students after the intervention. The first section the instrument required students to work with numerical expressions and equations. The second section focused on algebraic examples.

Findings

It was expected that students who received the conventional teaching and those who were engaged in the sorting activity benefitted from the teaching. But students who were engaged in the sorting task could state precisely what they could do with one set of objects but not with the other. For example, students were able to state that they could factorise expressions but not solve them. For equations they could factorise as well as solve the equations. Students found the hands-on activity interactive as they could discuss the mathematics involved in carrying out the mathematics task. They learn and remember the mathematics built into the sorting activity better than if they were to listen to the teacher. They enjoyed the learning process.

Conclusions

In this paper, sorting activities are presented as a means to help improve performance of novice learners with basic algebraic activities. The preliminary findings reported in the four Singapore studies show that sorting activities has the potential to help learners improve their central executive functions as well as their literacy skills, and hence their learning of basic algebraic manipulations. More rigorous research is needed to verify these findings. Nevertheless sorting activities are a useful means for teachers to use as a review tool to encourage students to reflect upon what they have learnt at the end of a section.

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Appendix A

For each of the following, put a tick in the relevant or correct box.

	Expression	Equation	
		Identity	Contradictions
1) 7 = 3 + 4			
2) 5 + 4 – 3			
3) 3 x 2 + 4			
4) 3 = 3			
5) 2 x 5 = 5 x 2			

6) $18 \div 3 = 18 \text{ x } \frac{1}{3}$			
7) 5 = 5+1			
8) 9 + 3 = 12 + 5			
9) $12 \div 2 = 4 + 2$			
10) 3 x 5 = 30 $\div$ 2			
11) $\frac{2}{3} - \frac{1}{2}$			
12) $\frac{1}{5} + \frac{1}{3} = \frac{8}{15}$			
4 (5 + 10) = 4 x 5 + 4 x 10			

	Expression		Equation		
	Like terms	Unlike terms	Identity	Conditional Equation	Contradictions
1) $3a + 3\,a$					
2) $3a + 2a$					
3) $3a – 2a$					
4) $3a + 2b$					
5) $3a – 2b$					
6) $2(a + b)$					
7) $4a\,(a + b)$=0					
8) $5t^2$=t					
9) $3p(p+2) = 3p^2+ 6p$					
10) $x = x + 1$					
11) $(x + 2)(x + 1) = x^2$+3x+2					
12) y x $0 = y$					

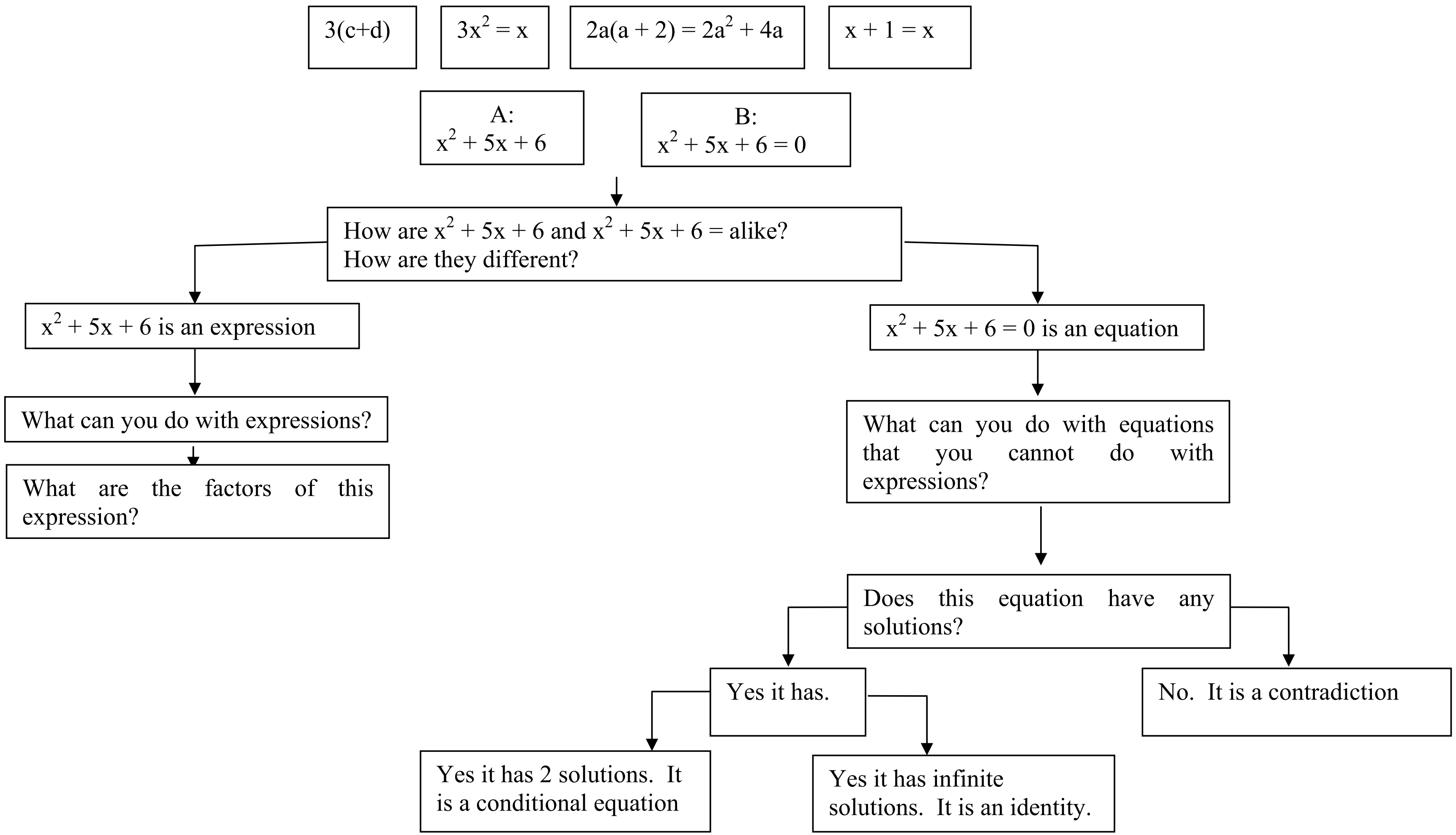
13) $\frac{y-1}{2} = \frac{2}{y-1}$					
14) $\frac{5x^2 - 20}{10x^2 + 10x - 20}$					
15) $3(x-5) - 1 = 7 - (1-x)$					

Can you solve the following?

	No	Yes	If Yes, show your working
1) $3(x + y)$			
2) $3x^2 = x$			
3) $2a(a+2) = 2a^2 + 4a$			
4) $4a = 0$			
5) $(x + 2)(x + 1)$			
6) $x + 1 = x$			
7) $x + 3x + 2 = 0$			

Appendix B: Sorting flowchart for expressions and equations.

Sort these expressions and equations



School Math Teacher Training 2.0

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Abstract

Some researchers state the effects on the Internet usage to the way students learn. If this observation is true, the way they learn school math changes too. In particular, they prefer to discover understandings by themselves and they prefer to learn math from computers. These changes force modern math teachers to teach for understanding and utilize digital technology. Therefore modern pre-service and in-service math teacher professional development programs should equip the math teachers with deep mathematics understanding and digital literacy.

Introduction

When TV technology was introduced to education in the last century, the technology was seen as beneficial to students. In particular, (Strommen, E.F., Lincoln, B., 1992) states the role of technology in active learning.

'Technology takes a special place in the CDLE (child-driven learning environment) as a powerful tool for children's learning by doing.' (Strommen, E.F., Lincoln, B., 1992)

The situation in this century is the same that is educators see the benefits of technology in education. However, instead of TV technology which is analog, in this era, educators see the possibilities of the utilization of digital technology. Moreover, different from the analog technology introduction into education practices in the last century that was seen as an efficiency means, in this era, the digital technology together with Web 2.0 are seen as necessity. In particular, because some researchers like (Carr, 2010) believe that our students nowadays learn differently than their teachers. The students nowadays do not like to read the whole books anymore. Carr continues to state that the constant Internet usage weakens the students' ability to concentrate. If his observation and theory are true, it means that they require new ways to learn. They also require new media to learn. School math is no exception.

In this paper, it is explored the idea that the implementation of technology is actually supporting the application of the classical learning theories like Piaget's constructivism. This observation is interestingly similar to the one done in the last century, like (Nicaise, M., Barnes, D., 1996, May). The only difference is the type of technology utilized. The urgency is also different. In last century, technology usage was needed to allow the distribution of quality education in a large scale or broader targets. In this century, the urgency to utilize the technology, the digital one in particular, is required because the learners need it. In short, modern students in this century, called students 2.0, call for a new way to learn math. Therefore, modern math teachers should be prepared with knowledge, skills, and passion to teach math. Thus, automatically, professional development for math teachers should provide these three important components.

Students 2.0

This particular era in education is very interesting. Our K-12 students nowadays mostly were born after 1993. They are popularly called GenZ or Net Generation. Since from the very beginning their life has been cramped with digital media and gadgets, some also call them Digital Natives. Their attitude toward information is different from ours. In part, this is because search engines like Google play a very big role in their education experience.

'Google has succeeded in making the Internet a far more efficient informational medium' (Carr, 2010).

Even though many statements in his book are not supported by hard evidences, by common sense and our own limited observations, Carr's statement in his book on the change of learning behavior has some validity. He claims that the way our students learn has changed because of Google. Because of Google, our students put the priority not on the new information, but on the way to find it. Thus, instead of memorizing the new facts, our students memorize the means on finding those particular facts. Of course this is in contrast with how we used to react when finding important information before the era of search engines. We used to memorize the information as accurate as we could, because we realized that it was not easy to find the information again.

If his claim is true, it means that our present students approach math learning differently too. In the old days, some traditional math lessons stressed so much on formula memorization and complicated calculations. We as students at that time saw this type of math lessons as normal. It was normal to memorize tons of math formula, even though we did not understand how to discover the formulas. It was also normal to do tedious, complicated, and meaningless calculations. This can be understood, because at our time, information was not that easy to obtain. We had to search the appropriate references using card techniques in real libraries. We had to compute by hand all computations. However, this digital world is totally different now. To search certain information and particular math formulas, we could find them using our smart phones in our hands now. And, the finding and selecting process is done in an instant, easy, and cheap way. Not only that, if we reach some information that we do not intend to search for, the search engines may suggest to us alternative sources of information. We used to rely on our real teachers and real librarians to obtain this kind of assistance. Now, computers and Internet with its smart search engines fill in their jobs. Similarly, when our students nowadays face boring, complicated, and routine numerical calculations, their normal reaction would be: "Why should we do these calculations, when a cheaper-than-BigMac calculator can do these faster and with much higher accuracy?" We on the other hands do not have some solid arguments to go against their reasoning. Their arguments are as valid as they should be.

Another characteristic GenZ have is their openness. They are very open to express their emotion. If they like or dislike something, they will instantly say it. Various Web 2.0 technologies support this characteristic. They are socially connected stronger than ever

with their peers. Using social media applications like Twitter, Facebook, Tumblr, etc, our students develop their own social network.
Students belonging to GenZ are also multi-taskers. They like to do many things at once. One can observe this characteristic by monitoring the way how they surf the Internet. They can open several tabs or windows simultaneously. Moreover, they are very smart and quick learners. However, they have some tendency to prefer to scan articles, instead of reading them line by line. It is possible to be like that because they live totally in a text world, they refine their reading skill, not to read thick books, but to interpret short text or paragraphs.
Students 2.0 also learn best using video, music, and interactive media. They learn best when they can control their learning. They construct the knowledge by themselves. They also like to obtain instant feedbacks. Not only they are not tolerant like the previous generations, but they also search for instant success. These characteristics must be understood well by School Math Teachers 2.0.

School Math Instruction 2.0

After observing the changes in the way students learn, one can see that to engage students to math learning activities nowadays becomes so vital. Moreover, the math concepts they learn should be meaningful. It would be very risky to teach unconnected math concepts to them using chalk and talk approach. Therefore, math teachers in this era should have deep understanding on the math they teach and also know its connection to real life. Without sufficient math understanding on the part of the teachers, it would be very difficult to engage the students into abstract concepts like algebra or geometry. The next condition to teach math well in this era is the understanding on the utilization of technology. In particular, math teachers nowadays, we call them School Math Teachers 2.0, should have passion in doing math and at the same time IT literate.
There are many elementary school math teachers at present that did not have major in math. In particular, because most elementary school teachers are class teachers, they are not specialist teachers. So, in general, elementary school math teachers do not have formal education in college-level math for more than one year. Even some high school math teachers have no college-level math background courses beyond first year calculus. They are usually natural science or engineering graduates. The problem with this situation is that mathematics is seen as merely a tool. Because they learn to use math in their disciplines, mathematics is valued by its functions only. One should realize that mathematics is useful; therefore it is true that it has real functions across disciplines and in real life. There is nothing wrong with this point of view. However, mathematics is also a form of art. It has some esthetics perceptions.
Space and time nowadays are the limitation of school teachers. No technology can solve this limitation conditions. It becomes more and more expensive to build extra space to learn. It also becomes very hard to find sufficient amount of time to create an actual learning community. Therefore, real school space and real school time must be substituted with the virtual math lesson and it must be done without school time limit. Here, information technology and Internet play important roles.
Thus, in short, school math teachers 2.0 must be equipped with

1. understanding on mathematics concepts they teach and the ability to see them from advanced points of view;
2. Know how on utilizing computers, software, and basic Internet applications.

The first point is a must. If a math teacher knows how to utilize information technology very well, but does not understand math topics, it is a recipe for chaos. It will be worse than if a school math teacher is just not good, for the teacher can broadcast her/his misunderstandings and confusions easier and faster with the new technology.

Math Teacher Professional Development 2.0

Since nowadays school math teachers must have deep understanding on math concepts, it is becoming more and more urgent to equip the pre-service and in-service math teachers with appropriate education and training on the math contents. In pre-service teacher preparation institutions, curricula stressing on the basic math concepts are needed. The math contents must on one hand be relevant to the school math they are going to teach, but on the other hand the way to interpret the K-12 math concepts must be from very advanced points of view.

For example, the teacher preparation institutions must teach number system concepts in a very deep approach. They must provide courses discussing fractions, both from the algebraic point of view as the field generated from the ring of integers and from analysis point of view as a dense topological space. Even though they are not going to teach these advanced ideas to their students, their understanding on these advanced concepts will help them to design meaningful lesson activities. Therefore, when school math teachers teach fractions, even to elementary school students, they will be guided by enduring understandings on fractions, like:

1. Every non-zero fraction has an inverse, that is the inverse of $\frac{p}{q}$ is $\frac{q}{p}$ as long as p is not zero.
2. For every pair of fractions a, b, where $a < b$, there is always a fraction $\frac{p}{q}$, such that $a < \frac{p}{q} < b$.

Concepts like fractions are often misunderstood by our students. These misunderstandings or misperceptions should be studied very well by the school math teachers, before they are in service. Unless ones learn basic abstract algebra and real analysis in college, these understandings will not be easy to appreciate. Let alone, conveying the passion to do math to their students. One cannot implement classical learning theories like Piaget's constructivism theory or Bruner's CPA theory, unless she or he understands the math concepts first. If a math teacher does not master the math concepts, she or he will definitely use teacher-centered teaching approach.

The approach "relevant yet very advanced" like above is the basic principle used in the master of math for teaching program at Institut Teknologi Bandung. The program, intended for in-service school math teachers, focuses on both strengthening their mastery on 7-12 school math concepts from an advanced point of view and rejuvenating their passion in doing math. Similarly, modern professional development for math teachers, called in this paper as School Math Teacher Professional Development 2.0, should focus first on developing both the teachers' mastery of math concepts and their passion in doing math.

The next capability our math teachers 2.0 should develop from their professional development or formal training is digital literacy. Because of characteristics of Students 2.0 have, PD 2.0 in school math should also equip the teachers with know-how of modern digital technology. In particular, Math Teachers 2.0 would teach math better, if they master math applications like GeoGebra, Mathematica, etc. Using computer applications like these, creating dynamic worksheets becomes very easy nowadays. Ones do not need computer tech experts to help school math teachers creating these dynamic worksheets anymore.

Utilizing modern math applications, school math teachers nowadays can make dynamic and interactive worksheets in a short time. Moreover, since applications like GeoGebra are open source types, teachers all over the world contribute or share to develop the application and they are easy to legally obtain or download them.

More importantly, incorporating this information technology, School Math Teachers 2.0 can implement and realize classical learning theories, like Piaget's constructivism theory, for example. All modern teachers know that if students construct their knowledge or understanding by discovering math ideas themselves, their conceptual understanding will be more meaningful and sustained. Therefore, the mathematics education challenges nowadays consist of implementing classical learning principles using modern technology. Dynamic worksheets may help the students discover and conjecture math properties that they used to be told by the teachers in the previous century.

Another characteristic of the students 2.0 that one should remember is their preference to work with machines than with human beings. They prefer learning from computers or recorded videos than listening to real teachers themselves. Therefore, using the dynamic worksheets above, the school math teachers 2.0 can record the interactive usage of the worksheet complete with their voice. This recorded video can be uploaded to their blogs, and the students can learn from them.

An Illustration on How Dynamic Worksheet May Realize Piaget's Constructivism

In this section, an illustration is given for explaining how classical learning theories can be realized easily using modern computer application like GeoGebra. What is proposed here has some resemblance to the idea stated in (Goos, 2010).

> …the way in which dynamic geometry software allows students to transform a geometric object by 'dragging' any of its constituent parts to investigate its invariant properties. Through this experimental approach, students make predictions and test conjectures in the process of generating mathematical knowledge that is new for them. (Goos, 2010)

As an illustration here, the topic used to describe the idea is linear equation concept $y = mx + c$. This concept is usually learned by the seventh graders. The school math teachers 1.0 might explain explicitly the main two components of linear equation: the gradient and the y-intercept. They might also tell exactly what the geometric meaning of these two parameters. That is the former describes how steep the line is, and the latter describes where the line intersects the y-axis. One question that is always asked by school math teachers 2.0 is whether these facts must be told. Is it possible the students themselves discover these two important conceptual understandings?

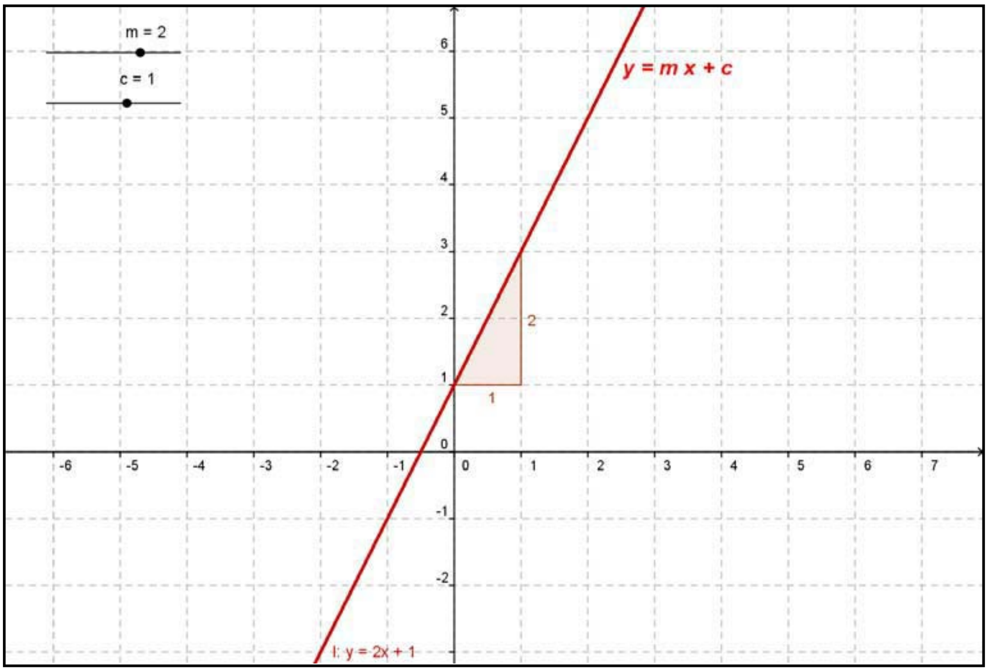

Figure 1 Line Equation

The picture in Figure 1 is the actual screen shot of GeoGebra. After creating this dynamic worksheet and posting it on their personal blog, school math teachers 2.0 could ask their students to visit their blog and play around this worksheet. The students should observe the geometric effects on the changes of these two parameters. The students should observe the geometric effects of the line by changing the value of m and c using the sliders on the top left.

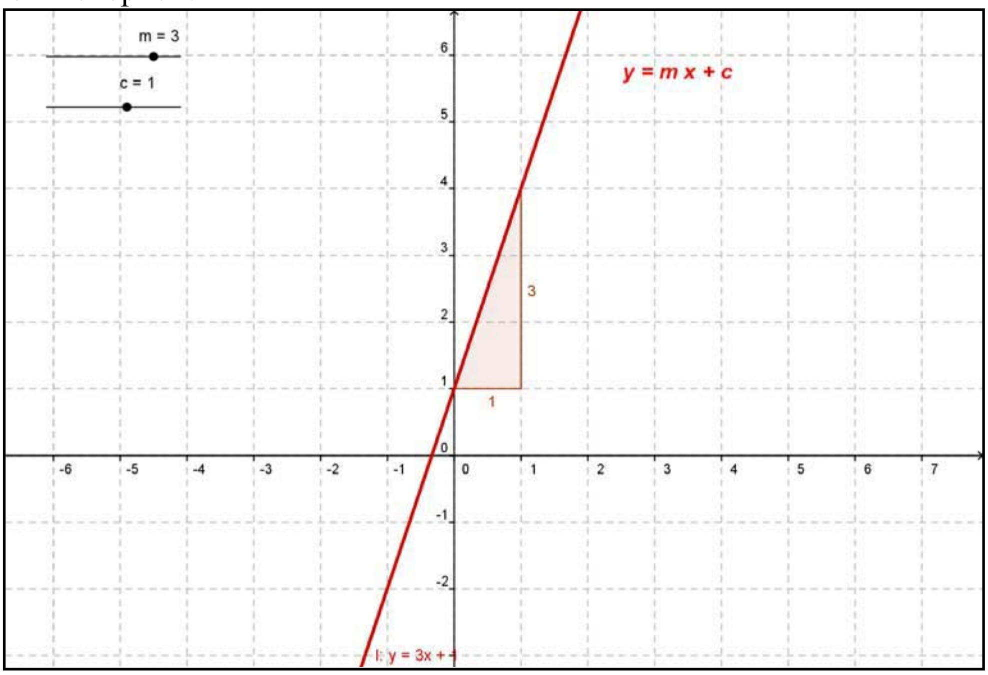

Figure 2 Linear Equation with Higher Gradient Value

Students could observe when the value $m = 3$, for example, how different the line is from the previous one. Thus, the students should evaluate and describe the difference between Figure 1 and Figure 2 and others. Observing the geometric effects on the changes of the two parameters m, c was not so easy without computer aided teaching like this.
While the students are playing on the dynamic worksheets, the teachers guide and facilitate the learning process by questioning them. Listening and questioning are the best teaching tools left for teachers 2.0. By sequences of questions, school math teachers 2.0 should guide the students to uncover, among other things, two enduring understandings:

1. The higher the value m, the steeper the line;
2. The value c determines where the line intersects the y-axis.

Knowledge and understandings like these usually must be explicitly told by teachers 1.0, but using the dynamic worksheets, students 2.0 can uncover these understanding themselves.

Using GeoGebra to Strengthen the Teachers' Understanding on Math

In the math teacher training and professional development programs, technology-based instruction should be practiced extensively. Below is an illustration on how the technology is introduced in the graduate of math for teaching program conducted at Institut Teknologi Bandung (ITB), Bandung, Indonesia. On the course Symmetry and Transformation at the master of mathematics for teaching program at ITB, the teacher participants were asked to make dynamic worksheets using GeoGebra or any other applications. The dynamic worksheets are required to represent and interpret several theorems in the course handbooks.
As an illustration, during the course Symmetry and Transformation, students were asked to create the dynamic worksheets interpreting the following theorem taken from the course handbook (Usiskin, Z., et al., 2003).

Theorem 7.9 (b) (Two-Reflection Theorem for Translation):
If line m and line n are parallel, then $r_m \circ r_n$ is the translation with direction perpendicular to the lines m and n, and with magnitude twice the distance between m and n, in the direction from m to n.

In the theorem above, the symbols r_m, r_n denote the reflection with respect to m, n, respectively.
One participant created a worksheet and posted it to his blog. The Figure 3 below shows the screen shot of the worksheet.

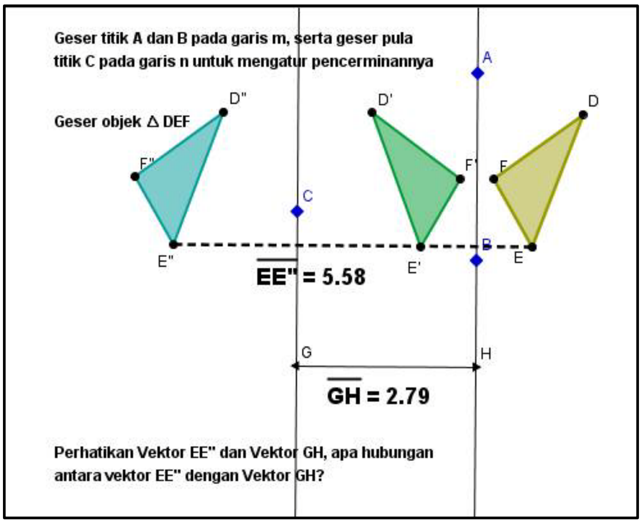

Figure 3 Two-Reflection Theorem for Translations

After creating and posting it, he explained that because he made this dynamic worksheet, he understands the theorem more. Moreover, using this worksheet, when he later on teaches the subject to his students, he could use the worksheet and guide his students to conjecture this property of the composition of two reflections. Therefore, he could implement the integration of the Piaget's constructivism in actual geometry topics.

Conclusions and Future Works

One sees that nowadays, the students 2.0 can uncover the math enduring understandings by themselves if the school math teachers 2.0 own three required components; namely, deep understanding on math concepts, passion in doing and learn math, and literacy in digital technology. Therefore, one big challenge to both in-service and pre-service math teacher training institutions nowadays is to design programs that may facilitate the school math teachers to develop the above all three components. One should remember though that the classical learning theories become more relevant today than ever. The modern digital technology is to help to increase the effectiveness of the math learning.

The students 2.0 learn in a way very different from the teachers. The students 2.0 also have different preference in learning. They are socially connected to each other and need to express freely their emotions. These changes force school math teachers to change the way they teach. The school math teachers 2.0 are the ones who can adapt to the style of the students 2.0 and take advantages from these changes. So the math teacher training and professional development institutions in this era must anticipate the changes. At the same time, future study focusing at these changes is required. There must be series of researches on designing the learning methods compatible with the changes.

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MATHEMATICAL LITERACY

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Abstract

In the past century, literacy was related to written communication skill, which included an ability to read and write using letters. This means that one is illiterate when he/she cannot read or write. This is an implication of the need of people to survive in their culture and civilization. Nowadays, reading, writing and arithmetic skills are not sufficient to deal with complicated problems in our daily life. People should also know the relationship between two or more objects, refer to the existing premises and the principle used, and then a temporary conclusion and a final conclusion are drawn. All this steps are called mathematical reasoning. Based on this development, where reasoning is a must for all people, the meaning of mathematical literacy, of course, does not only cover reading, writing, and arithmetic. By adding reasoning into existing literacy aspects, we can see that mathematical literacy can be viewed as knowledge and skills needed to be able to survive financially, socially, economically in global culture and civilization.

Keywords: Mathematical literacy; mathematical competency.

A. Mathematical Literacy

In Cambridge Advance Learner's Dictionary "Literate" is: (1) able to read and write; and (2) having knowledge of a particular subject, or a particular type of knowledge. This means that mathematical literacy means an ability to read and write mathematics, as well as having knowledge of mathematics.

In the past century, literacy was related to written communication skill, which included an ability to read and write using letters. So, we consider that one is illiterate when he/she cannot read or write. This is an implication of the need of people to survive in their culture and civilization.

Nowadays, reading, writing and arithmetic skills are not sufficient to deal with complicated problems in our daily-life situations. In mathematics, people need to know the relationship between two or more objects. Based on the rules or theorems or principles which have been proved, one has to conclude to create a temporary conclusion. By referring to the existing premises and the principle used, then other conclusions are resulted. This process is repeated until final conclusion can be drawn. Add fun and value to your mathematics lessons

Allan Leslie WhiteAll this steps are called mathematical reasoning.

Based on this development, where reasoning is a must for all people, the meaning of mathematical literacy, of course, does not only cover reading, writing, and arithmetic.

By adding reasoning into existing literacy aspects, we can see that mathematical literacy can be viewed as knowledge and skills needed to be able to survive financially, socially, economically in modern culture and civilization.

Mathematical literacy is an individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgements and to use and engage with mathematics in ways that meet the needs of that individual's life as a constructive, concerned and reflective citizen (OECD, 2003).

In this meaning, mathematical literacy is used to give emphasis to mathematical knowledge, to be applied in daily life or real world problems. To support this, fundamental mathematical knowledge and skills are required. Mathematical literacy, therefore, contains knowledge of mathematical terminology, facts, and procedures (including algorithmic operations and the use of some methods). The problem we may encounter would be solved by combining all these essential components of mathematics.

The literacy is also related to the word "the world", which means that it is contextual and connected to the individual real worlds. To some extent, mathematical concepts, structures and idea, are created and used as tools to organize all phenomena in real world and converted in symbolic manipulations.

Based on the meaning of the aforementioned mathematical literacy, the use of mathematics for solving problems is always there, when someone is dealing with mathematics. People who have mathematical literacy are involved in communicating, assessing, and appreciating mathematics. So, mathematics becomes important for preparing their further study as well as for their recreational purposes.

The extended meaning of the above statements regarding mathematical literacy, involves posing (constructing a question), formulating, solving, and interpreting the problem based on the contexts. There might be a lot of contexts related to the problems. Some of them can be pure mathematics, or contexts without mathematical structure. In this case, the structure should be identified, analyzed, and explained.

To be mathematically literate, we need to possess all these competencies to varying degrees, and need confidence in using mathematics under their ability and should be comfort with quantitative ideas. It is also desired to be able to appreciate mathematics from historical, philosophical, and societal points of view.

Mathematical literacy is not easy to understand and to teach, as mathematics is not identical to memorizing or rote learning. In mathematics we need to understand the concepts; so memorizing facts is not sufficient. Further, in mathematics, apart from facts, there are other important aspects involved: principles, procedures, algorithms, and insight. Learning or memorizing algorithm is important, but insight is the more important aspect, that is an essential component of mathematical understanding. Insight is an understanding of quantitative relationship and the ability to identify those relationships in an unfamiliar context; its acquisition involves reflection, judgment, and experience. In fact, there so many countries have begun to take seriously the problems associated with overemphasizing algorithms but neglecting insight.

According to de Lange (2003), mathematical literacy consists of more than executing mathematical procedures and possessions of basic knowledge that would allow a citizen to be able to live in a difficult situation, and by having just enough of something they need. Mathematical literacy is mathematical knowledge, methods, and processes applied in various contexts in insightful and reflective ways. Mathematical literacy is the important literacy (as it can affect the other areas) that includes *numeracy*, *quantitative literacy* and *spatial literacy.* Each of these types of literacy empowers the individual in making sense of and understanding aspects of the world and his/her experiences.

Mathematical literacy is less formal and more intuitive, less abstract and more contextual, less symbolic and more concrete. Mathematical literacy focuses more attention and emphasis on reasoning, thinking, and interpreting as well as on other mathematical competencies.

The definition of mathematical literacy does not focus attention to minimal knowledge of mathematics. It also covers doing mathematics and applying the concepts of mathematics in other disciplines or solving problems in our daily life, from usual ones to unusual ones, from the simple ones to the complex ones.

The competency rely on the work of Niss (1999) and similar formulations can be found in the work of many representing many countries: (1) Mathematical thinking and reasoning, (2) Mathematical argumentation, (3) Mathematical communication, (4) Modelling, (5) Problem posing and solving, (6) Representation, (7) Symbols, and (8) Tools and technology.

To function as a mathematician, a person needs to be literate. It is not common that someone familiar with a mathematical tool fails to recognize its usefulness in a real-life situation (Steen, 2001: 17). Neither is it uncommon for a mathematician to be unable to use common-sense reasoning (as distinct from the reasoning involved in a mathematical proof).

Is that the competencies needed for mathematical literacy are actually the competencies needed for mathematics as it should be taught? In the past, literacy and what is learned in mathematics classes were largely disjointed. Now, however, they should be taught of as a largely overlapping.

In the Netherlands, people are moving away from the strictly algorithmic way of teaching mathematics. Some mathematical abilities or competencies are clustered: (1) reproduction, algorithms, definition, and so on; (2) encompasses the ability to make connections among different aspects (de Lange, 2003).

Character building, as one of the current issues in Indonesia, can be considered as crucial affective aspects which can encourage students to do mathematics. All these aspects can give important contribution to students' mathematical literacy. In mathematics, character building of affective aspects covers curiosity, the desire to find the solution, self efficacy, self confidence, enthusiasm, mathematical disposition, motivation, interest, honesty, and diligence.

B. Mathematization, Mathematical Reasoning, and Mathematical Problem Solving

Mathematics education is required as a basic tool for developing further knowledge and skills in mathematics needed to survive financially in our jobs and occupations. This demand gives implication to arithmetic as a mainstream of mathematics teaching in elementary schools which teach students addition, subtraction, multiplication, and division operations on numbers numerically.

Almost all contents in elementary school mathematics are numeric oriented, which means that almost all concepts are presented in numbers. Consequently, all material of mathematics presented in elementary school, as well as in Junior and Senior High Schools, are presented in numbers. This situation results in community perception, which concludes that mathematics is always about numerals or numbers. In terms of traditional system of teaching, numbers are considered as objects to be manipulated under certain operations. There were not enough efforts to give interpretation on what the number depicts. Most of the cases, students do not know how to comment on the meaning of their computation results. Many students find difficult in solving worded problems, in which mathematical models should be presented before they come to the solution of the problem.

Based on the aforementioned explanation/description, the notion of mathematical literacy in modern era can be considered as an entity knowledge, understanding, and skills needed by anyone to effectively function in modern era. These skills also cover all skills which have been developed in mathematics school which comprises basic arithmetic operation on numbers.

In modern life, arithmetic skills which consist of addition, subtraction, multiplication, and division operation, need to be combined with mathematical reasoning, communication, and problem solving skills. Mathematical representation, which some people think as a part of communication skills, is also important for people who learn mathematics, in order that they can present a model derived from daily life problem. By these efforts, we solve our real world problems using mathematical concepts and procedures. In this situation, we "do" mathematics. This type of competency can be considered as "mathematization", as people model a problem or phenomenon in their life and solve it by the means of mathematics. Here people "mathematize" general problem into a more specific problem in mathematics notation. There are a number of problems we encounter in our daily life, and it is our task to convert it into mathematical problem. The problem is how to represent real world problems in mathematics. Once they have been presented in mathematical models, computers can be utilized to do computation operation.

The other mathematical competency to be put into account is the skill of relating mathematical notions (mathematical ideas) and modern life problem. Our students need to be convinced that mathematics is really important in solving their daily problem as there are many real world problems, which can be simplified and solved by using mathematical ideas and concepts. This means also that mathematics teaching should be presented to the students by connecting mathematical idea and its real use, and the students can see this connection clearly. By these efforts, mathematics will be considered as a useful and worthwhile skill in students' life.

Mathematizing has 5 aspects (OECD, 2003): (1) Starting with a problem situated in reality; (2) organizing it according to mathematical concepts; (3) gradually trimming away the reality through process such as making assumptions about which features of the problem are important, generalizing and formalising; (4) Solving the mathematical problems; and (5) making sense of the mathematical solutions in terms of the real situation.

All these 5 steps describe the meaning of "doing mathematics", where mathematical concepts, facts, and procedure are used in some works and occupations. People who are well informed and fully engaged in solving certain real world problems make use of "mathematization". This process should be accomplished by all students who learn mathematics.

Mathematics is not only for mathematics. Nowadays, all developed countries, as well as developing countries (including Indonesia) need smart people (as their citizen) who are able to deal with more and more complex problems. Mathematics is used not only for computation purposes, but also for giving argumentation or presenting claims, which needs logic, to ensure that their way of thinking is correct. Reasoning in logic is widely used, not only in mathematics, but also in linguistic. In linguistic some principles of rules of inferences and rules of replacements are widely used to back up the statements they pose, when argumentation is presented or a debate is conducted.

The students' mathematical literacy can be seen from mathematical knowledge and skill they show when they are solving mathematical problems. All mathematical problems should be connected to their previous background and learning experience. This means the context of the problem has to be familiar to the students. Otherwise they fail to interpret the result of their computation.

In PISA, solving a problem is given under some mathematical contents: quantity, space, shape, change and relationship, and uncertainty. PISA also define mathematical competency as mathematical process that students use when they attempt to solve problems. Three clusters of competency are defined: the reproduction cluster, the connection cluster, and reflection cluster. In these three clusters the following aspects are observed: (1) Thinking and reasoning; (2) Argumentation; (3) Communication; (4) Modelling; (5) Problem Posing and Solving; (6) Representation; (7) Using symbolic, formal and technical language and operations; and (8) Use of aids and tools.

C. Memorization

In the past, good in mathematics means good in memorization or rote learning. In their early schooling time, students were trained to memorize the result of products between two numbers. As a matter of fact, this activity of memorization is certainly needed to speed up our computation and reasoning. The main point is, we have to understand the process why we come to the results. There is nothing wrong, of course, to remember some facts, provided that we know its process and its usage in other mathematical concepts. In fact, it will be very difficult for students to remember many facts without understanding. If students always try to memorize all information without understanding, all the facts will easily disappear from their memory, as without this competency their retention will be in short-term period.

Mathematical literacy is not a final journey. It is not the final destination. Rather, it is a process which grows during a student does mathematics. All students in their schooling time develop their mathematics skills, which from time to time, will be even richer and richer. Teachers need to convince their students that mathematical proficiency should be developed continuously, as we will be faced with more complex problems. People with high mathematical proficiency can do much better, as they are skilful and well experienced.

In terms of the nature of mathematics, as an organized structure, abstraction, generalization, patterns, facts, procedures, students should comprehend all these contents, as all these things will assist them in dealing with further concepts they might face.

Based on Bloom Taxonomy, students have to possess competency indicated in Cognitive Domain, as well as Affective Domain and Psychomotor Domain. In cognitive domain students should be able to recall facts, concepts, principle, and procedures; to understand concepts; to apply formulae, procedure, and principle in a new situation; to analyze concepts; to synthesize concepts; and to evaluate and give judgement accompanied by correct reasoning.

Students should also be able to do mathematical representation, where they express their ideas in terms of symbols, notations, or mathematical expressions. By having these skills, they can communicate with other students, teachers, as well as mathematician.

Further, students should be able to see the connection between concepts in mathematics, concepts in mathematics and other disciplines, concepts in mathematics and their use in daily life (real world). So, understanding concepts is not sufficient, as reasoning, application and problem solving are also important for students.

When students do mathematics, they need to solve problems. Once the result from computation process finished, they have to be able to interpret they have which are depicted by all numbers resulted.

D. Students' Proficiency in PISA

Indonesia, for the first time, involved in Programme for International Assessment (PISA) in 2000. In 2003, Indonesia again attended this international assessment, together with the other 48 countries who also involved in this activity.

Basically, there are three groups of competency that PISA concerns: group of knowledge, students' background, and school background. In terms of group of knowledge, students are tested in reading, mathematics, and science.

The evaluation on mathematical literacy aims to identify students' competency in identification, understanding, and application of some basic mathematics facts and procedures for solving problem in daily life.

PISA also analyzes the students' background including students' demography, their social-economy status, students' expectation, and their motivation and discipline. All these data are integrated with school demography which includes school organization, staffing patterns, models of teaching, and academic atmosphere.

The study of PISA is worthwhile to identify students' literacy level in some countries; to design a benchmark for improvement purposes; and to understand the strength and the weakness of the education system in all participating countries.

PISA measures students' competency in 3 main aspects: (1) the content (structure) obtained by students; (2) the process the students do in presenting argumentation; and (3) students' reaction when they are faced to their daily life problem to be solved by mathematical models and computations.

The main aspects evaluated in PISA consist of definition, content, process, and situation dimension. The content (of mathematics) includes change, relationship, space, and shape. In addition, students should be skilful in computing. In the year to come PISA will also include probability. PISA gives students 210 minutes to finish their work in mathematics. The total time, together with Science and Language, is 390 minutes.

In PISA there are 6 proficiency levels used: (1) answering questions in general contexts, identifying information and solving problem using routine procedure; (2) interpreting and identifying situation in context which need direct inference; (3) execute procedures exactly, using representation from different sources, expressing the reason they uses, and communicating the interpretation and all reasoning; (4) effectively working with models and concrete contexts they have, select and integrate all types of representation and observe its relation with real world using sound argument and inference; (5) working with a model in complex situation, understanding all constraints they might have, select, distinguish, and evaluate some strategies to solve complicated problems related to the model, using deep and wide reasoning, connecting mathematical skill and the situation they face, doing reflection process and attempting to communicate what they have in their mind, applying deep understanding in new situation using new strategy and approach, formulating and communicating what they find, interpreting and presenting argumentation.

E. Concluding Remark

People need to be mathematical literate in dealing with problems they face in modern life. Mathematical literacy is important for people in dealing with their work and jobs. People in modern era, not only need understanding of arithmetic, but also reasoning, and problem solving, as they have to solve more and more complex problems. Reasoning in logic is widely used. Rules of inferences and rules of replacements are widely used to back up the statements we pose, when argumentation is presented or a debate is conducted. All mathematical problems should be related to our previous background and learning experience. We have to be familiar with the context of the problem.

In school, mathematical literacy is a process which grows during a student does mathematics. All students during their schooling time develop their mathematics skills, which from time to time, will be even richer and richer. Teachers need to convince their students that mathematical proficiency should be developed continuously for solving more complex problems. Students with high mathematical proficiency can do much better, as they are skilful and well experienced in solving the problems in their real world situations.

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PLENARY PAPERS

PRIMARY 6 STUDENTS' MODEL DEVELOPMENT IN THE MATHEMATICAL MODELLING PROCESS

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Abstract

In the last decade, mathematical modelling has become more prominent in the field of mathematics education. The Ministry of Education (Singapore) has distinctively included *Applications and Modelling* into the mathematics curriculum in 2007 suggesting the importance the role that mathematical modelling plays in the curriculum. During the formative period, Chan (2011) was involved in a case study research of Primary 6 students' engagement with mathematical modelling tasks situated in a problem-based learning setting. Mathematical modelling in the research was based on a models-and-modelling perspective (Lesh & Doerr, 2003). This paper is part of that larger study and presents the students' model development as their conceptual representations and the related mathematical reasoning within the modelling process. The findings revealed the students' expressing, testing and revising of their models as part of their mathematical thinking within the modelling stages towards goal resolution.

Key words: Mathematical modelling, model development, problem solving

Introduction

In the 21st century, schools are largely aware of the need for students to develop skills and abilities that go beyond managing factual knowledge. These abilities include interpreting, describing, explaining, constructing, manipulating and predicting complex systems with conceptual tools and resources that they would generate towards managing and solving problems (English & Sriraman, 2010). The need to inculcate the said abilities in students suggests that current classroom pedagogies would require significant changes for impacting students' mathematical thinking towards the promotion of student communication, collaboration and conceptualization in the process of solving problems. As a forward looking future-oriented mathematics education curriculum, some researchers have proposed mathematical modelling as a pedagogical approach for students to draw on knowledge within and outside the mathematics classroom to solve problems (English & Sriraman, 2010; Lesh & Zawojewski, 2007). When mathematics activities include realistic mathematical modelling elements for making sense of real-world situation and giving meaning to such a situation, they provide students with opportunities for making connections between the mathematics utilized in real-world situations and their curricula mathematics. In 2007, the Ministry of Education (MOE, 2007), Singapore, included *Applications and Modelling* into the mathematics curriculum to advance this cause. The purpose of this paper is exemplify the modelling process of Primary 6 students involved in mathematical modelling tasks, in particular, to trace the model development of the students based on a selected modelling task.

Mathematical Modelling

Mathematical modelling has been variously defined and it also takes on differing perspectives in literature. While some see solving structured word problems as mathematical modelling, others liken it to traditional mathematical modelling due to the direct mapping of givens to operations that is limited in both interpretation and solution generation (English, 2003). In general, mathematical modelling begins with the confrontation of a real-world problem or situation, and it is the process of representing such a problem in mathematical terms as one attempts to find solutions to the problem (Ang, 2001).

This study adopts the theory of mathematical modelling from a models-and-modelling perspective (MMP). The MMP focuses on students' representational fluency through the flexible use of mathematical ideas when students make mathematics descriptions of the problem context and data (Doerr & English, 2003; Lesh & Doerr, 2003). Authentic contexts are the platforms for sense-making where students situate their reasoning in coming up with real-world solutions. Based on this perspective, students develop models (internal conceptual models) that are powerful but are under-utilized unless they are expressed externally through some representational media as they solve the problem (hence the modelling activity is also commonly known as "model-eliciting activity"). The process of model development is seen as the students' internal conceptual systems are continually being projected into the external world through spoken language, written symbols, graphs, diagrams, and experience-based metaphors. In a sense, the development of models is the process of *mathematizing reality* as contrasted with *realizing mathematics* (Lesh & Doerr, 2003). The other key element of the MMP is that the modelling process comprises several express-test-revise cycles that help students' refine their thinking beyond current conceptions.

Modelling activities have been found to help in the promotion of important mathematical reasoning processes such as constructing, explaining, justifying, predicting, conjecturing, and representing (English & Watters, 2005); reasoning aspects that are valued as a powerful way to accomplishing learning with understanding. Furthermore, studies have shown that children have been able to manage complex mathematical constructs irrespective of their academic mathematics achievement (Lesh & Doerr, 2003).

Method

Participants

The participants were Primary 6 students from two classes of the same neighbourhood schools. Each class comprised small groups of four or five students. Two groups from each class were selected by their respective mathematics teacher to be the target group for video-recording.

Modelling Task

The main study comprised 5 modelling tasks that were designed for the students to undertake over a period of 2 months. In this paper, as the purpose is to exemplify the model development of the students, only the Floor-Covering Task is used. The task was based on an adaptation of the modelling task from Gravemeijer, Pligge and Clarke (1998) and situated in the context of determining the most economic way of covering the floor of a study room given the different floor-covering materials that were priced differently. The details of the task given out as the task sheet are shown in the appendix. The students had to identify the key mathematics components from the task, conceive layout designs, establish key variable relationships and analyze their designs to determine which design was most economical as part of the decision-making process.

Data Collection and Analysis

The target groups were video-recorded during their engagement of the modelling task. Other sources of data included field notes, the students' written work and journals. The video data were transcribed and reviewed several times by two independent coders and the researcher based on a problem-solving coding scheme for reliability check. Using an interpretive framework developed for this study, the data were parsed as modelling stages and episodes for making inferences of the students' discourse to track the progression of the students' thinking for emerging models through modelling stages. The development of models was captured as the students' conceptual representations and their respective mathematical relations.

Findings

This section presents a walk-through of the modelling endeavour of a group of Primary 6 students towards exemplifying their model development for one of the modelling tasks. For a more comprehensive coverage in comparing two groups of the students' work, refer to Chan (2010).

Description Stage

Description refers to attempts at understanding the problem to simplifying it. This includes identification of goals, variables, and clarifying task details through drawing inferences from text, diagrams, formulas or whatever given data to make sense of the task details. Students also make assumptions from personal knowledge to simplify the problem.

Identification of goals

The following is a short excerpt depicting the students discussing the goal of the activity at the beginning of the modelling process.

S2	Why don't we calculate the best way and the cheapest way for each one first?
S4	Can I say something? They never state to find the cheapest way, so we can choose not to.
S3	Yes, "explain clearly mathematically your best choice" *(referring S4 to the task sheet).*
S4	They say the best choice, not necessarily the cheapest.
S3	What do you mean by the best choice?
S2	The cheapest choice lah. The one that you spend the least money

As evident from the excerpt, the students deliberated about what they wanted to achieve. The students were found to negotiate the meaning of "best choice". They clarified their goal by situating the best choice to be the "cheapest" where least money would be spent on covering the floor.

Identifying Variables

The students were involved in trying to identify the task variables that would help them situate the problem. They specifically mentioned the need to find "area" and knew that calculations had to be performed.

S1	We have to think out of the box to solve the problem. *(S1 begins to record)*
S2	Find out the area.
S1	What mathematics do we need?
S4	Area. Calculation.

Clarifying Task Details

Before the students went into solving the problem, they spent quite some time clarifying task details. In particular, one of the students, S3, was not able to visualize the dimensions of the carpet when unrolled. She could not make out if the 4m printed beside the carpet icon was fixed as well as how much to unroll.

S3	What do you mean by 4m?
S4	4m here *(using index finger to point at the side of the carpet icon)*
S3	This one can come up to what?
S4	This one is 4m. Really.
S4	Ms XXX, this is 4m right?
T	What did they say?
S3	Are you sure this is not 4m long?
S4	Because they write here 4m. This is 4m right?
S4	You mean 4m wide, right? *(pointing to the side of the carpet icon again).*
T	Yeah, this is the width, this is the floor. How are you going to cover it?
S4	4m here. This 4m cannot be changed.

The discourse is one of clarifying the dimensions with one another and even with the teacher. This suggests the persistence to know enough about the details before going into solving the problem.

Manipulation Stage

Manipulation refers to attempts at establishing relationships between variables and task details through constructing hypotheses, critically examining contextual information, retrieving or organizing information, mathematizing, or using strategies towards developing a mathematical model. The following examples show the various models that students conceptualized and revise along the way.

Conceptualizations of Layout Designs

This aspect saw the students engaged in modelling as a form of organizing. They had to determine ways to orientate the carpet to cover the floor, the dimensions needed, and also the amount of loose materials needed towards covering the remaining floor gap. One of the orientations they deliberated on was to unroll the carpet breadthwise against the floor leaving a gap of 0.3m by 3m. Interestingly, the students conceptualized two ways to cover the gap with the loose materials measuring 0.5m by 1m each (see Figure 1).

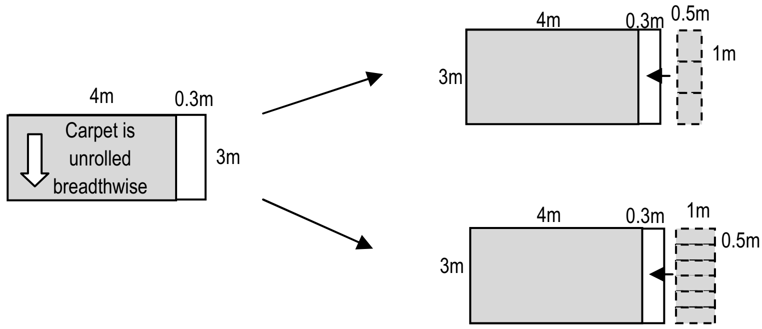

Figure 1. Floor-covering layout designs using the carpet

The students did not have much of an issue in deciding which option to go for. They decided on the first option as true to their goal because it required lesser loose materials and hence implied lower cost.

The students then conceptualized two ways to cover the floor with the mat, one of which is shown in Figure 2 where the mat was unrolled lengthwise. It had left two gaps for the students to consider patching up.

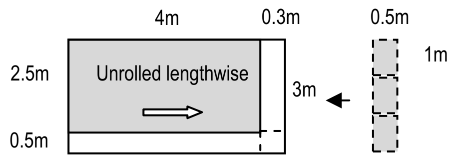

Figure 2. Floor-covering layout designs using the mat

It is interesting to note that the student who conceptualized this layout design unrolled the material up to 4m instead of 4.3m. This was questioned by a member as seen in the excerpt below:

S3	Why 4?
S2	4.3, but they don't sell in 0.3. They only sell per metre square.

The reply "4.3, but they don't sell in 0.3. They only sell per metre square" revealed how S2 could have confused the idea of area to only involve dimensions with whole numbers. Although the design still worked, it would not have been more cost efficient had the floor-covering material been unrolled all the way to 4.3m. The excerpt shows that when students solve problems in such settings, there were instances that ideas generated were scrutinised and defended. It also shows that the ideas adopted might not lead to the best solution outcome but nonetheless a workable solution. At this juncture, it is timely to add that the students eventually realized they could unroll all the way to 4.3m lengthwise, and they revised their model.

Finally, the students also worked out the layout design through tiling. The students were able to conceive the fitting of the tiles as an array of 9 by 6 (see Figure 3). The reason for not conceptualizing a 8 by 6 array was because the students were factoring the cost of the extra tiles to be used for the floor gap.

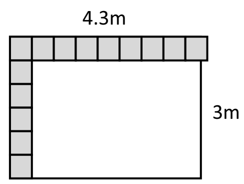

Figure 3. Floor-covering layout designs using the tiles

The above segment reveals that in a PBL setting, the students as they model layout designs were generating ideas about orientations, use of dimensions, and working with constraints towards making a fit to the floor and at the same time monitoring the costing involved. They were also negotiating and trying to justify their reasoning.

Other Conceptualizations
It was observed that students have been expressing, testing and revising their models for comparison towards goal resolution (see Figure 4).

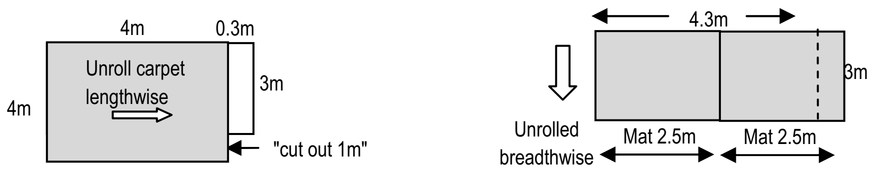

Figure 4. Other floor-covering layout designs

In the various conceptualizations, some of the designs were found to have infringed task parameters. These are considered part of the discourse where students would need to consider how to manage the situation and if the models would be viable. During the process, the above two designs were eliminated from their choice later on as they were found to be more costly. For the models that infringed the task parameters, they did not affect the students' final solution outcome.

Prediction Stage
Prediction refers to attempts at interpreting the models that they have conceived to ensure that they fit the parameters given or established. This includes affirming, verifying or making decisions to justify their cause that they have attained a workable model. A workable (functional) model is usually attained after several revisions have been made to the emerging model as meeting the task requirements. An example of the students' interpreting their results is shown in the following excerpt where they were found to be making decisions as they compared the different solutions (Method 1, 2, etc. in their language):

S3 Thus our conclusion is that (...) if we use mat, method 2, eh...mat method 2 how to make?

S2 Method 1 is like that right? And method 2 is like that, correct?

S3 Thus our conclusion is that if we use mat, Method 2, is the cheapest way although we have some pieces of mat left over.

Optimization Stage

Optimization refers to attempts at improving or extending their model solutions to achieve an ideal solution that is material efficient and yet maximizing value. The teacher plays an important part in extending the students' thinking towards optimizing their solutions. The students were initially satisfied that they had found that the mat floor-covering to be the cheapest and decided to stop work. The teacher however challenged their thinking further towards improving their solution.

S3	Because here is 3m, so we must buy 3 pieces.
T	Why 3 pieces?
S3	Because here 1m, 1m, 1m.
T	So if here is 0.5m, here will be…
S3	0.2.
T	OK, so here is 0.2, right? So you'll have these pieces. Can you use these remaining pieces?
S2	You mean you can cut along the loose pieces?
T	It’s a carpet. What did they say? You can "further cut". (*T points to the task sheet)* Think about it.
S3	We can cut it as small as possible.
T	Further cut. Think about it.
	(Pause as students try to figure out as they look at the task sheet)
S2	OK, let's go back to this one. Carpet. 4 times 3. So how do we do it? *(S1 begins to unroll the serviette again model the situation)*
S3	If she is saying that we can use the remaining loose carpet to fix instead of buying another loose carpet to fix, that means instead of buying another loose carpet, we can use the remaining to fix.
S2	0.3 here. 0.3 times 3. Here is 0.2, 0.2. Then here is 0.4.
S2	Then we only need 2.
S3	Yeah, we only need 2.

From the excerpt, the students explained to the teacher that they had used three pieces of the loose mat floor-coverings to patch the gap. The teacher then focused the students' attention to the actual dimensions needed to patch the gap and the amount of loose materials that would have been wasted. The students realized that they could cut the loose materials into smaller parts and even reused the potentially wasted parts. From there, they reanalyzed their designs for cutting the loose materials by working on the dimensions of the materials that would potentially be wasted to enhance the patchwork (see Figure 5).

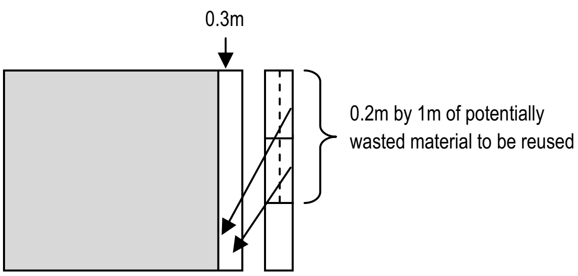

Figure 5. Making improvement to the model

They found that they only needed two pieces to patch the gap instead of three. With this idea of saving materials, the students were able to transfer their knowledge to optimizing the material-cost layouts for the carpet as well to check if it resulted in a lower cost.

Making Mathematical Relations

For each of the conceptual layouts that the students conceived, they worked out the costing as a form of mathematical relations to represent the situation. It was through comparing the costs that they determined which layout design for which material that was to be their choice. Figure 6 shows the mathematical relations the students made for each of the layout designs with respect to the areas needed and the associated costs.

<u>Carpet</u>
(1) 4 x 3 = 12
$12 x 12 = $144
Loose carpet → $144 + $12
Total cost → $144 + $12 = $156

<u>Mat</u>
(1) 25 x 3 = 15
15 x $11 = $165

<u>Tiles</u>
9 x 6 = 54
54 x $3 = 162

<u>Carpet</u>
(2) 4 x 4 = 16
$16 x 12 = $192
Loose carpet → $12
Total cost → $204

<u>Mat</u> √
(2) 25 x 4 = 10
10 x $11 = $110
2 x $5 = $10
4 x $5 = 20
$110 + $10 + $20 = $140

Figure 6. The mathematical workings to determine the costs of the layout designs

In Figure 6, the "Mat" material indicated with a tick represents the decision they made with respect to the choice of the floor-covering material as that model gave them the lowest cost.

Discussion and Concluding Points

The findings all point to a pedagogy that is vastly different from traditional problem solving. The different mathematical interpretations elicited during mathematical modelling reveal important aspects about the mathematical objects, relations, operations, and patterns underlying the students' thinking. It also shows that the students develop emerging models that are subject to constant testing and revisions. The early models are usually naïve as they are initial conceptualizations but as they are worked upon and tested, they are revised along the way towards becoming more stable models (Chan, 2009; 2010; English & Watters, 2004; Lesh & Doerr 2000). From the findings, the more pronounced emerging models reported in this study showed how the students use a consistent structure: Area of floor x cost per unit area of material. This structure is viewed as a summation of two parts: (Area of floor covering material for certain amount of floor area x cost per unit area of material) + (Area of floor gap x cost of amount of loose material). The conceptual representations coupled with the mathematical relations bears similarities to Gravemeijer's (1997) notion of mathematical modelling as a form of organizing and translating where models emerge through the organizing and the related mathematical procedures as translation. Recognizing the structure allows for a workable model to be developed but not necessarily an enhanced model, after all as in this modelling task the floor area is fixed. What varies therefore is the conceptualization of the layout designs which accounts for the different costs. This would demand the exercise of representational fluency towards reusing materials to achieve the goal of maximizing value.

As students worked on the modelling task, important mathematical knowledge and ideas are elicited, for example, the students' abilities to interpret, analyze, explain, hypothesize, conjecture, compare, and justify. It allows students to draw on their curricular knowledge to be used creatively and applied in a more authentic problem context. The mathematical reasoning and thinking involved also becomes more productive through the iterative cycles of expressing, testing and revising.

The models-and-modelling perspective does not insist that students get the correct answer. Students were found to develop models that had infringed task parameters for their lack of paying attention to task details. Sometimes students got stuck in the process due to some unwarranted assumptions or they impose of inappropriate constraints. The difficulties they encountered are to be seen as opportunities for refining initial ideas. What is important is that through the modelling process the students are able to edge towards what they believe would lead to their model solution. In this sense, as contrasted with traditional problem solving, the modelling perspective sees that "the process is the product" (Lesh & Doerr, 2003, p.3).

As schools search for more productive ways to help students in mathematics and problem solving, engaging in mathematical modelling activities is a promising platform to advance current practices.

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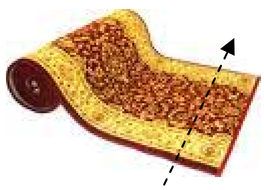
Appendix

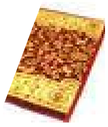
The Floor-Covering Problem

You have been asked by your mother to suggest a covering for the floor of your study room. The room is rectangular and measures 4.3m by 3m. There are three ways to cover the floor. You can use the mat, carpet, or tiles but they are of different costs. Explain clearly and mathematically your best choice and how you arrive at your decision. Drawing diagrams may make your explanation clearer. (Note that pictures are not printed to scale)

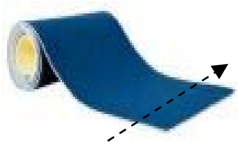


Carpet

Can be cut in only one direction as indicated by dotted arrow.

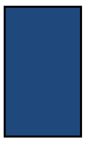


Loose carpet of 0.5m by 1m for patchwork at $6 per piece. Each piece can be further cut to fit size.



Mat

Can be cut in only one direction as indicated by dotted arrow.



Loose mat of 0.5m by 1m for patchwork at $5 per piece. Each piece can be further cut to fit size.

0.5m

Square Tile

If a tile cannot completely fit part of the floor space, professional help is required to cut the tile to fit.

ENGAGEMENT WITH MATHEMATICS: THE INFLUENCE OF TEACHERS

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Recent decades have seen growing concern over the lowering levels of engagement with mathematics in Australia and internationally. This paper reports on a longitudinal study on engagement with mathematics and explores the influences of teachers on the students' engagement with mathematics. Findings reveal that the development of positive pedagogical relationships between students and their teachers forms a critical foundation from which positive engagement can be promoted.

Introduction

In recent years there has been growing concern over the lowering levels of engagement with mathematics in Australia (Commonwealth of Australia, 2008; State of Victoria Department of Education and Training, 2004; Sullivan & McDonough, 2007; Sullivan, McDonough, & Harrison, 2004) and internationally (Boaler, 2009; Douglas Willms, Friesen, & Milton, 2009; McGee, Ward, Gibbons, & Harlow, 2003). The issue of lowered engagement levels in mathematics during the middle years of schooling (Years 5 to 8 in NSW) has the potential to cause wide-reaching consequences beyond the obvious need to fill occupations that require the use of mathematics. Lowered engagement with mathematics can lead to reducing the range of higher education courses available to students through exclusion from courses that require specific levels of mathematics. Students who discontinue studying mathematics can potentially limit their capacity to understand life experiences through a mathematical perspective (Sullivan, Mousley, & Zevenbergen, 2005).

One of the most significant influences impacting on engagement in mathematics is the teacher and teaching practices, or pedagogy (Hayes, Mills, Christie, & Lingard, 2006; NSW Department of Education and Training, 2003). This paper is derived from a longitudinal case study on engagement with mathematics during the middle years of schooling. In this study a group of 20 students experienced a range of mathematics teachers and pedagogical practices in their final year of primary school and the first two years of secondary school. Data was collected from the group across the three school years through individual interviews and focus group discussions.

This paper is an investigation of the influences of teachers and their practices on the participants' engagement with mathematics. The theoretical framework underpinning this paper is based on current theories and definitions of engagement, and literature defining 'good' teaching of mathematics. A brief overview of the literature is now provided.

Engagement

Seminal Australian research into student engagement, the Fair Go Project (Fair Go Team, NSW Department of Education and Training, 2006) focussed on understanding engagement "as a deeper student relationship with classroom work" (p. 9). The Fair Go Team found students need to become 'insiders' within their classroom, feeling they have a place and a say in the operation of their classroom and the learning they are involved with. Students have a need to identify themselves as 'insiders' as well as to be identified as 'insiders' by their teachers, students and all stakeholders.

There are other definitions of engagement that should also be considered. Some view engagement only at a behavioural level (Hickey, 2003), where others view it as a multidimensional construct (Fredricks, Blumenfeld, & Paris, 2004). Fredricks et al. (2004), define engagement as multi-faceted and operating at operative, affective, and cognitive levels. Operative engagement involves the idea of active participation and involvement in academic and social activities, and is considered vital for the achievement of positive academic outcomes. Affective engagement includes students' reactions to school, teachers, peers and academics, influencing willingness to become involved in school work. Cognitive engagement includes the idea of investment, recognition of the value of learning and a willingness to go beyond the minimum requirements. In terms of engagement with mathematics, engagement occurs when students are procedurally engaged within the classroom, participating in tasks and 'doing' the mathematics, and hold the view that learning mathematics is worthwhile, valuable and useful both within and beyond the classroom.

In an investigation into the reasons students are choosing not to pursue higher-level mathematics courses, McPhan, Moroney, Pegg, Cooksey and Lynch (2008), claim "curriculum and teaching strategies in the early years which engage students in investigative activities and which provide them with a sense of competence are central to increasing participation rates in mathematics" (p. 22), yet attempts to investigate the lack of engagement with mathematics have failed to find good reasons for students' difficulties. It is claimed students who are engaged with school are more likely to learn, find the experience rewarding and continue with higher education (Marks, 2000).

'Good' teaching and mathematics

The teaching practices employed within mathematics classrooms cover a wide spectrum ranging from 'traditional', text book based lessons, to contemporary or 'reform' approaches of problem solving and investigation based lessons, or a combination of both. When recalling a typical mathematics lesson, many students cite a traditional, teacher-centred approach in which a routine of teacher demonstration, student practice using multiple examples from a text book and then further multiple, text book generated questions are provided for homework (Even & Tirosh, 2008; Goos, 2004; Ricks, 2009).

An alternate approach to teaching mathematics reflects a constructivist perspective where students are given opportunities to construct their own knowledge with a focus on conceptual understanding rather than instrumental understanding. Such an approach promotes problem solving and reasoning and is consistent with Australian frameworks for quality teaching (Newmann, Marks, & Gamoran, 1996; NSW Department of Education and Training, 2003).

Although there are arguments for using either or both approaches, there is strong support for an investigational, contemporary approach to teaching and learning mathematics (Anthony & Walshaw, 2009; Boaler, 2009; Clarke, 2003; Lovitt, 2000). Open-ended, rich tasks transform students' beliefs about problem solving and alter the culture of mathematical engagement. Evidence suggests that providing students with engaging mathematical tasks supported by appropriate teaching strategies leads to sustained improvement in learning outcomes (Callingham, 2003).

Much research has been conducted on effective teaching of numeracy and mathematics, with a particular emphasis on the pedagogical content knowledge (PCK) required for effective teaching of mathematics (Askew, Brown, Rhodes, Johnson, & Wiliam, 1997a; Delaney, Ball, Hill, Schilling, & Zopf, 2008; Hill, Ball, & Schilling, 2008; Schulman, 1986). In support of the need for strong PCK it can be argued that teachers with higher mathematical qualifications do not necessarily produce strong learning outcomes in their students as a result of weak understandings of how students learn and the pedagogies that are appropriate for particular mathematics content (Askew, Brown, Rhodes, Wiliam, & Johnson, 1997b).

In recent years the national mathematics teaching professional association, the Australian Association of Mathematics Teachers (AAMT) (2006), developed a set of standards that reflects current literature on effective teaching of mathematics and represents national agreement of teachers and stakeholders on the required knowledge, skills and attribute of quality teachers of mathematics. Data informing this paper were analysed against the backdrop of the above literature on engagement, effective teaching and current teaching standards. The following is a brief description of the methodology used in the study.

Methodology

The participants in this case study were originated from a Year 6 (the final year of primary schooling in New South Wales) cohort in a western Sydney catholic primary school. The students were identified through Martin's Motivation and Engagement Scale (High School) (2008), as having strong levels of engagement with mathematics. The instrument consisted of a 44 item Likert scale requiring students to rate themselves on a scale of 1 (Strongly Disagree) to 7 (Strongly Agree) and was adapted to be specific to mathematics. All students in the group of 20 made the transition together to the local catholic secondary college which had been in operation for only two years prior to the group's arrival. The participants had a diverse range of mathematical abilities and came from a range of cultural backgrounds, and most came from families with two working parents.

During the study the students participated in individual interviews during Year 6 and again in Year 8, and a series of focus group discussions at five points across the duration of the study. Teachers identified by the students as ‘good’ mathematics teachers were interviewed and observed during several mathematics lessons. The students formed three focus groups, a boys group, girls group and mixed gender group. Each interview and focus group discussion was based on the following set of discussion points/questions: (a) Tell me about school; (b) Let’s talk about maths; (c) Tell me about a fun maths lesson that you remember well; (d) When it was fun, what was the teacher doing?; and (e) What do people you know say about maths?

The data gathered were transcribed and coded into themes. In terms of the students’ perceptions of mathematics teaching, two major themes emerged as being influential on their engagement with mathematics: teachers’ pedagogical practices, those day-to-day routines that teachers implement in their teaching of mathematics, and the pedagogical relationships formed between teachers and students.

Results and discussion

During Year 6 the participants experienced pedagogies that included an emphasis on cooperative learning. The opportunities for interaction and discussion that this provided had a positive impact on the students’ engagement with mathematics, with one student saying: “You’ve got like more options to choose from rather than if you’re by yourself” and another: “working with partners is fun because you could find different strategies and you have fun and it’s easier.” It can be argued that the classroom practice of cooperative learning has positive results in terms of providing a safe environment in which the students are able to learn within a positive classroom culture. The ability to associate learning in mathematics with fun appeared to be a powerful influence on engagement, and the following quote summarised the collective feeling of most of the participants: “The group can work it out together to try and solve the problem and you’ve like learned something new or how to work out something.”

One Year 6 teacher, Mrs. L, who was identified by the students as the ‘best’ mathematics teacher, was described by several students as someone who enjoyed teaching and had a passion for mathematics. Alison believed this quality to increasing her own engagement: “She just puts a lot of enthusiasm in maths and makes it really fun for us. She gets all these different maths activities. She just makes it really fun for us and I quite enjoy maths now because of that.”

It appeared the Mrs. L’s enthusiasm for mathematics promoted positive attitudes and excitement towards mathematics, reflecting the findings from research (Askew et al., 1997b) and recommendations by the AAMT (2006). In addition to her passion for mathematics, the students witnessed Mrs. L. as appearing to have fun teaching. Tenille said: “It’s fun when the teacher, like, while you’re doing the work she also has fun teaching the maths as well.”

When the students moved on to their first year of secondary school, Year 7, they experienced a new set of pedagogies and a new group of mathematics teachers. In contrast to the teaching approaches used during their primary years, the students were expected to work on an individual basis, using computer-based interactive tutorials and mathematics textbooks. This caused a reduction in classroom interaction and discussion, and rather than having a single mathematics teacher, the students were provided with a rotation of four different teachers.

Although the provision of computer technology provided the opportunity for teachers to deliver a new and relevant way of teaching and learning (Collins & Halverson, 2009), they instead appeared to be used as replacements for teachers. Alison commented on this emerging idea among the students:

> … it's probably not the best way of learning because last year at least if you missed the day that they taught you, you still had groups so your group could tell you what was happening. Where now, we’ve got the computers and it’s alright because there is, um, left side of the screen does give you examples and stuff, um, but if you don’t understand it, it’s really, hard to understand.

It is reasonable to suggest that the website and textbook were not necessarily inferior resources. However, the data was showing that it was the way they were used in isolation from other resources that meant the students began to disengage from mathematics. During Term 2 of Year 7 the students were provided with the opportunity to participate in tasks that were more interactive and hands-on, consistent with recommendations from research (Boaler, 2002; Callingham, 2003; Lowrie, 2004). Several of the students commented on this change, with Fred saying: “It’s more interesting”. The students found the incorporation of concrete materials made their mathematics lessons more interesting, and the opportunity to work in groups during one particular activity made those lessons memorable, with Rhiannon giving this reason: “because we got to create the shape by using straws, in groups. Not by ourselves.” In addition to the benefits of being able to work collaboratively, George felt he and his group made more of an effort than usual: “it was good because we could make it ourselves and we could like put effort into it.”

When the students reached their second year of secondary school, Year 8, the school’s structure had been reviewed and during Term 2, the students were allocated one regular mathematics teacher per group. The newly formed mathematics classes appeared to increase the students’ engagement, allowing stronger teacher/student and peer relationships to develop. In terms of the resources that were used in the Year 8 lessons, there was less reliance on the students’ laptops and more emphasis on using text books. Kristie described a typical routine:

> Well, we just got our text book and the laptops don't come out in maths as much or at all, unless you've forgotten your text book or something like that. And, um, maths is good, we separated into groups and the teacher's out the front and he'll tell us what to do and you pretty much put your hand up if you need help, and he'll help you and then you have the text book out and you answer the questions in your maths book.

Although it has been found that a traditional approach to teaching mathematics may have a negative influence on student engagement, in this particular case the students saw it as an improvement on previous pedagogies and appeared to experience higher levels of engagement. One aspect of the teachers' pedagogies that had a positive effect on the students' engagement was the students' perceptions of an improvement in teacher explanations. George made this comment which reflected the feelings of many of the students: "I think maths has improved because the teachers go through it with you more, whereas last year they would just set you a task and leave you with it." Billy, a student who had difficulty maintaining his engagement with mathematics added: "Sir just writes stuff on the board and then he explains it really good and we learn about stem and leaf graphs. He teaches it really good and other teachers just write it down and say 'go do that'."

During the final focus group discussions, Alison made a comment that was reflective of the group's feelings once they were assigned their regular teachers and were able to begin building positive pedagogical relationships: "The teachers know where we're coming from and what we need to learn and they learn, not what the group needs, but what we need." The data shows that the students appeared have begun to re-engage with mathematics because they felt the teachers knew them in terms of their mathematics learning needs. The opportunity to establish positive pedagogical relationships with teachers appeared to provide students with a sense of belonging, an important aspect of an effective mathematics classroom (Boaler, 2009).

Implications and conclusion

The biggest influence on engagement with mathematics for these students appeared to be that of their teachers. This influence can be viewed at two interconnected levels. The first level includes the pedagogical practices employed by the teacher, and the second, the pedagogical relationships that occur between the teachers and students. That is, the connections made between the teachers and students, and the teachers' recognition of and response to the learning needs of his or her students. Although this study has limitations in terms of the selective nature of the sample, it is suggested that the development of positive pedagogical relationships forms a critical foundation from which positive engagement can be promoted and this may be applicable to a wider student population.

The findings discussed in this paper imply many students in the lower secondary years of schooling are still dependent on high levels of interaction within the mathematics classroom. Repetition of the current study within different school contexts would be of benefit in further exploring the concept of engagement with mathematics. Further studies on engagement with mathematics during the later years of schooling and beyond into tertiary education would be beneficial in terms of investigating whether pedagogical relationships remain as important for older students. Although student achievement and its relationship to engagement levels was not a focus of this study, such an exploration would also be worthwhile for future research.

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Towards mathematical literacy in the 21st century: Perspectives from Indonesia

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Abstract

The notion of mathematical literacy advocated by PISA (OECD, 2006) offers a broader conception for assessing mathematical competences and processes with main focus on the relevance use of mathematics in life. This notion of mathematical literacy is closely connected to the notion of mathematical modelling whereby mathematics is put to function for solving real world problems. Indonesia has participated as a partner country in PISA since 2000. The PISA trends in mathematicsfrom 2003 to 2009 revealed unsatisfactory mathematical literacy among 15-year-old students from Indonesialeggedbehind the average of OECD countries. In this paper, exemplary cases to examine and to promote mathematical literacy at teacher education level will be discussed. Lesson ideas and instruments were adapted from PISA released items 2006. The potentials of such tasks will be discussed based on case studies of implementing these instruments with samples of pre-service teachers in Yogyakarta.

Introduction

The notion of mathematical literacy advocated by the Programme for International Student Assessment (PISA) has gained wide acceptance globally. Mathematical literacy goes beyond curricular mathematics and covers a broader conception of what constitutes mathematics. The main focus of PISA assessment is on measuringpotentials of 15-year-old students inactivating their mathematical knowledge and competencies to solve problems set in real-world situations. PISA (OECD, 1991) definition of mathematical literacy captures this:

> Mathematical literacy is an individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgements, and to engage in mathematics in ways that meet the needs of that individual's current and future life as a constructive, concerned and reflective citizen. (OECD, 2006, p. 72).

Indonesia has participated as a partner country in PISA since the start of PISA in 2000. The trend from PISA results in mathematics from 2000 to 2009 consistentlyrevealed poor performance. Indonesia is ranked among the lowest performing countries that performed below the OECD average. Figure 1 presents changes in some of countries performance from PISA 2003 to 2009.The comparison between 2003 and 2009 results showed that Indonesian 15-year-olds improved their performance by 11 score points.However, a worrying note from the 2009 PISA results was that almost 80% of Indonesian samplesperformed below the baseline of level 2 of mathematical literacy. At Level 2, students are expected to show ability to use basic algorithms, formulas, procedures or conventions and use direct reasoning and interpretations of the results (Table 2).The fact that majority of Indonesian students performed below the baseline shows a serious problem with maintaining basic skills of mathematics. Clearly there is a strong impetus to address this problem by improving the quality of mathematics teaching and learning so that more students are mathematically literate.

Figure 1 Change in mathematics performance from 2003 to 2009 (OECD, 2010, vol. 5, p. 61)

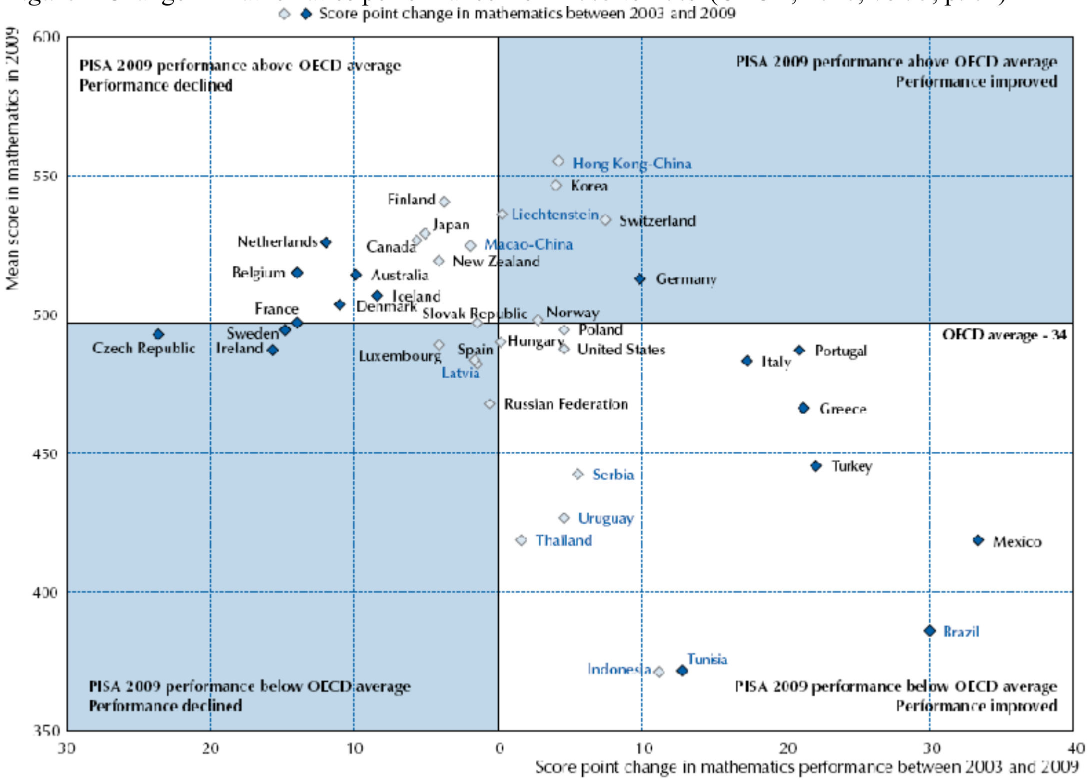

Table 1*Mean scores of mathematical literary of Indonesian samples in comparison to OECD average* [From OECD 2000, Table 3.3., p. 287; OECD 2003, Table 2.5c, p. 356 ; OECD, 2010, Table V.3.1., p. 156]

Country	**Mathematics literacy mean scores (S.E.)**			
	2000	**2003**	**2006**	**2009**
Indonesia	367 (4.5)	360 (3.9)	391 (5.6)	371 (3.7)
OECD average	**500 (0.6)**	**500 (0.6)**	**497 (0.6)**	**499 (0.6)**

Concern over lack of mathematical literacy among Indonesian students has prompted initiatives to raise more awareness on mathematical literacy. In primary school levels, a reform movement to place more emphasis on teaching mathematics connected to real life with PMRI (Sembiring, Hoogland, Dolk, 2010). Professional development sessions for secondary school teachers have started a few years ago to carry forward the realistic approach of teaching mathematics. Last year, the national council of research and development in Jakarta, 'Balitbang', offered a grant to construct test items for secondary schools similar to PISA items using Indonesian contexts. Some of the items are available from http://pisaindonesia.wordpress.com/aktivitas-pisa-indonesia. Presently, mathematical literacy contests for secondary school students are being held concurrently in 7 cities in Indonesia to improve mathematical literacy of secondary school students. In addition to such events, a sustainable program at teacher education level is needed to build capacity of future teachers in planning and carrying out lessons that support the development of mathematical literacy.

Mathematical literacy and mathematical modelling

The notion of mathematical literacy is closely connected to the notion ofmathematical modeling (Kaiser &Willander, 2005; de Lange, 2006; Stacey, 2009).Mathematical modeling involves cyclical processes which start with a problems situated in 'real-world' contexts which are translated and formulated as a mathematical problem.The process of formulating mathematical problems from real-world problems involves simplifying the real-world situations by making assumptions in order to derive mathematical solutions. This process is often referred to as '*mathematisation'* process (de Lange, 2006). The cycle of mathematical modeling ends with interpretations of mathematical solutions in reference to the real-world situations. Evidently, both mathematical modeling and mathematical literacy place the functionality of mathematics in solving real-life situations at the center of mathematical learning.The descriptors for the top levels (i.e., level 6) of proficiency in mathematics explicitly pinpointability to work with models for complex situations and to generalize and utilize information based on the models (Table 2).

Table 2 *Descriptions of mathematical literary of students at level 2, and 6* [From OECD 2010, p. 130]

Level	Lower score limit	What students can typically do
6	669	At Level 6 students can conceptualise, generalise, and utilise information based on their investigations and modeling of complex problem situations. They can link different information sources and representations and flexibly translate between them. Students at this level are capable of advanced mathematical thinking and reasoning. These students can apply this insight and understandings along with a mastery of symbolic and formal mathematical operations and relationships to develop new approaches and strategies for attacking novel situations. Students at this level can formulate and precisely communicate their actions and reflections regarding their findings, interpretations, arguments, and the appropriateness of these to the original situations.
2	420	At Level 2 students can interpret and recognize situations in contexts that require no more than direct inference. They can extract relevant information from a single source and make use of a single representational mode. Students at this level can employ basic algorithms, formulae, procedures, or conventions. They are capable of direct reasoning and literal interpretations of the results.

Real-world contexts and situations are integral elements of mathematical literacy.However, there is no conclusive voice as to whether real-world contexts in mathematical problem affordor inhibit students' mathematical learning (Rittle-Johnson &Koedinger, 2005; Feijs& de Lange, 2004; Sembiring, Hadi&Dolk, 2008, Widjaja, Dolk, &Fauzan, 2010).Contexts might present a barrier in solving mathematical problems. Movshovitz-Hadar, Zaslavsky, and Inbar (1987) noted that contextual mathematical problems demand linguistic skills which present a barrier on mathematical performance. Prior studies revealed that contexts might not be activated by students due to a tendency for direct translation from a problem into mathematical formulas (Busse, 2005; van den Heuvel-Panhuizen, 1999). However, real-world contexts carry a lot of potentials for learning, it allows for multiple pathways to derive at mathematical solutions. Hence the use of real-world contexts cultivates flexible thinking (English, 2010; Lave & Wenger, 1994).Similarly, Widjaja, Dolk and Fauzan (2010) found that meaningful contexts allowed students to relate with their personal experiences which afforded them to solve problems at different levels of mathematical sophistications.

Mathematical tasks

Two contextualized tasks adapted from PISA 2006 released items will be discussed (Appendix A, Appendix B). These tasks were assigned to cohorts of Indonesian pre-service teachers as part of their module on '*Teaching strategy of mathematics in secondary school*'. The goal is to expose pre-service teachers to contextualized mathematical tasks and mathematical modeling processes. This exposure is important as future reference in fostering their students'mathematical literacy. The goal of task 1is to find combinations of pizzas, given a condition to choose maximum of 2 toppings. Task 1 was given as a written quiz to be solved in 20 minutes for a class of Indonesian pre-service teachers(Figure 2).

Task 2was adaptedas a modeling task to investigate a relationship between a person's leg length and his or her pace length. Task 2 can be considered as an extension of the original item whereby a relationship between pace length and the number of steps per minute was given as a mathematical formula. Task 2 was assigned as a group work (of four pre-service teachers) to be completed in 2 weeks. Choice for method of investigation and location for data collection are left open for groups to decide.

Masalah Pizza

[Picture taken from: http://en.wikipedia.org/wiki/Pizza]

Yummy Pizza Shop ingin mengembangkan usahanya. Toko pizza ini menawarkan tiga ukuran pizza: kecil, sedang dan besar.

Pelanggan dapat memilih ***8 topping berbeda***: keju, jamur, nanas, sosis, seafood, ayam, sapi dan sayuran.

Setiap pizza disajikan dengan saus tomat dan taburan keju.

Harga pizza ditentukan oleh ukuran dan banyaknya topping yang dipesan.

Ari ingin memesan pizza ukurang sedang untuk makan malamnya. Karena keterbatasan anggaran, dia hanya dapat memilih ***maksimum 2 topping berbeda***. Berapa banyak variasi pizza yang dapat Ari pilih? Jelaskan jawabanmu selengkap mungkin.

Tuliskan dalam lembar terpisah yang disediakan.

[Problem adapted from: Sáenz, C. (2009). The role of contextual, conceptual and procedural knowledge in activating mathematical competencies (PISA). *Educational Studies in Mathematics, 71*(2), 123–143.]

*Figure 2.*Task 1 is adapted from PISA 2006 released item and Sáenz (2009) as a written quizitem

Hubunganantarapanjang kaki danpanjanglangkah

Apakahadahubunganantarapanjang kaki danpanjanglangkahseseorang?

1. Tentukanfaktordanvariabeldalammasalahini
2. Kumpulkan data untukmembantu kalian menemukan model matematika. Catat data yang dikumpulkan
3. Apa model yang dapatmenjelaskanhubunganantarapanjang kaki danpanjanglangkah? (Cobacariapakahada model yang sudahbaku).
4. Jelaskanmetode kalian dalammenemukanhubunganini.
5. Setelahmenemukanpenyelesaian, interpretasikan model yang kalian temukan.
6. Selidikikembaliasumsi yang kalian buat di awaldanberikanmasukanuntukperbaikan model kalian.

*Figure 3.*Task 2: A modelling task to investigate the relationships between pace and leg length

Findings

Pre-service teachers' responses to Task 1 showed that pre-service teachers came up with different interpretations for the condition of "*maximum two different toppings*" as illustrated in a few samples given in Figure 4. A variety of strategies were displayed, e.g., make a list, create a diagram, and use a formula. Both solutions displayed knowledge of relevant formula to solve the problem as well as correct listing of combinations of pizza with two toppings. However, the solution in Figure 4b might suggest that this pre-service teacher lack did not interpret back her solution to the real-world context. The most common incorrect interpretation was disregarding the possibility of having only 1 topping. In this case, only 28 combinations were identified. Some pre-service teachers applied the formulaefor finding combinations without reference to the given contexts. For instance, one pre-service teacher found 30 combinations by adding $C^{8}_{2} = \frac{8!}{2!(8-2)!} = \frac{8!}{2!(6)!} = \frac{8\times7}{2} = 28$and $C^{2}_{1} = \frac{2!}{1!(2-1)!} = \frac{2!}{1!(1)!} = 2$. This is an example of what Busse labeled as 'mathematically bound'.

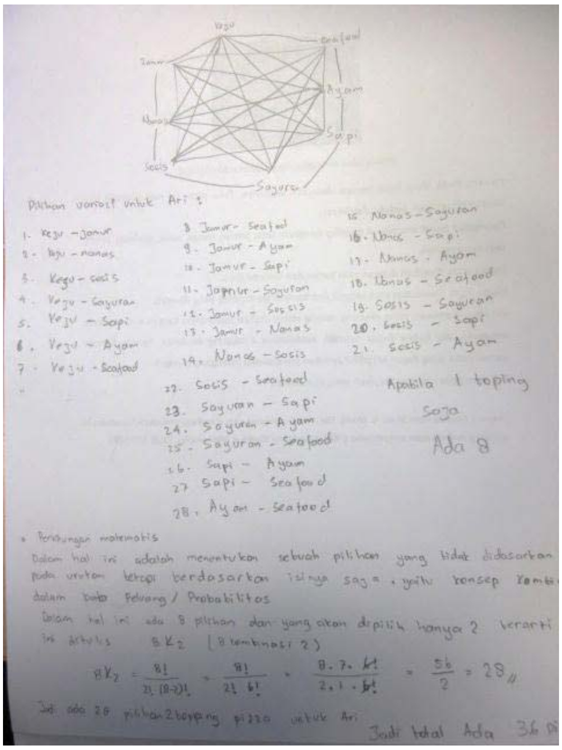

Figure 4a

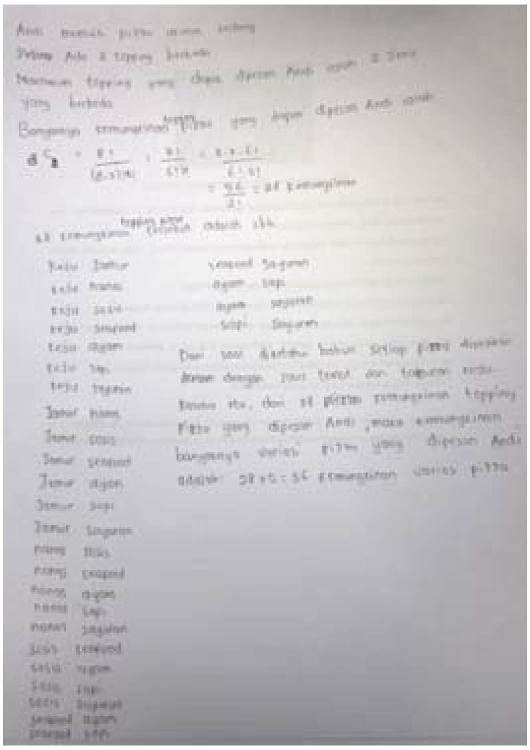

Figure 4b

*Figure 4.*Samples of responses to Task 1

Implementations of task 2 with groups of pre-service teachers revealed potentials and challenges faced by pre-service teachers. As illustrated in Figure 5, different ways of investigating the relationships between leg length and pace length were observed. One group decided to collect data by measuring the footsteps of people who walked on the beach. Another group chose to collect data on campus but required volunteers (classmates) to step their feet on paints and walk along the white cloth to obtain a more accurate measurements of pace lengths. The pace length were calculated after taking average of four footsteps. Both groups noticed that there were variations among data and it was not a straightforward linear relation based on plotting of the points. A person's moodwhen walking (e.g., no hurry or in hurry), and locations (e.g., beach or campus) were offered as reasons for non-uniform pace lengths. An assumption such as constant pace length was not made but an average of pace lengths was taken instead.Using the line of best fit, a linear model to explain the relationship between leg length and pace length was derived. Different linear models were offered, $y = 0.526\,x + 12.86$ by the group who collected data on the beach and$y = 0.641\,x - 7.138$, by the group who collected data on campus, with x represents leg length and y represents pace length.

*Figure 5.*Investigations of relationships between pace and leg length

Conclusions

Two tasks from PISA 2006 items were adapted to be use with Indonesian pre-service teachers. The initial finding suggested that contextualized tasks provide opportunities for various strategies. Such tasks allow pre-service teachers to experience the potentials power of mathematics in real-world contexts. Introducing pre-service teachers with contextual tasks and mathematical modeling as part of their training is expected to build capacity of the future teachers to in planning and carrying out lessons that support the development of mathematical literacy. Commitment to place more emphasis on learning processes which present mathematical problems in real-world settings as part of teacher training program is strongly needed. Providing pre-service teachers with such learning experience during their training will better equipped them to make use of their mathematical knowledge and skills in their life.

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Appendices

Appendix A: PISA 2006 released item (OECD, 2006, p. 71)

M510: Choices

Question 1: CHOICES *M510Q01*

In a pizza restaurant, you can get a basic pizza with two toppings: cheese and tomato. You can also make up your own pizza with **extra** toppings. You can choose from four different extra toppings: olives, ham, mushrooms and salami.

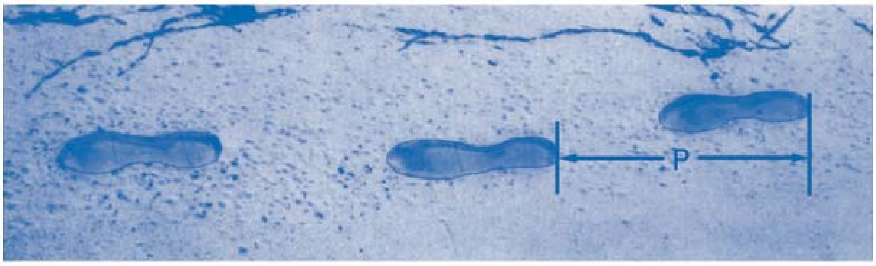
Ross wants to order a pizza with two different **extra** toppings.

How many different combinations can Ross choose from?

Answer: ....combinations.

Appendix B: PISA 2006 released item (OECD, 2006, p. 8)

MATHEMATICS EXAMPLE 26: WALKING



The picture shows the footprints of a man walking. The pacelength P is the distance between the rear of two consecutive footprints.

For men, the formula, $\frac{n}{P} = 140$, gives an approximate relationship between n and P where,

n = number of steps per minute, and

P = pace length in metres.

Question 1: WALKING

Bernard knows his pacelength is 0.80 metres. The formula applies to Bernard's walking.

Calculate Bernard's walking speed in metres per minute and in kilometres per hour. Show your working out.

TOWARDS MATHEMATICAL LITERACY IN THE 21ST CENTURY: PERSPECTIVES FROM SINGAPORE

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Abstract

The Organization for Economic Cooperation and Development (OECD) postulates that a major focus in education is to promote the ability of young people to use their knowledge and skills to meet real-life challenges (OECD, 2006). PISA, an international standardised assessment of students' (aged 15) performance in the literacies of mathematics, science, and reading, was developed by the OECD in 1997 to evaluate the achievement of students who are about to finish their key stages of education (Anderson, Chiu, & Yore, 2010). The concept of mathematical literacy has been defined and interpreted in various ways as recorded in the curriculum documents around the world. This paper will share perspectives from Singapore on how mathematical literacy is interpreted in the mathematics curriculum through the use of three tasks: interdisciplinary project work, applications, and modelling. It will surface challenges to improving the mathematical literacy of students when using such tasks.

Introduction

A major focus in education is to promote the ability of young people to use their knowledge and skills to meet real-life challenges (Organisation for Economic Co-Operation and Development [OECD], 2006). In other words, equipping learners with *literacy* relevant to day-to-day real-world competencies is perceived to be an important current goal in education. PISA, which refers to the "Programme for International Student Assessment" from the OECD, assesses and compares students' reading, scientific, and mathematical literacy. Since 2000, PISA tests are run every three years to elicit the knowledge and skills of 15 year-old students because they are nearing completion of their compulsory schooling. Students' familial and institutional backgrounds are factors considered in providing explanations for differences in performance during PISA among countries. PISA defines literacy to include various "competencies relevant to coping with adult life" (Anderson, Chui, & Yore, 2010, p. 374). In particular, *mathematical literacy* is:

> the capacity of an individual to identify and understand the role that mathematics plays in the world, to make well-founded judgements and to use and engage with mathematics in ways that meet the needs of that individual's life as a constructive, concerned and reflective citizen. (OECD, 2006, p. 21)

Hence, teaching for mathematical literacy is discussed within a broader, utilitarian view of mathematics whereby students are provided opportunities to engage with different types of mathematical problems, especially those relating to real-world contexts. Indeed, proponents for the incorporation of problem solving tasks involving real-world contexts in the mathematics curricula (e.g., Gravemeijer, 1994; Ng & Stillman, 2009;

Van den Heuvel-Panhuizen, 1999; Zevenbergen & Zevenbergen, 2009) have long argued for the importance of cultivating students' ability to work with tasks relating to real life. In relation, other educators (e.g., English, 2008; Galbraith, 1998) espoused the need for students to draw upon their repertoire of *interdisciplinary learning* rather than tapping upon subject-specific knowledge and skills for more holistic mathematical learning through making connections between school mathematics and real-world problems which are often interdisciplinary in nature. According to Stacey (2009), the current concept of mathematical literacy proposed by PISA is related to several other concepts ingrained in mathematics education. Among them is *mathematical modelling*, a process of representing real world problems in mathematical terms in an attempt to understand and find solutions to the problems (Ang, 2010).

The Singapore mathematics curriculum framework (Curriculum Planning and Development Division [CPDD], 2006) shown in Figure 1 highlights mathematical problem solving as central to mathematics learning. Teachers are encouraged to use a wide range of problem-solving situations, including non-routine, open-ended, and real-world contexts in their mathematics classrooms. Some of these tasks can also be interdisciplinary where subject-specialist teachers work in collaboration during task implementation within a class. Although mathematical concepts form the foundation of this pentagonal framework, skills and processes are perceived to be the pillars of this framework. Real-world problem solving tasks involving *applications* and *modelling* are one of the latest infusions in the processes component of the framework which recognises the importance of mathematical reasoning, communication, and connections. It is postulated that such tasks provide platforms for the analysis of mathematical situations, construction of logical arguments, as well as links between mathematical ideas, between school-based subjects, and between school mathematics and everyday life (English, 2008). Nonetheless, the syllabus documents did not set out to distinguish between applications and modelling tasks, perhaps contributing to the limited use of modelling activities in Singapore schools (Ng, 2011a). Though both involve the use of real-world contexts, Stillman, Brown, and Galbraith (2008) articulated the differences between applications and modelling tasks in the following ways. Application tasks are commonly evident in situations where the teacher looks for real-world contexts to match specific taught mathematical knowledge and skills for use. In contrast, each modelling task starts with the real-world context where any of a variety of mathematical knowledge and skills can surface during mathematisation (de Lange, 2006) of the context for model development in problem solving.

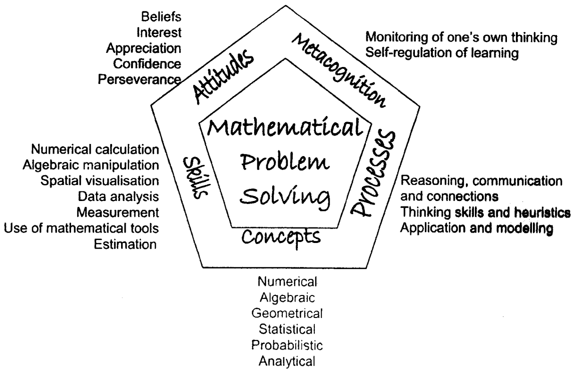

Figure 1. Framework of the Singapore Mathematics Curriculum (CPDD, 2006)

The purpose of this paper is to share selected perspectives from Singapore on how mathematical literacy is interpreted in the mathematics curriculum through the use of three tasks: interdisciplinary project work, applications, and modelling. Implications will be drawn from these interpretations to suggest future developments in teacher education for mathematical literacy as there are still challenges to be overcome.

Mathematical Literacy in Interdisciplinary Project Work

Interdisciplinary Project Work (PW) has been implemented in Singapore primary, secondary, and pre-university institutions since 2000 (CPDD, 1999). As from 2005, students' performance in PW has been part of entry requirements to universities in Singapore (MOE, 2001). PW is an applications task embedded in real-world context which draws upon the integrated use of at least two areas of discipline-based knowledge and skills for problem solving. There were two main impetuses for widespread introduction of PW in Singapore schools. Firstly, each PW makes explicit connections between content knowledge and skills of its anchoring school-based subjects. PW encourages more holistic learning in preparation of students for work in a knowledge-based economy (Tharman, 2005) where work-related real-world problems are often interdisciplinary in nature (Sawyer, 2008). Secondly, PW promotes student-centred learning (Quek, Divaharan, Liu, Peer, Williams, Wong et al., 2006). Figure 2 shows an example of a PW task for year 7-8 students (aged 13-14) comprising mathematics, science, and geography as anchoring subjects (Ng, 2009).

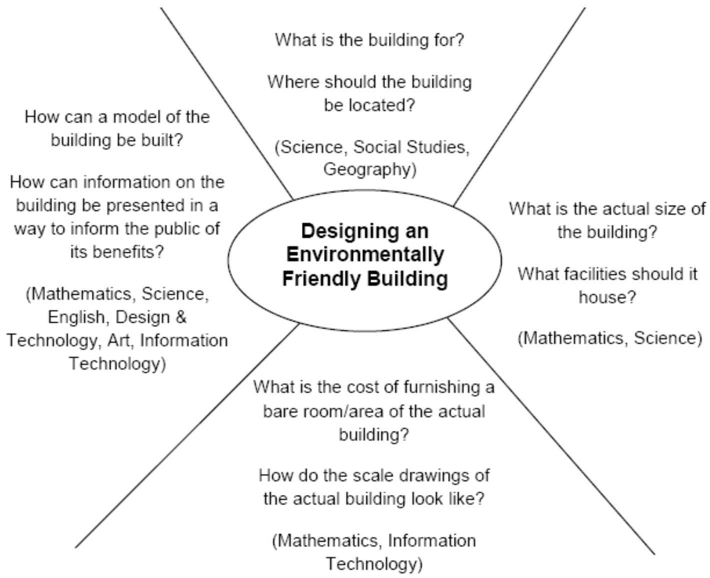

Figure 2. An example of PW involving mathematics (Ng, 2009)

The task required students to work in groups to design an environmentally friendly building in a selected location within Singapore and then construct a physical scale model of their building using recycled materials. It draws upon mathematical concepts and skills such as scale drawings, proportional reasoning, arithmetic, and measurement. Students were encouraged to make decisions on their building design and location based on mathematical calculations after considering scientific and geographic real-world constraints. Here, mathematical literacy is explored when students provided appropriate mathematical reasoning and arguments for their choices using their interpretations from their day-to-day experiences. Nonetheless, research into the mathematical thinking of the students in case-study groups reported in Ng (2011b) revealed that some students face challenges in mathematical literacy during real-world mathematical applications and decision making. One such example came from a year 8 group where each member drew independent scale drawings of the front, top, and side views of their eco-friendly house but using different scales and dimensions, not considering that all drawings of the house should come together to form a coherent image of the house.

Mathematical Literacy in Mathematical Applications Tasks

Mathematical applications tasks have been commonly used in Singapore classrooms. An example is shown in Appendix A (adapted from Foo, 2007) where year 7 students (aged 12-13) worked on calculating the budget for painting and waxing an office room, given a floor plan of it. The task was designed to elicit mathematical concepts and skills such as measurement, arithmetic and area, drawing upon students' real-world understanding of painting and waxing of a room as well as the sale packaging of paint and wax which are usually sold by the litre. Again, a challenge in mathematical literacy is detected in a student's work shown in Figure 3. Although the student has successfully worked out the floor area and the actual amount of wax needed to cover the floor, he had assumed that he could purchase 1.15 litres of wax without much consideration about real-world constraints.

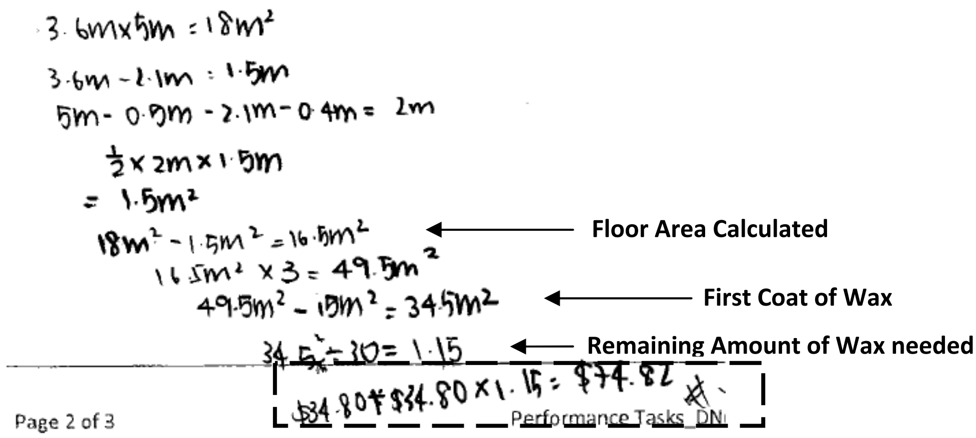

Figure 3. An example of a student's work on the Painting Task

Indeed, as much as challenges exist in student's display of mathematical literacy cited above, it was found that pre-service teachers at times faced the same challenges. Echoing the findings from Verschaffel, deCorte, and Borghart (1997), some postgraduate pre-service teachers who were undergoing teacher education for teaching of middle school mathematics (years 12-14) in Singapore were unable to critically examine the fallacy in realistic mathematical meaning-making in the given context as shown in Appendix B. Whilst many of the 15 respondents could detect that it was almost impossible to solve the problem, most were unable to present sound mathematical reasoning and arguments as to why this was the case. Only two of the respondents mentioned a lack of information on the dimensions of the flask and how its shape changed at different parts, hence removing the possibility of direct proportion reasoning.

Mathematical Literacy in Mathematical Modelling Tasks

Ng (2010) investigated the initial experiences of primary school teachers in their attempt at a mathematical modelling task (Appendix C) and found that many teachers in her sample size of 48 were constrained by their beliefs and conceptions of what mathematics was during their modelling process. They embarked on immediate translation of information provided in the context of the task into mathematical expressions or known algorithms in order to obtain unique answers to the problem. The teachers needed some time to accept that mathematical representations can also take the forms of tabulation of data, graphical, and written statements containing mathematical reasoning based on data, along with assumptions made and conditions set during their chosen approach. It appeared that the teachers' interpretation of mathematical literacy during contextualised tasks was still confined to abstract mathematical representations.

Implications and Future Directions for Teacher Education

This paper set out to present perspectives of mathematical literacy as interpreted in the Singapore mathematics curriculum through exploring the use of three tasks types: interdisciplinary project work (PW), applications, and modelling. Although proponents of mathematical tasks embedded within meaningful real-world experiences espoused that such tasks provide platforms for enhancing the mathematical literacy of students, there are challenges to overcome in teacher education so that the potentials of these tasks could be harnessed for stated educational goal.

One of the challenges is that of producing quality mathematical outcomes from the tasks which should incorporate a reasonable degree of mathematical accuracy within the appropriate choice of approaches used bound by real-world constraints. Another challenge relates to the preparation of students for mathematical arguments tapping upon various forms of mathematical representations. This is because PW and modelling tasks are deliberately open-ended to encourage multiple interpretations and solution pathways. Last but not least, a third challenge involves changing the mindsets of teachers towards a more encompassing view of what mathematics for the purpose of promoting mathematical literacy in students. Teachers can be encouraged to take a less prescriptive pedagogical approach which can limit the nature and variety of mathematical interpretations and representations in contextualised tasks.

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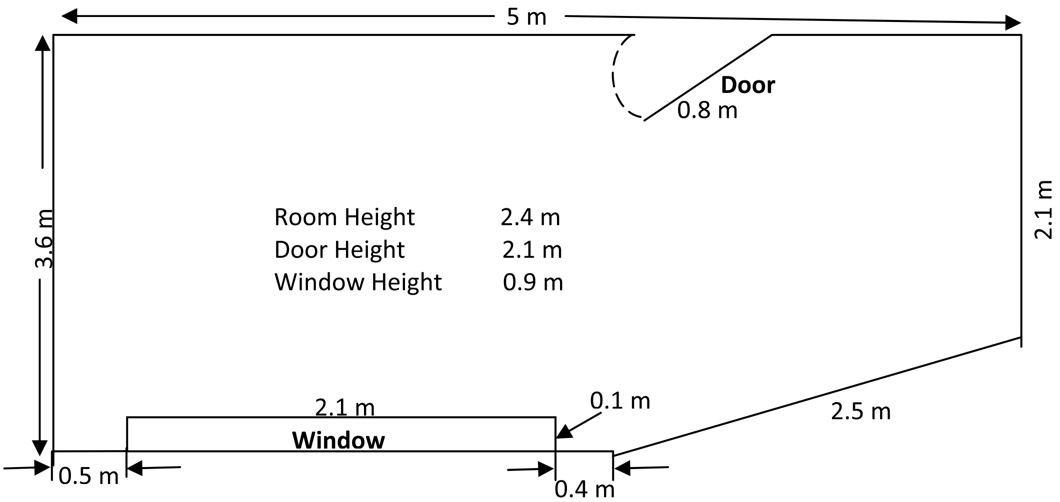
Appendix A

Painting Task

Suppose you are a painter and have been asked to give a quote for painting an office room. It is requested that only ceiling and wall should be painted in silken blue while the floor in the room need to be waxed.

A floor plan of the room is shown below:



Additional Information:

1 litre of paint covers 16 m^2;

1 litre of paint costs $19.90;

1 litre of wax covers 15 m^2 for first coat;

1 litre of wax covers 30 m^2 for subsequent coats;

A litre of wax costs $34.80.

Please help to provide the ***budget*** for both paint and wax:

Wax (3 coats are needed):

Paint (2 coats are needed):

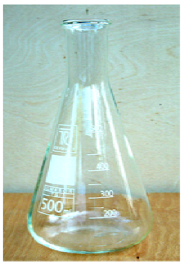
[Adapted from Foo, K. F. (2007). *Integrating performance tasks in the secondary mathematics classroom: An empirical study.* Unpublished Masters Dissertation, Nanyang Technological University, Singapore.]

Appendix B

Flask Task

Some students are given the following task to do:

Li Wei is having a Chemistry class. He has to fill a ***conical flask*** with water from a tap. The flask is being filled at a constant rate. If the depth of the water is 3.5 centimetres after 10 seconds, how deep will the water be in the flask after 30 seconds?



[Picture taken from http://upload.wikimedia.org/wikipedia/commons/thumb/0/00/Erlenmeyer_flask_hg.jpg/450px-Erlenmeyer_flask_hg.jpg

Here are some of their solutions:

Solution 1:

3 x 3.5 cm = 11.5 cm

After 30 seconds, the depth of the water in the flask will be 11.5 cm.

Solution 2:

3 x 3.5 cm = 10.5 cm

After 30 seconds, the depth of the water in the flask will be 10.5 cm.

Solution 3:

3.5 cm + 20 cm = 23.5 cm

After 30 seconds, the depth of water in the flask will be 23.5 cm.

Solution 4:

I can't get a precise answer!

Your task is to decide which solution is correct. Explain your choice. Also explain why you think the other solutions are incorrect. Please write your explanations on the next page.

[Task adapted from Verschaffel, L., deCorte, E., & Borghart, I. (1997). Pre-service teachers' conceptions and beliefs about the role of real-world knowledge in mathematical modelling of school word problems. *Learning and Instruction, 7*(4), 339-359.]

Appendix C

Youth Olympic Games Problem

Singapore will be hosting the first Youth Olympic Games (YOG) from 14 to 26 August 2010. It will receive some 3,600 athletes and 800 officials from 205 National Olympic Committees, along with estimated 800 media representatives, 20,000 local and international volunteers, and more than 500,000 spectators. Young athletes - between 14 and 18 years of age - will compete in 26 sports and take part in Culture and Education Programme. The Singapore 2010 Youth Olympic Games will create a lasting sports, culture and education legacy for Singapore and youths from around the world, as well as enhance and elevate the sporting culture locally and regionally. There are altogether 201 events featuring 26 different kinds of sports such as swimming, badminton, cycling, fencing, table tennis, volleyball and weightlifting. Singapore is well-known in the region for grooming young swimmers.

The Singapore Sports School is having difficulty selecting the most suited swimmers for competing in the Women's 100m freestyle event. They have collected data on the top five young female swimmers over the last 10 competitions. To be fair, codes are used to represent the swimmers until the selection process is over. The list of swimmers and their codes are only known to you and your group. As part of the Singapore Sports School YOG committee, you need to use these data to develop a method to select the two most suited women for this event.

Study the data presented below collected over two years (2007-2008). Records for Competition 1 are the most recent. Records for Competition 10 are the oldest.

*Women's 100m freestyle Results recorded (seconds)**

Competition No.	Time in 100 m Freestyle (Minutes and Seconds)				
	Swimmer A	Swimmer B	Swimmer C	Swimmer D	Swimmer E
1	00:56	00:49	01:02	00:57	00:55
2	00:55	DNC	00:59	01:05	DNC
3	DNC	00:56	DNC	00:59	00:55
4	00:58	01:01	00:57	DNC	01:04
5	DNC	00:57	DNC	00:58	00:49
6	00:59	DNC	DNC	00:56	00:57
7	01:00	DNC	00:59	00:56	00:57
8	00:59	00:56	00:55	00:58	00:57
9	00:59	00:56	DNC	DNC	00:58
10	00:59	00:57	00:57	DNC	00:58

*DNC: Did not compete. *Best time across heats, semi-finals and finals.*

(1) Decide on which two female swimmers should be selected.
(2) Write a report to the YOG organizing committee in Singapore to recommend your choices. You need to explain the method you used to select your swimmers. The selectors will then be able to use your method to select the most suitable swimmers for all other swimming events.

[Adapted from English, L. (2007). Modeling with complex data in the primary school [Electronic Version]. *Thirteenth International Conference on the Teaching of Mathematical Modelling and Applications*. Retrieved July 22-26, from http://site.educ.indiana.edu/Papers/tabid/5320/Default.aspx.]

The roles of Indonesian Mathematical Society (IndoMS) in the Development of Mathematical Sciences and Mathematics Education in Indonesia

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Presented at:
the 1st International Symposium on Mathematics Education Innovation
18-19 November 2011
SEAMEO QITEP in Mathematics,
Yogyakarta, Indonesia

Abstract:

In this paper, firstly it will be shortly explained the Indonesian Mathematical Society (IndoMS) as one of scientific organizations in Indonesia from 1976 till 2011. Secondly, it will be described the roles of IndoMS in the development of mathematical sciences and mathematics education in Indonesia such as: national conference in mathematics, national conference in mathematics education, international conference, journals, association of Indonesian mathematics teacher, mailing list, mathematics taxonomy based on the group's areas of expertise, and the roles of mathematics in sciences and technology.

Introduction

IndoMS (**Indo**nesian **M**athematical **S**ociety) or formerly known as "*Himpunan Matematika Indonesia*" is a forum for mathematicians and the users of mathematics as well as people who have interest in enhancing mathematics in Indonesia. The Society is a scientific, nonprofit, non-governmental and professional organization. It was established on July 15th,1976 in Bandung, West Java. In 2010 IndoMS has 1.283 active members consisting of university teachers, mathematics, statistics and mathematics education researchers from 33 Indonesian universities, and school teachers from elementary and high schools. Several IndoMS founding fathers are Moedomo, Bana Kartasasmita, Ahmad Arifin, M. Ansyar, Lee Peng Yee, and Soeparna Darmawijaya.

The objectives of the Society are

i. to enhance and extend mathematical knowledge,
ii. to enhance and extend education in the Mathematical sciences, and
iii. to increase the role of mathematics in Indonesia.

To achieve these objectives, the Society has activities such as:

a. hold meetings for the purpose of hearing and discussing communications from members of the Society and others on mathematical subjects as well as mathematical education;
b. publish such communications on mathematical subjects as a means of providing information and knowledge for members as well as the public;
c. improve the competences of members of the Society through education and training;
d. promote mathematics and its application to the public in supporting the national development in Indonesia;

e. build mathematical cooperation with other mathematical societies around the world;
f. receive donation in order to support activities of the Society;
g. conduct other related activities according to the mission of the organization.

IndoMS has established 9 provincial officers to stimulate and enhance mathematical activities in the country. The branches are:

1. branch Special Territory of Yogyakarta and Central Java,
2. branch Banten, Special Territory of Jakarta, and West Java,
3. branch East Java,
4. branch South and West Sulawesi,
5. branch South Kalimantan,
6. branch South Sumatera,
7. branch Nanggro Aceh Darussalam and South Sumatra,
8. branch East Nusa Tenggara,
9. and branch of middle Sumatera.

Presidents from 1976-Present:

1. 1976 – 1982 : M. Ansjar (ITB)
2. 1982 – 1991 : Bana Kartasasmita (ITB)
3. 1991 – 1997 : Soeparna Darmawijaya (UGM)
4. 1997 – 2000 : R.K. Sembiring (ITB)
5. 2000 – 2002 : Ichary Sukirno (UNPAD)
6. 2002 - 2006 : Sri Wahyuni (UGM)
7. 2006 – 2008 : Edy Tri Baskoro (ITB)
8. 2008 – 2012 : Widodo (UGM)

Current Officers (2010-2012)

- President :Widodo (UGM)
- Vice President for Publication Affairs : Budi Nurani (UNPAD)
- Vice President for Education : Zulkardi (UNSRI)
- Vice President for Foreign Cooperation Affairs : Ch. Rini Indrati (UGM)
- Vice President for National Cooperation Affairs: Mashadi (UNRI)
- Executive Secretary :Indah Emilia Wijayanti (UGM)
- Treasurer :Herni Utami (UGM)
- Chair on Information Management System: Irwan Indrayanto (UGM)
- Mailing List Coordinator: Wikaria Gazali (UBINUS)

The roles of IndoMS in the Development of Mathematical Sciences and Mathematics Education in Indonesia:

A. National Conference in Mathematics and National Congress

Since 1976, IndoMS has already 15 times organized National Conference in Mathematics and National Congress:

I. 1976: Institut Teknologi Bandung (ITB), Bandung, West Java
II. 1978: Universitas Gadjah mada (UGM), Yogyakarta, Central Java
III. 1979: Institut Teknologi Sepuluh November (ITS), Surabaya, East Java
IV. 1982: Universitas Sumatera Utara, Medan, North Sumatera
V. 1983: Universitas Indonesia, Jakarta
VI. 1991: Universitas Padjadjaran, Bandung, West Java

VII. 1993: ITS, IKIP Surabaya and Universitas Airlangga, Surabaya, East Java
VIII. 1996: Universitas Hasanuddin (UNHAS) Ujung Pandang, South Sulawesi
IX. 1997: Universitas Sumatera Utara (USU), Medan, North Sumatera
X. 2000: Institut Teknologi Bandung (ITB), Bandung, West Java
XI. 2002: Universitas Negeri Malang (UM), Malang, East Java
XII. 2004: Universitas Udayana (UNUD), Denpasar, Bali
XIII. 2006: Universitas Negeri Semarang, Semarang, Central Java
XIV. 2008: Sriwijaya University, Palembang, Sumatera.
XV. 2010: Manado State University, North Sulawesi
XVI. 2012: The 16th National Conference in Mathematics and National Congress will be hold at **July 3-6, 2012** in Padjajaran University, West Java.

B. National Conference on Mathematics Education

Since 2006, IndoMS also has already 4 times organized National Conference in Mathematics Education:

I. 2006: in Madania School in West Java,
II. 2007: in Education University of Indonesia (UPI-Universitas Pendidikan Indonesia), about 700 participants
III. 2009: in State University of Medan (UNIMED-Universitas Negeri Medan), about 500 participant and
IV. 2011: in State University of Yogyakarta Indonesia (2011), July 21-23, about 450 participants.
V. 2013: There are three universities as candidates for the place of National Conference in Mathematics Education, i.e. Sebelas Maret University Solo Central Java, State University of Malang East Java, and State University of Gorontalo Selawesi.

C. International Conference

Starting in 2009 IndoMS organize International Conferences. **IICMA2009** is **I**ndoMS **I**nternational **C**onference on **M**athematics and its **A**pplications 2009, and it will be hold once every four (4) years.

General objectives of the conference are to:

a. facilitate researchers and user of mathematics to exchange ideas and discus research results and development of mathematics internationally
b. increase the number of international publication in the fields of mathematics and its applications (including mathematics education) in Indonesia.

D. Journals of the IndoMS

To accommodate and publish research results in mathematics and mathematics education IndoMS have been published:

1. Journal of the Indonesian Mathematical Society (**JIMS**) http://www.jims-a.org/. It has been accredited by the General Directorate of Higher Education DIKTI (4 times), the last one with SK NO.:51/DIKTI/Kep/ 2010. Firstly Published in 1992, and it will be the International Journal at the end of 2011.
   Chief-Editor : Handra Gunawan (ITB Bandung)
   Executive-Editor : Ch. Rini Indrati (UGM Yogyakarta)
   Managing-Editor : Budi Nurani (UNPAD Bandung)

2. Indonesian Mathematical Society Journal on Mathematics Education (**IndoMS-JME**) **http://jims-b.org/.** Firstly it has been Published in 2010.
   Chief-Editor: Zulkardi (UNSRI, Palembang, Indonesia)
   Managing-Editor: Wanty Widjaya (Univ. Sanata Darma Yogyakarta Indonesia, visiting Professor in National Institute of Education-NIE Singapore).

   IndoMS-JME was launched in the opening of the Fifteenth of National Conference on Mathematics (KNM15) in State University of Manado, North Sulawesi, on 31st July 2010. This journal is started in order to facilitate not only the members of IndoMS but also all mathematics teachers/teacher educators in publishing their research reports related to the mathematics education. This journal is designed and devoted to IndoMS members especially mathematics school teachers, teacher educators, university students (S1, S2, and S3) who want to publish their research reports or their literature review articles about mathematics education and its instructional. Start from 2010, this journal is published two times a year, in July and January.
3. IndoMS Journal on Statistics (**IndoMS-JMS**) will launched and published on December 2011
   Chief-Editor : I.Wayan Mangku (IPB Bogor)
   Executive-Editor : Gunardi (UGM Yogyakarta)
   Managing-Editor : I. Nyoman Budiantara (ITS Surabaya)
4. IndoMS Journal on Industrial and Applied Mathematics (**IndoMS-IAM**) will launched and published in January 2012.
   Chief-Editor : Robert Saragih (ITB Bandung)
   Executive-Editor : Siti Fatimah (UPI Bandung)
   Managing-Editor : Wikarya Gazali (UBINUS Jakarta)

E. Association of Indonesian Mathematics Teacher

At the National Congress UNNES in Semarang Central Java July 2006, IndoMS has established Association of Indonesian Mathematics Teacher (AGMI-Asosiasi Guru Matematika Indonesia).

F. IndoMS Mailing List (indoms@yahoogroups.com):

Since 2002, IndoMS organized a mailing list (indoms@yahoogroups.com). This mailing list has about 1.000 (one thausand) active members and coordinated by Mr. Wikaria Gazali from Bina Nusantara University (UBINUS) Jakarta. In this mailing list we are discussing everything related with mathematics, mathematics education, research, teaching and learning methods, informations about conferences, symposiums, workshops, etc.

G. Taxonomy of Mathematics

In the international world, is generally classified based on the research of mathematics Mathematics Subject Classification (MSC). The main purpose of the classification of research subjects in the MSC is helping the Researchers areas of research as much detail as possible so they can be used as a research database. Items classification of mathematical literature is made such that holds all the developments in the field of science today in one or more items with a clear classification of MSC. MSC was first developed in 1991 by two well-known mathematical journals at the time of

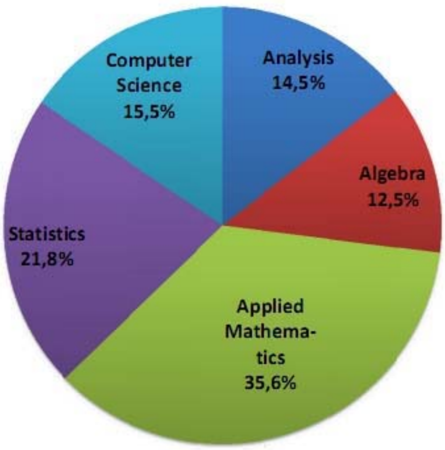
Mathematical Reviews (MR) and Zentralblatt MATH (Zbl). Furthermore, owing to the development of research in the field of mathematics, in 2000 with the revision of the editors of two journals, the result is referred to as the MSC2000. At the time this report was written, performed another revision of the classification MSC2001 to MSC2010, as listed in the site of the American Mathematical Society http://www.ams.org/ mathscinet/msc/. However, the study of taxonomy mapping MSC2010 is unfounded in the beginning of 2010. Nevertheless, the difference between two versions of the MSC is not too significant.

IndoMS Study about Taxonomy of Mathematics in Indonesia

In this study, the taxonomy mapping refers to the mathematics research MSC2000. However, an overly detailed MSC2000 (seehttp://www.ams.org/ mathscinet/msc/msc.html) is considered too difficult for the respondent. So for the purposes of the study of taxonomy, used a shorter version of MSC2000. This version was obtained by excluded some of the details of roots. However, this simplification does not change the grouping field of research.

Taxonomy based on the group's areas of expertise in Indonesia (Existing in 2004)

At the end of 2004, IndoMS have tried to map a field research in Indonesia, based on the group's areas of expertise (*KBK-Kelompok Bidang Keilmuan*) at universities in Indonesia which has a major in Mathematics and mathematics education. There are five major groups of fields of applied mathematics, computer science, statistics, analysis and algebra.

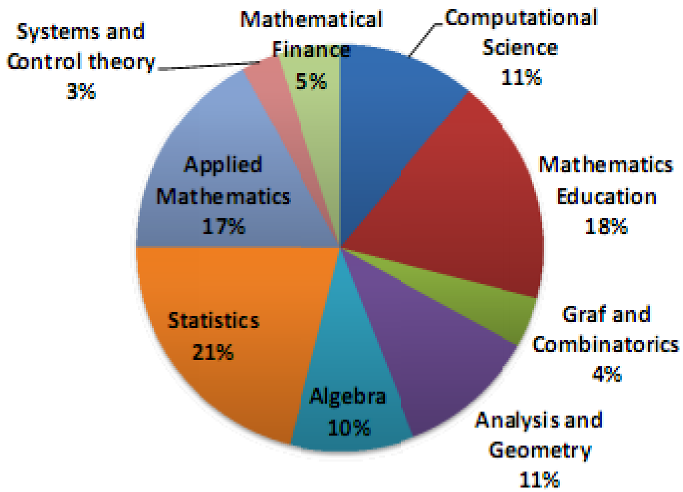


Fugure: Taxonomy based on the group's areas of expertise (2004)

From figure above, the mathematical research in Indonesia is dominated by the applied mathematics research by 35.6% followed by 21.8% (statistics), 15.5% (computer science), 14.5% (analysis) and 12.5% (algebra). It can be concluded that the direction of the development of mathematical research is correct, means that the study of applied mathematics can always be improved so that it is capable of supporting the potential of the industry. In the implementation of the research of applied mathematics have always supported by statistics, computer science, analysis and algebra, thereby increasing the applied mathematics research by itself is already promoting the four other areas.

Taxonomy based on the group's areas of expertise in Indonesia (2009)

In 2009, IndoMS had a study by spreading questioners to 1097 IndoMS members. But from 1097 questioners we had only 236 resondens asfollows:

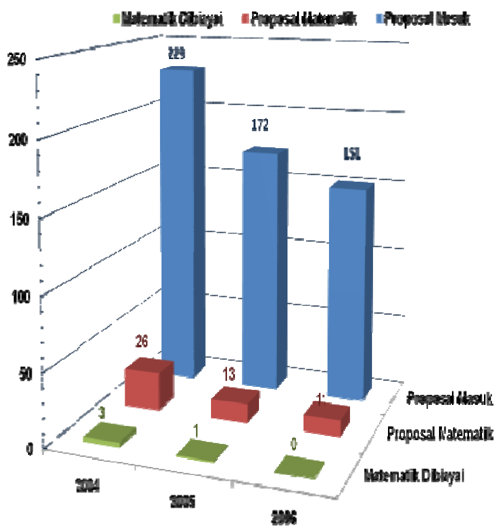


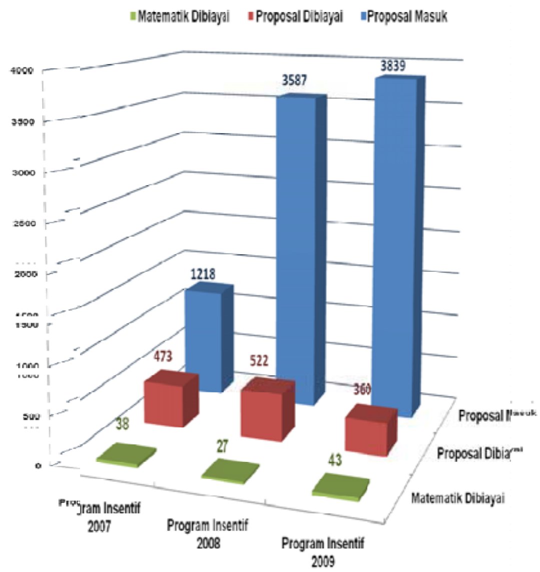
From the picture above, based on the results of IndoMS survey of mathematical research in Indonesia, the largest is the field of Statistics (21%). Research in the field of mathematics education was ranked second (18%). The applied field research ranked third (17%). When compared with the figure before (2004), where applied tops the list of 35% more, then this could be explained that taxonomy based MSC2001 more detail than ever before, which is a combination of the applied research which is not included in the taxonomy 2004. Therefore, based on this result, in 2010 IndoMS proposed to enlarge group's areas of expertise in Indonesia from 5 become 9 groups as follows:

1. Graf and Combinatorics
2. Analysis and Geometry
3. Algebra
4. Statistics
5. Applied Mathematics
6. Systems and Control theory
7. Mathematical Finance
8. Computational Science
9. Mathematics Education

H. The roles of Mathematics in Sciences and Technology in Indonesia

One of the KNRT (Indonesian Ministry of Research and Technology) roles in providing the budget competitively for the research for basic sciences generally and mathematics especially was in the form of the incentive programs. Incentive programs between the year 2004-2009 showed the increase in the role of mathematics. This was seen in the diagram bellow: In the KNRT budget in 2004 showed only had 26 proposals that involved mathematics from 229 of the total proposal of the incentive program, and from 26 proposals of only 3 proposals that could be accepted. Eventhough in 2005 and 2006 it was decreasing, but in 2007 and 2008 it was increasing very fast. As the picture, in 2007 there were 473 proposals that involved mathematics from 1,218 of the total proposal of the incentive programs, and from 473 proposals having 38 proposals that involved mathematics could be accepted. Whereas during 2008 and 2009 respectively was 27 and 43 proposals that involved mathematics could be accepted. This showed that the role of Mathematics was significant in the development of SCIENCE AND TECHNOLOGY in Indonesia.





In the period of 2007 till 2009 there were 108 KNRT researches that involved mathematics directly in various scientific fields, and 16 of whom with the title of research:

1. Design and Implementation of Fishing Location Determination System with Knowledge Based Model Approach
2. Sea Wave Focusing as a Tool Engineering Tool of Wave Energy to Power Waves
3. Active Control System Using PZT Sensor-Actuator for Ship Structural Vibration Reducing
4. Development of Mobile Communications Encryption Tool with Custom Electronics Card Systems
5. Automation of Railway Signal Monitoring Methods with Utilization of GPS Tracking & Technology, GIS and GPRS
6. Modeling and Simulation of Genetic Regulatory Systems in Mybacterium Tuberculosis
7. Software for Determining the Elastic Modulus and Thickness of Soil Layers Based on Surface Waves
8. Full System Development and Stabilization Motion Cannon Barrel
9. Typical Development and Residential Design Guidelines for Earthquake Resistant Indonesia with Simulation Verification Tool Tables and Vibration Finite Element Numerical
10. Model Development and Optimization of System Software for Production and Transportation of Oil, Gas and Geothermal
11. Model Development and Optimization of Supply Chain Scenario Fuel Mix.
12. Numerical and experimental study design Midget for military applications in Indonesia

13. The development of dynamic geographic information system applications and thin-client mobile devices to overcome the problem of road transport
14. Optimization of Ship Production Capacity of Distributed Systems
15. Aspects of Mathematical Problem Transmission of HIV / AIDS in Indonesia
16. Transportation management systems to address congestion in urban areas by using a dynamic system

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PARALLEL PAPERS

(Abstract only)

DEVELOPING EFFECTIVE USE OF BLACKBOARD THROUGH BOARD PLAN

Afrial

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Abstract

Blackboard is one of the most basic forms and best used instructional media for emphasizing essential information and developing ideas as the class progress. Blackboard is used to write assignments, lesson objectives, and many other things to remind students. Teachers often use blackboard for drawing graphics, writing texts and formulas, but they never know about the effectiveness of using blackboard. As a basic form of instructional media, nowadays most teachers at schools begin to change it with modern instructional media, such as OHP, LCD Projector, video and so on. International School is the example; most teachers prefer using modern instructional media. Consequently, in the years later teachers will leave using blackboard as the media. In fact, the using of blackboard is very important and more effective, whether teachers use it in class or not, but teachers should not leave the board in blank. Teachers are expected to manage the board by making a "board planning" involved in teachers' Lesson Plan, as what Japanese teachers do that they rarely erase what they write on the blackboard. Everything written on the blackboard is for the purpose of recording, as it has been carefully planned in advance. Once a Japanese teacher described the importance of using blackboard "You should not erase what you write on the blackboard and you should not write on the board if you are going to erase it". Another Japanese teacher described "I try to organize the blackboard in such a way that my students and I can see and understand how the lesson progressed and what was talked about during the lesson and at the end of the lesson".

DEVELOPING THE CHARACTER IN MATHEMATICS LEARNING THROUGH THE USE OF PROBLEM BASED LEARNING APPROACH

Amalia Fitri

Pekalongan University, Mathematucs Education Study Program

The focus of the mathematic learning is previously on the cognitive domain only. However, as implied in the aims of the national education, especially in the constitution No, 20 , 2003, the teaching learning activities must develop the students' characters. Problem based learning is an innovative learning approach which involves the students in the problem solving activities, trains them to be independent in doing their tasks, construct their own knowledge, and present the learning result that they have at the end of their learning. Due to some characteristics it has, Problem based Approach is believed to be the approach which can develop the students' characters in the mathematics learning.

Key words : characters, mathematics education, problem based Learning.

A ROADMAP OF INDUSTRIAL MATHEMATICS ON B.Sc PROGRAM

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Abstract

In general, job market for B.Sc (S-1) holders in mathematics is still limited. Few of them become teachers, or pursue higher degree to be lecturers at higher education. Most of them have to find job outside education management. Often, fresh graduate students on mathematics fail on job test in industrial field. When they succeed, they still find difficulties in adapting themselves in industrial environment, especially in connecting their mathematical knowledge to industrial applications.

In this paper, the researchers will share experiences in developing a roadmap of some courses of S-1 Program in Mathematics to build knowledge and experiences in problem solving of industrial problems which need mathematics applications. Final project or *Tugas Akhir* is designed to build experiences in problem solving of industrial problems. Courses on Mathematics for Finance, Economics and Industry are intended to build knowledge on the applications and modelling in those fields. In general, this is more advanced than in Mathematical Modelling course. All of these require elementary mathematics understanding, such as calculus, statistics, numerical methods and analysis.

THE DEVELOPMENT OF MATHEMATICS TEACHING AND LEARNING 'MODELS': EXPERIENCE OF SEAMEO QITEP IN MATHEMATICS

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Abstract

The term 'teaching and learning model' here means an 'ideal' situation of mathematics teaching and learning as concrete examples that can be used and implemented by mathematics teachers in their classes. Two main questions have been stated as starting points: (1) What should be done and how to help and facilitate students to learn to think, to solve problem, to reason, and to communicate? (2) What should be taken into account and how to help and facilitate students to learn mathematics with understanding or learn mathematics meaningfully? Among others, there are some consensuses that the model should focus on problem solving approach and the mathematics learning processes should begin with an introduction to the problems in accordance with the situation (contextual problem). The research implemented three steps; designing lesson, trying out and revising, and trying out and finalizing.

EXPERIENCES IN BUILDING CLASS CULTURE AND CONTEXT USAGE IN MATHEMATICS LEARNING

Hanna Desi Suryandari

Abstract

This paper aims at sharing experiences in conducting mathematics learning by implementing different method with old paradigm (teaching and learning process paradigm), especially in classroom culture and context usage in mathematics learning. This paper talks about teachers' experiences in conducting mathematics teaching by implementing different method with old paradigm in Indonesia. Some differences will be generally explained starting from teaching and learning preparation to classroom teaching and learning process. This paper specifically talks about experiences in building class culture; on how students adjust with the new culture, and what are the obstacles in building the culture. Besides, the paper also covers learning experiences by implementing context, on how prepare mathematics learning with suitable and acceptable context and its role in mathematics learning.

The data of the research were collected from research entitled "Potret dan Kajian Proses Pembelajaran Matematika di Beberapa SD PMRI" in 2009, DIKTI and research entitled "Pengembangan Kegiatan Metakognisi dan Diskursif dalam Pembelajaran Matematika di Indonesia", a doctoral project of D. Novi Handayani (a student of Osnabruck University, German).

A THEMATIC LEARNING USING CARD FOR GRADE 2 STUDENTS IN SDN 001 TANJUNGPINANG BARAT

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Abstract

This paper describes the use of thematic approach using card in SDN 001 Tanjungpinang Barat. The teacher facilitated students with joyful activity using card when they learned mathematics. Through the implementation of classroom based action research (CAR), the teacher attempted to improve students understanding and positive view toward mathematics by using various learning media such as cards. According to the observation and evaluation in every cycle of classroom based action research, this study showed that a thematic learning using card for grade 2 students in SDN 001 Tanjungpinang is relatively effective for improving students understanding and attitude toward mathematics. This study suggests teacher to explore various teaching approach such as presented in this paper to accommodate students need in their learning, and to promote students centred learning.

EVALUATIVE STUDY OF BILINGUAL IN MATHEMATICAL LEARNING IN INTERNATIONAL STANDARD PUBLIC SCHOOLS IN DENPASAR REGENCY

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Abstract

This research aims at analyzing and evaluating bilingual in mathematical learning in international standard public schools in Denpasar through variable of context, input, process, and product. This research evaluative qualitative research that shows the procedure and process of program action. This research analyzes and evaluates the role of each factor according to the model of CIPP (Context, Input, Process, Product). The participants of the research were 20 mathematics teachers and 294 students of SMPN 1 Denpasar, SMAN 1 Denpasar, and SMAN 4 Denpasar. The data were analyzed by descriptive analysis to determine the quality of bilingual in mathematical learning by converting the raw scores to T-score and verified them using Glickman's prototype.

The research findings show that the quality of bilingual mathematical learning in international standard public schools in Denpasar is good, it is compatible with the variables of context, input, process, and product $(+ + + -)$. The symbol indicates that the variable of context is good, input variable is good, process variable is very good and product variable is bad. The cause of the bad product variable is the way students behave to bilingual in mathematical learning.

Based on the findings above, the researcher concluded that bilingual in mathematical learning in international standard public schools in Denpasar was categorized good. Afterwards, suggestion for those schools which conduct bilingual education on mathematics are: (1) improving and emphasizing the knowledge of teachers about programs, methods, and approaches used in bilingual learning, (2) improving teachers' ability and skill in scientific English especially in mathematics, (3) improving teachers' ability to design the adaptive curriculum, and (4) improving teachers' ability in bilingual mathematical learning based on the topics.

Keywords: Evaluative Study, Mathematical Learning, Bilingual Learning, Bilingual Mathematical Learning.

INTEGRATING DYNAMIC SOFTWARE AUTOGRAPH IN ENHANCING STUDENT'S CONCEPTUAL UNDERSTANDING AND MATHEMATICAL COMMUNICATION USING GUIDED INQUIRY APPROACH

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Abstract

The aim of the study was to investigate the effect of integrating Dynamic Software Autograph on students' conceptual understanding and mathematical communication using Guided Inquiry Approach. This experimental study was conducted at two Vocational Schools (tourism) of grade XI in Medan, Indonesia. The subjects were randomly selected. The main objectives of the research were to investigate: (1) whether the gain score of student's conceptual understanding through guided inquiry aided by Autograph software was better than those of students learned through conventional approach, (2) whether the gain score of student's mathematical comunication ability through guided inquiry helped by Autograph software was better than those of students learned through conventional approach, (3) the mastery of student's mastery learning (4) the student's learning activity during study. Instruments used in this research were : (1) test of conceptual understanding (2) test of mathematical comunication, and (3) observation sheet of students' learning activity. Those instrumens had been validated by the experts and tryout of those instruments showed that all items in both tests were valid. The correlation coeficient for the conceptual understanding test was 0,8 and for comunication test was 0,72. The results of research shows that: (1) the gain of student's conceptual understanding through guided inquiry aided by Autograph software (0,67) was higher that those of learned through conventional approach (0,53); (2) the gain of the score on student's mathematical comunicating ability through guided inquiry helped by Autograph software (0,67) was higher than those of student's learned using conventional approach (0,48); (3) the mastery of students learning through guided inquiry approach aided by Autograph was better than those of students learning using conventional approach; and (4) students' learned through guided inquiry approach aided by Autograph software was more actively engaged in learning (83%) than students learned using conventional approach (68,78%).

APPLICATION ASPECTS IN THE CURRICULUM OF MATHEMATICS EDUCATION PROGRAM MUSAMUS UNIVERSITY

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Abstract

Application of mathematics in daily life is very important, from business to engineering, also from natural sciences to social sciences. Hence, the application aspects of mathematics are becoming integral parts in the process of teaching and learning mathematics in elementary schools, high schools, and universities. This, in general, is implemented by training students to solve daily life mathematics problems. Students should model a given real life problem to a mathematical model, then solve the mathematical problems. Finally, they should interpret the mathematical solution into real life solution.

This paper presents application aspects in the curriculum of mathematics education program (S1) at Musamus University, Merauke, Indonesia. The curriculum is represented in courses of Mathematical Economics, Mathematical Engineering, Mathematical Modelling and Initial Value Problem, in which students should pass two of four courses. Moreover, some daily life problems related to the courses, such as Calculus, Statistics, and Linear Programming, are also discussed.

DIFFERENTIATED INSTRUCTION IN INDONESIAN REGULAR CLASSROOM: GENERATING AN OBSERVATION CHECKLIST

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Abstract

Only a small percentage of Indonesian gifted students are currently taught in selective schools or programs. By far, the majority of these children are taught in regular classrooms. Ideas about differentiated instruction are new to Indonesian teachers, but research in the rest of the world suggests that this is an essential teaching practice when catering for diverse students, in particular gifted students, in regular classrooms. This study will apply a document analysis research method to determine the most appropriate criteria to apply the design of a new checklist for Indonesian schools. This checklist can be used in Indonesian schools to identify how well teachers are differentiating instruction to cater for students of high academic ability in regular classrooms and will lay the foundation for future research.

INTERACTIVE LEARNING ON ONLINE TUTORIAL OF EDUCATIONAL STATISTICS COURSE TO IMPROVE STUDENT LEARNING MOTIVATION

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Abstract

Statistics education is one of the subjects offered by the Department of Mathematics Education-Open University (*UT*). Based on data in 2010, 57% of students got the value of D and E for the final exams. The problem was that they got difficulties to analyze and interpret data, and had no interaction with tutor in solving the problem.

Online tutorial (*Tuton*) is a learning support service provided by Open University via internet. However, the implementation of online tutorial for the course has not been running as expected. From the conditions observed, participation of students in accessing the material in online tutorial was not optimal. This was probably due to its grain was less attractive, initiation material was presented only in the form of text, not interactive and difficult to be understood, and interaction between tutors and students in a discussion forum was very low.

In this paper, the authors search for an innovation on interactive learning in the implementation of online tutorial. In this learning design, students are expected to learn independently in constrained space and time. Learning can be implemented by presenting material initiations that are packed in interesting and interactive ways by using flash program, utilizing discussion forums and social networking sites like Face Book and Twitter to discuss and post ideas, images, and videos easily, and providing links to additional learning resources that relate to material through blogs, diigo and flickers. Through this learning model, it is expected that students are motivated to follow online tutorial in order to enhance their learning achievement.

Keyword: interactive learning, online tutorial, motivation, statistic education

USING VIDEO AS A PROJECT-BASED LEARNING TO IMPROVE STUDENTS PRESENTATION SKILL IN MATHEMATICS

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Abstract

Nowadays, phenomenon in education is about how to teach school subjects through English especially for mathematics and science. This kind of teaching and learning is already applied starting from elementary level up to high school. This study investigates bilingual mathematics teaching and learning in SDN BI Tlogowaru. This study focuses on how to make students joyful in doing mathematics tasks in English. Technology is combined in order to make them enthusiastic and interested in accomplishing the tasks as their project. This study applied Classroom Action Research (CAR) to prove whether or not using video can improve students comprehension in learning mathematics and change their attitude towards mathematics. This study was conducted in one cycle with two meetings, and compared the videos done at school and at home as their project. The scoring rubric was announced to students to let them know the criteria they require in doing the project. The result showed that the videos done at home were well-prepared and arranged in a good order. They did not feel anxious anymore in learning mathematics and became more creative in presenting the project.

USE OF SHARING KNOWLEDGE ACTIVELY LEARNING MODEL WITH GUIDED DISCOVERY METHOD AT TUTORIAL STATISTICS GROUP STUDY AT OPEN UNIVERSITY IN BANJARBARU

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Abstract

This research aims at increasing the activity and learning through the application of sharing knowledge actively learning model with guided discovery method on statistics courses tutorial at Open University (*UT*) study group (*kelompok belajar*) in Banjarbaru. The subject of this study was S-1 students at Open University of Elementary Education Class A of Elementary School Teacher Education (*PGSD*), who took an educational statistics courses (PEMA - 4210) semester VII, academic year 2011.

Action research activities divided students into small groups, and in each meeting, the tutor provided Group Worksheet (*LKK*) containing materials summary to be studied by each group. Tutor only facilitated groups who had difficulties, reminded them to reread LKK, the tutor was not allowed to give a direct answer. During the research activities, it could be concluded that students learned well in groups, and they also helped each other to discuss the material that was less mastered by them. The results showed that the use of sharing knowledge actively learning model with guided discovery method was capable to provide assistance for students in learning statistical material, and the outcomes indicated that students of Elementary School Teacher Education (*PGSD*) S-1, Class A, who took education statistics (Pema 4210) semester VII, academic year 2011 were graduated 100%.

THE USE OF MAGIC CARD AS A TOOL IN TEACHING AND LEARNING ADDITION AND SUBTRACTION OF INTEGERS

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Abstract

In fact, not all students think that learning mathematics is easy; mathematics is abstract, sometimes confusing and complicated. Some students have problem in adding or subtracting two integers with different sign, difficulties also happen when they learn linear equations system, measurement, and any other topics in mathematics in which addition and subtraction of integers are needed.

Teachers should help students to solve problem in addition and subtraction of integers. A basic magic card is helpful to make students understand how to add or subtract integers; the card can translate an abstract thing into real and touchable one. Each card contains positive or negative symbol, therefore students can differentiate between positive and negative number. On the next level, students can expand the use of magic card by writing directly the symbol of the number and use different colours for positive and negative integers.

There are many methods to make students understand about addition and subtraction of integers. In this case, the researcher hopes that at least his students do not only learn addition and subtraction of integers as an abstract thing, but they can also play with magic card while learning. This activity is expected to balance their brain work. Thus, it is recommended to other teachers to develop the use of magic card in teaching and learning addition and subtraction of integers.

WORKSHOPS

The Travel Game: An Adaptable Context for Adding Mathematical Value to Lessons at any Level

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Abstract

It would be an unusual event for a child to complete their schooling without playing some type of learning game. Games are used in most key learning areas and particularly in mathematics. This workshop will discuss some of the many benefits and the teaching and learning issues of using games in the mathematics classroom. During the workshop participants will be actively involved and experienced in trying the Travel Game, which is a simple, highly adaptable, teacher made, easily constructed classroom game. Suggestions will be included for how it may be adapted for use in classrooms from K-12 and ranging across the various strands and levels of mathematics.

Engaging Young Students with Mathematics

Catherine Attard
University of Western Sydney

Engaging students in mathematics during the primary years is crucial in assisting them to develop a positive attitude towards and an appreciation of mathematics. This workshop will focus on using and creating mathematics tasks that are engaging, innovative, and represent 'best practice' for teaching mathematics. Participants will engage in a variety of hands-on activities that can be adapted to suit the needs of diverse learners.

EXEMPLIFYING A MODEL-ELICITING TASK FOR PRIMARY SCHOOL PUPILS

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Abstract

Mathematical modelling is a field that is gaining prominence recently in mathematics education research and has generated interests in schools as well. In Singapore*, modelling and applications* are included as process components in revised 2007 curriculum document (MOE, 2007) as keeping to reform efforts. In Indonesia, efforts to place stronger emphasis on connecting school mathematics with real-world contexts and applications have started in Indonesian primary schools with the Pendidikan Realistik Matematik Indonesia (PMRI) movement a decade ago (Sembiring, Hoogland, Dolk, 2010). Amidst others, modelling activities are gradually introduced in Singapore and Indonesian schools to demonstrate the relevance of school mathematics with real-world problems. However, in order for it to find a place in the mathematics classroom, there is a need for teacher-practitioners to know what mathematical modelling and what a modelling task is. This paper sets out to exemplify a model-eliciting task that has been designed and used in both a Singapore and Indonesian mathematics classroom. Mathematical modelling, the features of a model-eliciting task, and its potential and advice on implementation are discussed.

Key words: Mathematical modelling, model-eliciting task, features and implementation

Introduction

Research have found that modelling activities help children in the promotion of important mathematical reasoning processes such as constructing, explaining, justifying, predicting, conjecturing, and representing (Chan, 2008; English & Watters, 2005; Lehrer & Schauble, 2000); reasoning aspects that are valued as a powerful way to accomplishing learning with understanding. Furthermore, studies have shown that children have been able to manage complex mathematical constructs irrespective of their academic mathematics achievement (Lesh & Doerr, 2003). What is it about mathematical modelling that is asserted to be a promising reform effort in the mathematics classroom? This paper introduces mathematical modelling from a Models-and-Modelling Perspective (MMP) and shares the features of a modelling task (both from a Singapore and Indonesian context) towards understanding the potential of using such tasks. It will also discuss the potentials of the task and suggest teacher scaffolding strategies for task implementation.

Mathematical Modelling

There are various conceptions as to what mathematical modelling is just as there are various diagrammatic representations of the modelling process found in literature (e.g., Ang, 2006; Galbraith, Renshaw, Goos, & Geiger, 2003; Lesh & Zawojewski, 2007; Stillman, 1998). The common stance though is that mathematical modelling begins with a real-world problem or situation and the engagement process results in the representation of such problems in mathematical terms towards finding solutions to the problems. In this paper, we adopt a Models-and-Modelling Perspective (Lesh & Doerr, 2003) where the modelling task used is termed as a Model-Eliciting Task. Based on the MMP, students develop internal conceptual models that are powerful but are under-utilized unless they are expressed externally through some representational media as they complete the modelling task. The external projection of the models can be expressed spoken language, written symbols, graphs, diagrams, and experience-based metaphors. These representations are continually tested and revised as students aim to reach workable solutions. In this sense, the MMP focuses on students' representational fluency through the flexible use of mathematical ideas when students make mathematics descriptions of the problem context and data (Doerr & English, 2003; Lesh & Doerr, 2003).

The development of models takes place during the modelling process and is seen as involving *mathematizing reality* as contrasted with *realizing mathematics* (Lesh & Doerr, 2003). The mathematical modelling process involves several stages. Figure 1 shows a generic 4-stage process. The stages include (a) understanding and simplifying the problem, (b) manipulating the problem and developing a mathematical model, (c) interpreting the problem solution, and (d) verifying and validating the problem solution.

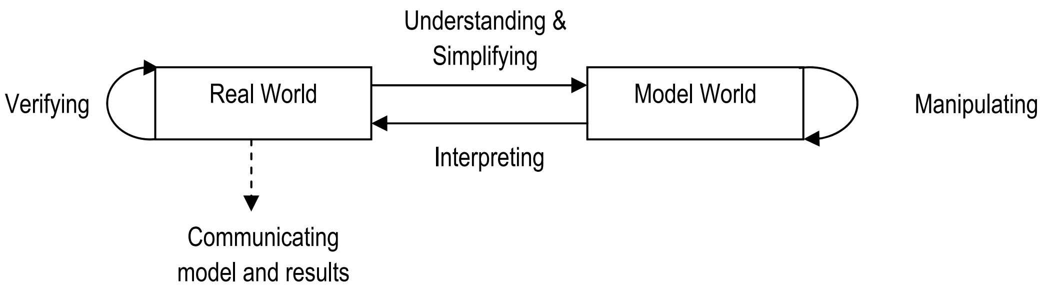

Figure 1. Modelling stages

Features of a Model-Eliciting Task

The mathematical modelling endeavour (involving a model-eliciting task) is commonly termed as a model-eliciting activity. During the modelling process, rich mathematical discourse and reasoning are made manifest during when students develop models. This is facilitated by the task when students confront it. As such, the design of the modelling task needs to ensure that the mathematics embedded can be surfaced when students are deeply engaged in it. Lesh, Cramer, Doerr, Post, and Zawojewski (2003) asserted a model-eliciting task is designed based on six instructional design principles: (1) the reality principle (i.e., elicits sense-making and extension of prior knowledge), (2) the model construction principle (i.e., warrants the need to develop a mathematically significant construct), (3) the self-evaluation principle (i.e., requires self- assessment), (4) the construct documentation principle (i.e., requires students to make visible their thinking), (5) the construct generalization principle (i.e., sharable, adaptable, reusable in other similar situations), and (6) the simplicity principle (i.e., the simplicities of the problem-solving situation). Thus the construct "model-eliciting" circumscribes a problem-solving situation, its mathematical structure, the problem-solving processes and the mathematical models generated that are invoked by the problematic situation. The features of a model-eliciting task thus are what the principles aim to promote.

An Example of a Model-Eliciting Task

Figures 2 and 3 show the "Bus Route Task" for grade 5 (aged 10-11) students implemented in Singapore and Indonesian classrooms respectively between August to early November 2011. The two versions are deliberately designed to be parallel tasks so as to investigate the nature of student mathematical thinking and teacher pedagogical decision-making in the two educational systems. The two tasks share the same goal of determining the most efficient bus route for travelling from point A to point B.

Determining the most efficient bus route

Ms Chang recently moved to Block 297C Punggol Road. She is going to start teaching at Punggol Primary school next week and needs to know how to travel to the school. However, the MRT is always too crowded for her to take and it also requires her to take a feeder bus which results in inconvenience. Ms Chang realizes that there are three bus services that ply different routes to her school. Help her to find the most efficient route to travel by bus from her home to the school. The location of her home is marked in the diagram. Currently the three bus services that are available for Ms Chang to choose are Service 124, Service 62 and Service 89. The routes for Service 124, Service 62 and Service 89 are marked as blue, yellow, and pink lines respectively on the map. The bus stops along each bus route are marked with stickers with corresponding colours.

Your task is to give Ms Chang a proposal consisting of the following:

1. How your group determines what is meant by the "*most efficient*" bus route
2. *Assumptions* about the problem your group made in order to help Ms Chang
3. The *mathematics* used to decide which route is the most efficient
4. How your group *justifies* that the selected route is the most efficient
5. The final *recommended* route for Ms Chang

For us to better understand your work, you can attach the following to your proposal:

(a) A map containing the chosen bus route.
(b) The information you found useful for this task

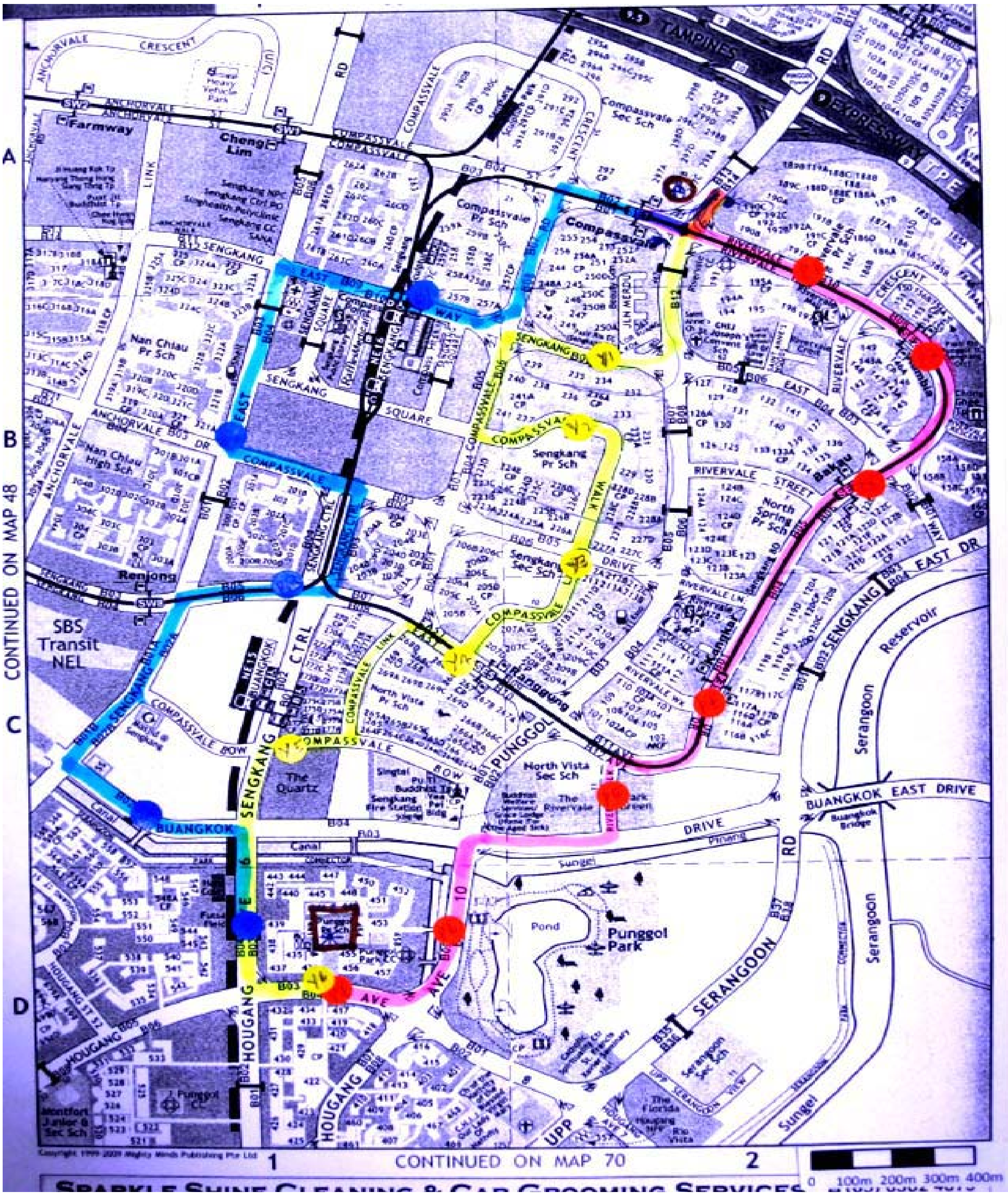

Figure 2. A sample of model-eliciting task (Singapore version)

Menentukan jalur bus paling efisien

Bu Mustari mendapat kunjungan tamu guru dari Singapore. Beliau ingin mengantar rombongan guru ini menggunakan bus dari terminal Jombor ke tempat kerajinan perak di Kota Gede. Ada 3 alternatif jalur bus yang dapat dipilih seperti tercantum pada peta. Ketiga alternatif jalur bus ditandai dengan warna biru, kuning dan merah muda pada peta. Pemberhentian bus di sepanjang jalur-jalur ini ditandai dengan stiker dengan warna yang sama dengan jalur bus.

Tugas kalian adalah membantu bu Mustari untuk memberi rekomendasi dengan menjawab pertanyaan berikut:

1. Bagaimana kelompok kalian menentukan jalur bus yang "paling efisien"?
2. Asumsi apa yang harus kalian buat untuk dapat membantu bu Mustari?
3. Bagaimana kalian menggunakan matematika untuk membantu Bu Mustari memilih jalur bus yang paling efisien?
4. Bagaimana kelompok kalian meyakinkan kelompok lain lewat penjelasan matematika bahwa jalur bus yang kalian pilih adalah yang paling efisien?
5. Rekomendasi akhir jalur bus yang paling efisien untuk bu Mustari.

Untuk membantu kelompok lain memahami penjelasan kelompok kalian, lampirkan dalam poster kalian:

(a) Peta yang memuat jalur bus yang kalian pilih

(b) Informasi penting yang kalian gunakan untuk menyelesaikan masalah

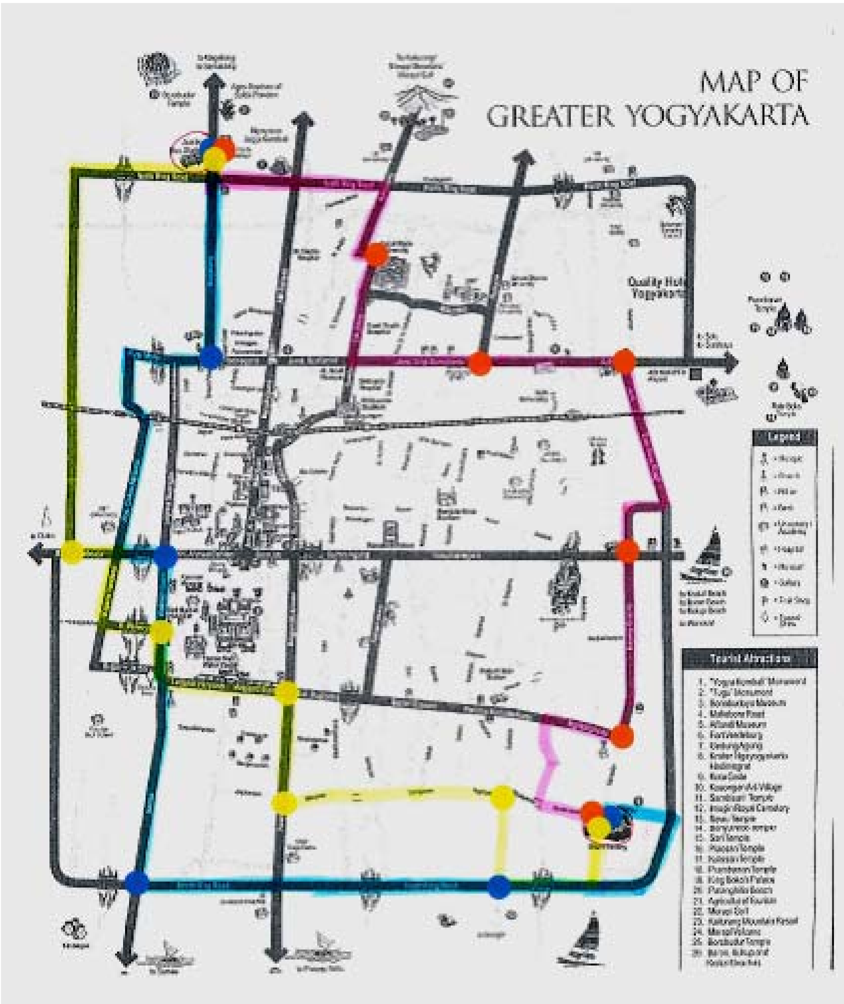

Figure 3. A sample of model eliciting task (Indonesian version)

Potentials of the Task

The tasks incorporated features of MMP and were presented to the students together with detailed maps of the respective areas mentioned in the task sheets. In particular, the information contained in these maps served as important data for use during the modelling process. For example, Figure 2 shows the map of Punggol area, as mentioned in the Singapore version of the Bus Route Task. Students participating in this task were to assist Ms Chang, a new teacher in their school, to select a most efficient bus route among three given colour-coded routes for travelling from Ms Chang's residence to the school. The starting point and ending point of travel were circled for ease of recognition. In addition, the positions of bus-stops along the three bus routes were marked by coloured stickers. The chosen map was deliberately a comprehensive one which also included information on the housing density along the routes, the scale measurement, and various land mark places within the area. Through participation in the tasks, students engaged in the negotiation of meaning within the context of the task, interpret the problem situation and the data given in the map using their everyday experience, and use their chosen relevant mathematical concepts and skills to represent a solution to the problem in a coherent mathematical model. The tasks present platforms for students to organise their approaches based on their interpretations, explain and describe their mathematical reasoning, make conjectures and justify them, as well as compare among various options of mathematical models.

On the mathematical front, the two tasks carried rich potentials to bring forth relationships among pertinent variables such distance, time, and costs interpreted within real-world contexts. As the tasks were open-ended in nature, it allowed for a diversity of mathematical approaches. There were multiple pathways to derive at the mathematical solution depending on interpretations, assumptions and choice of variables, as postulated by proponents of the MMP perspective (e.g., English, 2010a; 2010b). Students' knowledge of pertinent mathematics (i.e., length, time, distance, average) and their flexibility of use for determining a valid mathematical model in presenting a solution to the problem would be important during mathematisation of the given real-world situation.

The modelling tasks were set as a group work for students to complete over three hours on three consecutive mathematics lessons. This offered a good platform for students to develop their mathematical communications skills during group and whole class discussion (English, 2010b). Students were required to justify their solutions and check the reasonableness of their solution. It was also ideal that students revisit their assumptions and the conditions set in the task in order to check if the solution needed further refinement. However, numerous studies on mathematical modelling have identified that validating results and re-interpreting the results in the real-world context proved to be challenging within the time constraints of classroom implementation (English, 2010a; Galbraith & Stillman, 2006). The teachers involved in the implementation of the two tasks in the Singapore and Indonesian classrooms tried their best in helping their students refine the models within the constraints.

Suggestions to Task Scaffolding

This section will propose some teacher scaffolding suggestions for the implementation of the Bus Route Task which may apply across the Singapore and Indonesian classroom contexts. Firstly, it is important that the teacher sets aside time for students to discuss their interpretations of the context provided in the problem situation. For instance, students should be encouraged to brainstorm on what “efficient” means within the given context and how they would determine “efficiency” through use of mathematical measurements. At this stage, it is plausible that students would be able to suggest efficiency based on time, speed, and cost of travelling, well within their mathematical pre-requisites of the task. The scaffolding strategies of the teacher would be concentrating on questioning for ideas, exploration of possible choice of mathematical concepts and skills useful for the task, and helping students determine their next stage in the problem solving process. Students need to be reassured that multiple interpretations of “efficiency” are recognised and differing solution pathways for mathematical diversity are thus encouraged. They should also be allowed to put forth questions about task for clarifications.

Secondly, it is also important to help students understand the role of assumptions, conditions, and variables within the task. Assumptions are made about the context of the task (e.g., the bus travels at a constant speed, regular traffic conditions) so as to narrow down the focus for choice of appropriate mathematical approach. For example, it may be logical for students to assume regular traffic conditions (i.e., no vehicle break-downs, traffic lights are working) because data on traffic conditions vary day-to-day and it may not be readily available for the feasibility of classroom implementation. Conditions of the task need to be articulated and recorded because it would be necessary for students to decide on the parameters in which they would like to work within in order to develop a “mathematically valid” model as a proposed solution to the problem. For instance, one of the conditions in the Bus Route Task (Singapore version) where students can work with was to discuss the problem situation with the peak hour (i.e., 06 30 – 07 30) of travelling stipulated, based on the starting time of their school. This helps in decision making about the frequency of bus services, along with other real-world considerations of day-to-day bus journeys. Variables of the tasks include the ways in which efficiency is measured (e.g., time, speed, cost). Students can be prompted to explore how these could be determined from the given information (i.e., map) and whether more information needs to be collected for more accurate calculations. Furthermore, these variables are also related to each other. Students can work on how to develop a more sophisticated mathematical model holistically connecting the variables to help determine the most efficient route from the given three routes.

Last and not least, model refinement requires continue evaluation of the mathematisation process and critique of initial models based on the validity and applicability to the given situation. It is crucial for students to be encouraged to reflect and review their initial models in several ways. For one, the appropriateness of assumptions made, conditions set, variables chosen for exploration. These form the foundation of the mathematical model or representation of the students’ arguments and reasoning for their choice of the most efficient bus route. Students have to decide if the final decision made is a logical one, based on the above. Another way where students could review their initial model is to check their mathematical calculations for accuracy and reasonableness, whether these calculations present substantial support for their decision. Finally, students can reflect on their models to determine if it meets the requirements of the task. They need to recognise the possible limitations of their models for applications across other similar contexts (e.g., the most efficient train route) based on the parameters they had initially chosen to work with.

Conclusion

This paper serves three purposes: to (a) introduce the Model-Modelling Perspective (MMP) in mathematical modelling from a practice-oriented point of view, (b) exemplify a model-eliciting task (The Bus Route Task), and (c) discuss the potentials of the task and teacher scaffolding strategies. However, the implementation of modelling activities in classrooms can be challenging (Ng, 2011). Teachers who are used to prescriptive teaching approaches would need to explore other scaffolding approaches to more student-centred focus, encouraging multiple interpretations and solution pathways. Teacher beliefs about mathematics and how mathematics should be taught and applied may be challenged as mathematical models can take many forms; tables, graphs, and drawings, amidst abstract algebraic representations and calculations. In view of bridging the gap between the potentials of mathematical modelling and the recognition of these potentials by teachers, Ng (2011) recommends building teacher repertoire in two specific areas such as teacher questioning techniques during scaffolding and facilitating a conducive modelling climate in the classroom. The former recommendation has been elaborated in detail above based on the Bus Route Task. The latter recommendation is just as crucial for both teachers and students engaged in mathematical modelling as a positive climate which encourages inquiry, discussion, and fruitful mathematical arguments enhances sophisticated mathematical thinking.

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MATHEMATICS LEARNING AUGMENTATION: FROM BASIC TO COMPLEX APPLICATIONS

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Abstract

Learning Mathematics effectively and efficiently should be given importance by instructional leaders down to the teachers holding their classes. Teaching Mathematics should be fun and enjoying. There has been many innovations, creativity, techniques, strategies and approaches by teachers that can be introduced to make the class more interactive and students be able to love the subject. Activity sheets, workbooks, sourcebooks and textbooks are ideal sources of the activities for Mathematics lessons. But with the additional aid of manipulatives and other instructional materials we can be able to teach mathematical concepts more comprehensive and interesting to students. This is a workshop/demonstration of selected Mathematics concepts with simple activities using simple and locally made materials as strategies to augment learning and get more interested with the subject/s in varied levels of learners. There are five categories to be tackled during the workshop: origami, puzzles and dissections, topology experiments, tops and links between Science and Mathematics. The materials therefore are useful to increase the interest of students in learning Mathematics. The students will be able to create and discover links and connections between Science and Mathematics and learning process from the basic to the most complex applications.

Keywords: Innovation, Teaching Approaches, Teaching Strategies

PROCEEDINGS

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